Income Taxes, Spending Composition and Long-run Growth[*]

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Abstract

The focus of this paper is threefold. First, it reexamines the impact on long-run growth of changes in at-rate income taxes when a fraction of total government expenditures is used to provide public services that affect the productivity of privately held inputs. Second, for a given tax policy, this paper studies the impact of government expenditure composition on the rate of economic growth. Third, since demographics follow an overlapping generations structure and scal policy affects the economy s productivity, the paper features the role of productivity as a mean of redistributing income across generations. The economy is analyzed numerically and policy experiments are carried out.

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1 Introduction

1.1 Overview

Economists have long recognized that scal policy may affect economic growth. In the last three decades, numerous papers have studied how and to what extent taxation, spending, transfers, and other aspects of scal policy affect growth performance. Part of this literature concentrates on the study of the equilibrium relationship between scal policy and growth. From this analysis a broad support for the hypothesis that income taxes are detrimental for growth has emerged (Rebelo, 1991; Jones and Manuelli, 1992; Easterly and Rebelo, 1993; references therein). The mechanism through which this seems to take place is intuitively simple: an increase in the capital income tax rate decreases the rate of return to the investment activities of the private sector and leads to a decline in the rates of capital accumulation and growth. Even on normative grounds there seems to a be a case against income taxation. Particularly, it is argued that eliminating tax rates on capital income could lead to increases in growth and welfare (Chamley, 1986, Lucas, 1990).[1]

More recent theoretical work, however, puts forth provocative evidence that income taxation may affect growth positively. In the US, for example, the empirical support that low capital income taxes may foster growth seems less clear than has been proposed. Capital gains seem to be relatively unresponsive to changes in taxation in the long-run and the time series for the personal savings rate and the capital income tax rate, despite short-run divergence, seem to be positively correlated (Uhlig and Yanagawa, 1996; references therein).

[1] Chamley and Lucas provide theoretical and quantitative evidence supporting this line of thought. Lucas uses a framework similar to that of Chamley, but incorporates human capital into the model.

When government spending is allowed to affect private decisions to acquire education and accumulate human capital, simultaneous reductions in capital income taxes and government spending on education reduce the long-run growth rate, though it could be argued that such relationship is not quantitatively significant (Glomm and Ravikumar, 1998). Despite large bodies of work in both the growth and optimal fiscal policy literatures, many issues remain unsettled. Whether government size affects growth remains a controversial issue, especially in the absence of *stylized* facts (Temple, 1999). In addition, the questions of composition of government spending and its effects on the rate of growth remain open.

In the next section, I describe an endogenous growth model and study two important aspects of fiscal policy. First, I reexamine the impact on long-run growth of changes in flat-rate income taxes when a fraction of total spending affects private decisions to invest. I refer to this category of spending as *public services* (or *productive spending*), and it refers to government expenditures on the maintenance of (or additions to) the stock of infrastructure such as highways, educational facilities, hospitals, water and sewers, communication systems, and others; improvements in the legal system (law and order); enforcement of property rights, etc.[2] Secondly, this paper examines the impact of spending composition on the rate of economic growth for tax revenue can also be allocated to the purchase of consumption goods and to transfers. It is assumed that public services are provided without user charges and although part of these services may be subject to congestion effects, the latter are ruled out to keep the analysis as simple as possible.

It is found that the long run growth effects of income taxes are generally ambiguous, even

[2] For a review of the literature on how public spending affects the productivity of private factors see Gramlich (1994) for the case of spending on infrastructure and Barro (1991) and Alesina et al. (1996) for the case of law and order.

when a fraction of spending is allocated to productive services that affect the productivity of privately held inputs. How scal policy affects growth depends a great deal on the sensitivity of savings to changes in long run interest rates. While it is the case that for a large set of interest rate elasiticies reducing the share of total government outlays devoted to the provision of public services unambiguously reduces growth, the same thing cannot be said of income tax policies. In order to shed some light on how tax policy affects growth, a simple numerical analysis is provided and some policy experiments are carried out. Of course, this requires restricting the model to particular functional forms for preferences and technology.

Since the main interest of this paper is the study of permanent or long run effects of scal policy, an endogenous growth framework is used. Endogenous growth models have the virtue that they allow for capital accumulation (physical or human) to persist along the balanced growth path.[3] In the previous neoclassical paradigm, although growth studies also concluded that it translated into lower growth rates, income tax policy could affect growth only in the transition path toward steady state, for steady state growth could only be sustained through exogenous forces (technical progress and population dynamics).[4]

The idea that government expenditures may have a positive impact on growth needs little justi cation if any. Its roots can be traced back as far as Rosesntein-Rodan (1943) inquiry on the role of government in the development process. For a long time economists have debated the importance of productive public services in the generation of resources in the economy, and some have argued that such spending is not inessential from the point of view of society. Hansen (1955), for example, stresses the importance of public spending by saying: I am

[3] There is a large and important body of literature. An incomplete but in uential list is Lucas (1988, 1993), Romer (1986, 1990), and Shell (1966, 1967). For a comprehensive review of the literature on endogenous growth see Aghion and Howitt (1998).

[4] See King and Rebelo (1993), who study transitional dynamics and growth in the neoclassical framework.

convinced that economists have been grossly negligent as a profession in failing to examine the grave de ciencies in our society. Many of these cannot be overcome except by public investment in our material and human resources.

In the empirical front, studies nd that public investment induces an increase in the rate of return to private capital, thereby stimulating private investment (Aschauer, 1989; Easterly and Rebelo, 1993; Gramlich, 1994; and Morrison and Schwartz, 1996). That is, public spending may be complementary to private capital. Bene ts of such spending involve improved security, time saving, improved health, a cleaner environment, etc., which are often difficult to measure or not included in official measures of national output. It should be mentioned that there is also work contending whether public services foster growth, arguing that either the relationship of government spending and growth is fragile in the cross-section or that high spending lowers the level of income.[5]

1.2 Related Literature

The demographic structure of the model herein is based on Samuelson (1958)-Diamond (1965) s overlapping generations paradigm, a framework extensively used for the study scal policy and its effects on growth (Summers, 1981, Auerbach and Kotlikoff, 1987, references therein). With regard to this analysis, however, the closest theoretical work are the papers by Jones and Manuelli (1992) and Uhlig and Yanagawa (1996), while Aschauer (1989), Easterly and Rebelo (1993), and the work they cite provide empirical support for some of the results.

Jones and Manuelli (1992) show that within the realm of overlapping generations, for the class of one sector growth models with convex technologies, there cannot be equilibria

[5] See for example Barro (1991), Hall and Jones (1997), Levine and Renelt (1995), and Morrison and Schwartz (1996). Interestingly, at least in Hall and Jones (1997) and Levine and Renelt (1995), the spending referred to is expenditures on consumption goods.

with positive long-run growth. The intuition for this is that young generations lack sufficient income to purchase a sufficiently large capital stock from the elder. Thus, for all interest rates, the limiting growth rate is zero.[6] As a result, this kind of models must be modi ed as to display positive equilibrium growth. Jones and Manuelli show several ways through which growth can be restored. One alternative is to introduce a government that taxes all sources of income and then redistributes it in the form of lump-sum transfers. Using this *intergenerational redistributive* policy as a mean of restoring positive equilibrium growth, they then move to analyze its growth effects.[7] They nd that if the government s objective is to attain maximum equilibrium growth, policies that redistribute all tax revenue to the young are best. By modelling explicitly spending composition, this model nds an alternative redistributive channel: total factor productivity. Numerical exercises in Section 5 show that if a large fraction of total outlays is allocated to transfers at the expense of public services it could have severe adverse effects on growth even when transfers accrue only to the young. While this does not constitute a criticism to Jones and Manuelli, it clearly extends their work.

Uhlig and Yanagawa (1996) present an overlapping generations model in which growth is also endogenous in the sense that it does not depend on exogenous increases in technology. In fact, growth in their model is the result of technological spillovers across rms. Furthermore, they assume the existence of a government which taxes capital and labor income in order to nance a xed amount of government expenditure on consumption goods. These features enable them to shift the tax burden between capital and labor incomes, which combined with

6 Previous no growth results are Boldrin (1992) and Kotlikoff and Summers (1981).

7 Jones and Manuelli (1992) have two models in their paper, a one-sector growth and a two-sector growth model. When comparing this model to theirs I am always refering to their one-sector version.

the demographic structure may lead to a positive relationship between capital taxation and economic growth.[8] This model differs from theirs in at least two ways. The first is that in this analysis the government plays an active rather than passive role in the sense that it is the provision of public services what eventually sustain growth. Second, the model does not require shifting the tax burden across generations in order to implicitly redistribute income toward those agents with high propensity to save. As already explained, the mechanism responsible for such redistribution is the incidence of public policy in total factor productivity.

It is clear from the literature that whether used to restore growth or to study the equilibrium relationship between income taxation and the growth rate, government policies that result in direct or indirect intergenerational income redistributions matter. The redistributive mechanism implied in the analysis herein not only adds to the understanding of government policy, but also provides new insight into the growth process. Those papers and this consider different channels by which redistribution takes place and should be perceived as complementary.

The organization of the paper is as follows. Section 2 introduces the economy by specifying preferences, endowments, production, the composition of government expenditures, and how are these expenditures financed. Section 3 defines the equilibrium concept for this economy and shows that this economy is characterized by steady-state growth. Section 4 takes the polar case in which individuals are endowed with time only in youth and no transfer from the government take place and studies how fiscal policy affects long-run growth. A numerical example using U.S. data illustrates the mechanisms at work by considering a policy that

[8] The redistribution occurs because at any date labor income accrues mostly to the young generation and capital income mostly to the old generation.

increases income taxes while increasing the share of tax revenue allocated to the purchase of consumption goods. This exercise suggests that from the point of view of the long run growth rate it is worse to finance expansions of government consumption with capital rather than labor income taxes. Section 5 extends the model in the sense that individuals are also endowed with time when old and the government devotes part of its tax revenues to finance transfers. Section 6 concludes.

2 The Basic Framework

This economy is populated by identical individuals who live for two periods. Generations of individuals overlap. Thus at any point in time there are young as well as old individuals in the economy. Population is stationary, so one can think of a representative agent per generation. The representative individual of generation t has lifetime utility given by

$$U\left(C_t^t, C_{t+1}^t\right) \tag{1}$$

where C_j^i denotes consumption of generation i at date j. It is assumed that the function U is strictly quasiconcave, twice continuously differentiable, and strictly increasing in each of its arguments.

Each agent is endowed with $0 < \lambda \leq 1$ units of time when young and $1-\lambda$ units when old. Not exhibiting preferences for leisure, individuals supply their time endowment inelastically in exchange for which they receive a competitive compensation in the form of wages, denoted by w. There are two assets in this economy: capital K, which is used in production, and private loans b. At any point in time individuals who own capital in the economy receive rents, r. If capital income is taxed at a rate τ_K and labor income is taxed at the rate τ_L, the budget constraints for the representative individual of generation t are given by

$$C_t^t \leq (1-\tau_L)\lambda w_t + T_{1,t} - K_{t+1} - b_{t+1} \tag{2}$$

when young and

$$C_{t+1}^t \leq (1-\tau_L)(1-\lambda)\, w_{t+1} + \left[r_{t+1}(1-\tau_K) + 1 - \delta\right] K_{t+1} + T_{2,t+1} + R_t b_{t+1} \tag{3}$$

when old, where T_1 and T_2 denote transfer payments from the government to the young and the old respectively, and R_t stands for the interest rate factor between dates t and $t+1$.[9]

Note that after it is used in production, capital depreciates at rate $\delta \in [0,1]$, and then it is sold to the young. Thus, the law of motion of capital is given by

$$K_{t+1} = (1-\delta)K_t + X_t \tag{4}$$

where X_t denotes investment at date t. It will be assumed that at the beginning of time there is a capital stock outstanding, $K_1 > 0$.

Production is assumed to be carried out by a large number of perfectly competitive firms, each operating for two periods. Towards the end of the first period, firms borrow (real) capital from the young. At the beginning of the second period, they combine the acquired capital with labor hired and repay the capital plus interest to the now old creditors. The production function of firm i is represented by

$$y_{ti} = F(k_{ti}, l_{ti}, G_{st}) = Bk_{ti}^{\alpha}\left(G_{st} l_{ti}\right)^{1-\alpha} \tag{5}$$

where k_{ti} is the capital employed at date t by firm i, l_{ti} is the labor hired at date t by firm i, $B > 0$ is a technological parameter, G_{st} is the aggregate flow of public services at date

[9] So if i is the interest rate, then $R = 1 + i$.

t, and $0 < \alpha < 1$. The price of the consumption good is normalized to 1. The technology in (5) emphasizes what was previously explained, that G_s comprises services that in a way maintain the "social fabric".

Clearly, it has been assumed that the technology exhibits constant returns to scale in private capital and labor taken together. Therefore, payments to capital and labor exhaust output implying zero equilibrium profits. Firms do not invest in publicly provided services because they cannot internalize their benefits. It has also been assumed that the technology exhibits constant returns to scale in capital and government expenditures taken together. This assumption plays a critical role and enables the analysis of long-run effects of policy without the study of transitional dynamics. Finally, it will be assumed that there is no exogenous growth in technology. While individual firms' technology exhibits increasing returns to reproducible and non-reproducible factors together, below it is shown that there are constant returns to scale at the social level.

It is noteworthy that there is a scale effect under this specification of production. Since it can be spread in a non-rival fashion over all producers, aggregate public services G_s - and not per capita - is the important variable determining capital per capita (see Antinolfi et al., 1998). It is also worth noting that in equilibrium aggregate labor will be fully employed and equal to 1.

To close the model, the government is specified as a fiscal authority which collects taxes and spends a fraction ϕ of the revenues on government consumption G_c, a fraction γ on transfers Γ, and the remaining revenue, $1 - \gamma - \phi$, on public services G_s. Total government spending is denoted by G. It is assumed that the young receive a fraction η of total transfer payments and the old receive $1-\eta$.[10] As explained, total spending is nanced by imposing at-rate taxes on capital and labor incomes. It is assumed that the tax rates are announced at the beginning of time and that the government commits fully and credibly to these tax rates, therefore evading the fundamental issue of time-consistency of scal policy. Furthermore, it will be assumed that individuals cannot be taxed more than their capital and labor incomes. Since the government does not issue any debt, its budget must be always balance:

$$G_t = G_{ct} + G_{st} + \Gamma_t = \tau_L w_t + \tau_K r_t K_t \tag{6}$$

3 Equilibrium

Since rms act competitively, in equilibrium pro ts will be zero and the rst-order conditions of each rm s maximization problem are such that the marginal products of labor and capital equal the wage and rental rates respectively. These rates have been denoted by w and r respectively. Thus, we have

$$w_t = (1-\alpha)B{k_{ti}}^{\alpha}{l_{ti}}^{-\alpha}G_{st}^{1-\alpha}, \qquad r_t = \alpha B{k_{ti}}^{\alpha-1}l_{ti}^{1-\alpha}{G_{st}}^{1-\alpha}$$

By using equation (6) and exploiting the fact that in equilibrium all rms employ the same capital/labor ratio, it can be easily shown that the weighted average of the tax rates (where the weights are given by the factor shares), de ned by $\tau \equiv \alpha\tau_K + (1-\alpha)\tau_L$, is the share of national output spent by the government. Given the policy parameters aggregate output is given by

$$Y_t = AK_t \tag{7}$$

[10] To talk about growth, there must be some *productive* spending. This places a mild restriction on γ and ϕ, namely $\gamma + \phi < 1$.

where $A \equiv B^{\frac{1}{\alpha}} \left[(1-\gamma-\phi)\,\tau\right]^{\frac{1-\alpha}{\alpha}}$. Thus in equilibrium input prices can be rewritten as

$$w_t = (1-\alpha)Y_t, \qquad r_t = \alpha A \tag{8}$$

Note that A gives a measure of total factor productivity for this economy. Clearly, when total government outlays remain constant, total factor productivity declines when the composition of expenditures changes in favor of government consumption and/or transfers. Similarly, given a particular composition of expenditure by the public sector, total factor productivity is larger the higher the tax rates since an increase in revenue raises all spending categories including public services.

In equilibrium it must be that the net returns from investing in physical capital and private loans must be equal. This is a standard non-arbitrage condition given by

$$R_t = r_{t+1}(1-\tau_K) + 1 - \delta \tag{9}$$

In light of (8), it follows that in equilibrium the rental rate and thus the interest factor do not depend on time. Exploiting this time-independence, the trade off between consumption and investment can be obtain from utility maximization and is given by[11]

$$\frac{\partial U}{\partial C_t^t} = \beta \frac{\partial U}{\partial C_{t+1}^t} \left[\alpha A(1-\tau_K) + 1 - \delta\right] \tag{10}$$

Goods market equilibrium is given by

$$S_t = K_{t+1} \tag{11}$$

In addition to the goods market, there are markets for labor, capital, and private borrowing and lending in this economy, which must also clear in equilibrium. Since by assumption

[11]It should be speci ed that the initial old consume the initial return on capital plus the amount of her transfer, $C_1^o = R_o K_1 + T_1$.

all of the labor in the economy at time t is supplied inelastically to firms, the demand for labor determines the wage rate at which the labor market clears. Because firms behave competitively, in equilibrium all firms employ the same capital labor ratio, and the demand for capital by the firm determines the rental rate that clears the capital market. In addition, the borrowing and lending market clears, since in equilibrium aggregate net borrowing per generation equals zero. We are now in position to formally define an equilibrium in this economy.

Definition 1 *Given a fiscal policy, a competitive equilibrium under balanced budget is sequences of quantities* $\left\{C_t^t, C_{t+1}^t, K_{t+1}, X_t, Y_{t+1}, b_{t+1}\right\}$ *and prices* $\{w_t, r_t, R_t\}$ *consistent with: (i)Utility maximization:* $\left\{C_t^t, C_{t+1}^t, K_{t+1}, b_{t+1}\right\}$ *satisfy (10), (2) and (3); (ii)Profit maximization: (8) is satisfied; (iii)Markets clearing: (11) holds; (iv)Balanced budget: (6) is satisfied; and (v)Non-arbitrage: (9) holds.*

A few remarks are in order. Given the level of K_t, equilibrium input prices (and thus marginal productivities) are both increasing on the weighted average of the tax rates (and in each tax rate taken separately) since higher taxes increase the flow of public services to the economy. By the same token, equilibrium input prices decline in γ and ϕ. Furthermore, the equilibrium wage rate is increasing in the aggregate capital stock. Finally, notice that the marginal condition (10) depends on τ_L even though individuals do not derive utility from leisure (through A).

In order to make arguments more precise and since one of the goals is to examine the quantitative implications of the model with regard to long run growth, consider the utility function

$$U(C_t^t, C_{t+1}^t) = \frac{\left(C_t^t\right)^{1-\sigma}}{1-\sigma} + \beta\frac{\left(C_{t+1}^t\right)^{1-\sigma}}{1-\sigma} \tag{12}$$

where $\sigma > 0$.

For this utility function, it could be shown that the marginal condition (10) together with the lifetime budget constraint, and the fact that transfers are given by $T_{1,t} = \eta\gamma G_t$ and $T_{2,t+1} = (1-\eta)\,\gamma G_{t+1}$ lead to optimal savings of the form[12]

$$S_t = S(R)\left[(1-\tau_L)\lambda w_t + \eta\gamma G_t\right] + \left[1 - S(R)\right]\left[\frac{(1-\tau_L)(1-\lambda)w_{t+1} + (1-\eta)\,\gamma G_{t+1}}{R}\right] \quad (13)$$

where $S(R) \equiv \frac{\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}{1+\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}$ is the saving rule.

It is clear from (13) that the relationship between savings and the interest factor is not simple and depends on all other parameters in the model. Using (11) and (13) along with (8), one obtains a rst order difference equation in the capital stock. Since it was assumed that the initial old are endowed with some capital, $K_1 > 0$, one can pin down the economy s growth factor g, which is given by

$$g = \frac{(\beta R)^{\frac{1}{\sigma}}\left[(1-\alpha)(1-\tau_L)\lambda + \eta\gamma\tau\right]A}{R + (\beta R)^{\frac{1}{\sigma}} + \left[(1-\alpha)(1-\tau_L)(1-\lambda) + (1-\eta)\gamma\tau\right]A} \quad (14)$$

which makes use of the de nition of $S(R)$ given above.

Provided that $g > 1$, and given $K_1 > 0$, the economy s relevant variables, capital, consumption, investment, government spending, output, and wages, all grow at the constant rate g. Notice that for the growth rate to be positive, savings must be sufficiently high. Clearly, this cannot be guaranteed for all possible tax rates τ_K and τ_L. To make sure that tax policies are consistent with long-run growth mild restriction on parameters must be imposed (see below).

[12] Using (2), (3), and (9) one obtains the lifetime budget constraint, which is given by $C_t^t + \frac{C_{t+1}^t}{R} = (1-\tau_L)\lambda w_t + T_{1,t} + \frac{(1-\tau_L)(1-\lambda)w_{t+1} + T_{2,t+1}}{R}$

4 The Baseline: Retirement Age and No Transfers

Consider rst the polar case in which the elder lack a time endowment and total outlays by the government are divided between purchases of consumption goods and provision of public services. That is, individuals are endowed with time only in youth ($\lambda = 1$) and the government does not allocate a fraction of its revenue to transfer payments ($\gamma = 0$). In this case, life-time income consists only of net real wages earned in youth.

Given the assumptions above, this version of the model is similar to that of Uhlig and Yanagawa (1996), but differs from theirs in a few important regards. First, the sources of growth are different. Although both models take advantage of an AK framework, Uhlig and Yanagawa employ technological spillovers to sustain positive growth along the equilibrium path, whereas this model uses public services to have balanced growth paths, providing the government with an active rather than passive role in the economy. Second, tax revenues in this model are used to nance different categories of spending rather than a xed level of government consumption. As it will turn out, the composition of spending is important in assessing growth performance. Third, in this model intergenerational transfers occur mostly through productivity changes rather than by shifting the tax burden across generations directly.

I will now study conditions under which the equilibrium interest rate is bounded away from zero along the balanced growth path since this is necessary for positive long run growth. As already mentioned, not all tax policies are consistent with positive equilibrium interest rates, thus some parameter restrictions are needed. For a positive steady state interest rate, it must be the case that $\tau_L < 1$, otherwise life-time income is zero and individuals are unable to invest in the first place. Similarly $\tau_K < 1$, otherwise the equilibrium interest rate is negative. Further restrictions are needed and these can be placed on either tax rate or on both. I choose to impose the burden on the labor income tax rate as to have more freedom in varying the capital tax rate. Thus I restrict the labor income tax rate by imposing a minimum tax rate at which the interest rate is bounded away from zero even for $\tau_K = 0$. This implies considering policies such that $\tau_L \in (\underline{\tau}_L, 1)$, where $\underline{\tau}_L \equiv \frac{1}{(1-\alpha)(1-\phi)} \left(\frac{\delta}{\alpha B^{1/\alpha}} \right)^{\alpha/(1-\alpha)}$. Given the spending composition parameter $\phi \in (0,1)$ the following assumption provides parameter restriction that guarantee the existence of policies with $\tau_L \in (\underline{\tau}_L, 1)$.

$$\delta^\alpha < B\alpha^\alpha (1-\alpha)^{1-\alpha} (1-\phi)^{1-\alpha} \tag{15}$$

Fiscal policies satisfying (15) will be referred to as policies *consistent* with steady state growth. Note that this assumption is not restrictive insofar as the technological parameter B remains free.

4.1 Fiscal Policy, Long run Interest rate, and Long run Growth

I now proceed to examine how the long run interest rate is affected by fiscal policies consistent with steady state growth. As it will turn out, how policy affects long run interest rate will be of consequence in the overall assessment of how policy affects long run growth.

Using (9) and the definition of A, the interest rate can be written as

$$i = \alpha B^{1/\alpha} \left[(1-\phi)\,\tau \right]^{(1-\alpha)/\alpha} (1-\tau_K) - \delta \tag{16}$$

By inspection the interest rate is monotonically decreasing in ϕ and monotonically increasing in τ_L. But in contrast with many AK models of growth in which A is a parameter and the capital tax rate affects the interest rate only through the capital tax factor, the equilibrium interest rate is nonmonotonic in τ_K. To see this, notice that given a consistent scal policy

$$\frac{\partial i}{\partial \tau_K} = (i+\delta)\left(f(\tau_K) - h(\tau_K)\right) \tag{17}$$

where $f \equiv \frac{\partial A/\partial \tau_K}{A} = \left(\tau_L + \frac{\alpha}{1-\alpha}\tau_K\right)^{-1}$ and $h \equiv (1-\tau_K)^{-1}$. In this setting f is the rate of change in total factor productivity due to changes in capital taxation (and thus in public services for a given ϕ) and h is the rate of change in the capital tax factor or in the degree of distortion due to capital taxation. When f is larger than h, increasing the capital tax rate brings about increments in productivity that increase per unit capital rents which more than offset the per unit capital income losses associated with higher taxation and thus the interest rate rises. The contrary occurs if the prevailing tax is too high. This is summarized as follows.

Proposition 2 *Consider a consistent scal policy. Then there exist $\widehat{\tau}_K$,$\overline{\tau}_K \in (0,1)$ with $\widehat{\tau}_K < \overline{\tau}_K$ such that the interest rate is a concave function of τ_K reaching a maximum at $\widehat{\tau}_K = (1-\alpha)(1-\tau_L)$ and $\overline{\tau}_K$ is given by*

$$F(\overline{\tau}_K) \equiv \alpha B^{1/\alpha}\left[(1-\phi)\left(\alpha\overline{\tau}_K + (1-\alpha)\,\tau_L\right)\right]^{(1-\alpha)/\alpha}(1-\overline{\tau}_K) - \delta = 0$$

In addition, for any $\tau_K \in (0,\overline{\tau}_K)$ the interest rate is positive and negative otherwise, with $\overline{\tau}_K$ increasing in τ_L and decreasing in ϕ (proof in the Appendix).

Figure 1 shows how $\widehat{\tau}_K$ is determined and how it responds to changes in τ_L. Other things equal and from the point of view of factor productivity, a lower τ_L increases the marginal tax effect of capital taxation since the interest rate declines at all capital income tax rates ($f(.)$ shifts up). Thus lower τ_L translates into larger $\widehat{\tau}_K$ while $\overline{\tau}_K$ falls. An increase in ϕ has the same effect except that $\widehat{\tau}_K$ remains unchanged. Figures 2 and 3 show the effects on the interest factor of different policies for the parameter values given in Table 1.[13] Figure

[13] Table 1 are the parameter values used in the policy experiments with which this section ends.

2 plots the interest factor R for different values of ϕ and τ_K and Figure 3 does the same for different values of ϕ and τ_L.

Remark 3 *Given the labor income tax and the composition parameter, the direction of change in the interest rate depends on the ratio of government revenue from capital taxation to the net of tax capital income of the private sector at the prevailing capital tax rate (relative size of government). Note that* $\partial i/\partial \tau_K \gtrless 0 \Leftrightarrow \widetilde{A}_{\tau_K} \gtrless \frac{G_K}{I_K}$*, where* $\widetilde{A}_{\tau_K} \equiv f \cdot \tau_K = \frac{\partial A}{\partial \tau_K}\frac{\tau_K}{A} = \left(\frac{\tau_L}{\tau_K} + \frac{\alpha}{1-\alpha}\right)^{-1}$ *is the elasticity of productivity with respect to the capital tax rate,* $G_K = \tau_K \alpha A$ *is the per unit of capital revenue of the government, and* $I_K = (1-\tau_K)\,\alpha A$ *the per unit of capital net of tax capital income that accrues to the private sector.*

I now turn to study the effects on growth of different policies. Empirical work shows that government expenditures on consumption goods are detrimental for long-run growth (see Hall and Jones, 1997 and Levine and Renelt, 1997). Recall that in this version of the model increasing the share of total outlays allocated to government purchases corresponds to a reduction in the share of spending used in the provision of public services. It will be reasonable then to expect that increasing ϕ will affect growth negatively. This will be consistent with empirical ndings. Berndt and Hansson (1992), for example, nd that spending on infrastructure increases productivity signi cantly and leads to lower labor requirements for rms. Nadiri and Mamuneas (1994) nd that spending on infrastructure have a signi cant positive effect on total factor productivity. Easterly and Rebelo (1993) also show that government services (transport and communications) are positively related to growth. In addition, Barro (1991) and Alesina et al. (1992) nd that certain measures of political unrest are negatively related to growth, which suggests that expenditures in categories such as law and order may be conducive to higher growth.

From (14) the growth factor is now given by

$$g = \frac{(\beta R)^{\frac{1}{\sigma}} (1-\alpha)(1-\tau_L) A}{R + (\beta R)^{\frac{1}{\sigma}}} \tag{18}$$

For a given tax policy, how an increase (decrease) in the share of outlays allocated to consumption goods (public services) affects growth depends on two forces. On the one hand, increasing ϕ leads to a decline in total factor productivity which in turns exerts downward pressure on the growth rate by lowering the real purchasing power of consumers. On the other hand, it affects long-run interest rates, which could lead to lower savings and lower growth or could attenuate and even dominate the productivity effect depending on the value of the intertemporal elasticity of substitution for consumption. This is summarized as follows.

Proposition 4 *If the elasticity of intertemporal substitution for consumption is larger or equal to unity (σ □ 1), then for a consistent tax policy a larger share of total outlays allocated to government consumption (ϕ) will unambiguously result in lower growth (proof in the Appendix).*

The intuition is that by reducing productivity, a marginal increase in ϕ reduces the real return on capital and thus the interest rate. If the interest elasticity of savings (which depends on σ) is sufficiently high, then lower productivity translates into lower real income and lower savings while lower interest rates also lead to lower savings, unambiguously decreasing growth. From the proposition above, one can easily nd sufficient conditions for a negative relationship between ϕ and g.

Corollary 5 *An increase in the share of outlays allocated to government consumption leads to lower growth across equilibria if and only if the interest elasticity of savings, $\varepsilon = \frac{1/\sigma - 1}{1+\beta^{1/\sigma}(1+i)^{1/\sigma - 1}}$, satis es the following condition:*

$$-\omega < \varepsilon \tag{19}$$

where $\omega \equiv \frac{1+i}{\delta+i}$.

It will be assumed in the remaining of this section that (19) holds. With respect to capital taxation, there are two channels through which the capital income tax rate affects growth. First, changes in capital taxation affect productivity which directly affects real income. Secondly, movements in the capital tax rate change assets real rates of returns. How changes in the interest rate affect growth will depend on the interest elasticity of savings ε. This analysis does not preclude the possibility that for an increase in the capital income tax rate the economy evolves along a path in which growth is higher as in Uhlig and Yanagawa (1996), but this will necessarily depend on the prevalent capital tax rate ($\tau_K < \widehat{\tau}_K$).

Proposition 6 *Suppose that the elasticity of intertemporal substitution is larger or equal to unity (σ □ 1). Then for a given composition of spending, an increase in the capital income tax rate will lead to an increase in the growth rate if the prevalent tax rate is relatively small (proof in the Appendix).*

Take for example an economy in which the expenditure composition remains unaltered. In this case, an increase in tax revenue increases the ow of public services to the economy. If the latter are nanced through capital taxation, it is actually the elder who pay for the tax increase for in equilibrium they own all the capital in the economy. For a given capital stock, this policy results in higher total factor productivity raising the (real) income of young agents. Faced with higher purchasing power and anticipating higher income taxes when old, they will be induced to save more in order to have a smoother path of consumption over their lifetimes. This is the *direct* effect of this policy. But there is also an *indirect* effect that depends on how the policy affects the equilibrium interest rate and on the agents response to changes in the intertemporal price of consumption. It is possible that the latter effect lessens and even offsets the former. If both work in the same direction then the economy could evolve along a higher balanced growth path.

Other things equal, the growth effects of capital taxation are generally ambiguous. For instance, it could be shown that for sufficiently large elasticities of substitution, capital taxation displays a Laffer effect on the growth rate in the sense that to attain a given rate of growth there are two tax regimes that could implement it.[14] That higher capital tax rates lead to higher growth is not always the case. In fact, it occurs if and only if the interest rate elasticity is strongly restricted.

Corollary 7 *An increase in the capital income tax rate will unambiguously lead to higher long run growth across equilibria if and only if the interest elasticity of savings satis es the following condition*

$$-\omega < \varepsilon < 0$$

In the absence of labor leisure choices, it could be argued that studying how labor income taxation affects growth is not so interesting. However, as it was mentioned before, although labor does not bear any disutility for agents, changes in labor taxation affect productivity which in turn determines real income and interest rates, thus affecting intertemporal consumption decisions along the equilibrium path. It could easily be shown that under (15) for $\varepsilon \geq 0$ the labor income tax rate has a Laffer effect on long run growth, likewise for the case in which $-\omega < \varepsilon < 0$, albeit mild parameter restrictions.

4.2 Quantitative Analysis

Since in general both labor and capital income taxes have ambiguous effects on the growth rate, a simple quantitative exercise using US data can illustrate how different tax policies could affect growth. I concentrate on different income tax policies used to nance an increase in total government outlays of nearly 1% of GDP (5% increase in τ) while the fraction of

[14] It can easily be shown that in such a case the threshold rate at which the growth rate attains a maximum is larger than $\widehat{\tau}_K$.

spending allocated to the purchases of consumption goods is simultaneously raised by 5% (5% increase in ϕ). There are different combination of tax policies that could achieve this objective, but I look at the polar cases in which the increase in total outlays is fully financed through either capital or labor income taxes. If this policy is financed with capital (labor) income taxes, the corresponding tax rate must be increased by approximately 13.6% (8%).

In this highly stylized economy, a period is interpreted as 25 years. Cooley and Prescott (1995) report that for the period 1952-1992, the steady-state aggregate capital-output ratio averages 3.32. After modifying this ratio to fit the period length chosen, we have that $Y/K = 7.8125$. The rate of depreciation of capital δ is fixed so that long-run interest rate is 3% per annum. Following Uhlig and Yanagawa (1996), the income labor share, α is fixed at 0.4. The technological parameter B is chosen as to satisfy $A = Y/K$.

Capital and labor income tax rates are set at 35% and 40% respectively. The choice of the capital tax rate is taken from Hendricks (1999), who uses *IRS* data to compute the value for τ_K. The choice of labor income tax rate matches US total outlays as a fraction of GDP over the period 1960-1995, which averaged 0.38. For the same period, government final consumption expenditure as a percentage of GDP averaged 17.4%, suggesting a benchmark value for ϕ of 0.53. Since social security transfers as a percentage of GDP are assumed to be zero, the composition parameter is adjusted accordingly so that $\phi = 0.6$. Figure 4 shows the time series for total outlays (Tot. Outl.), final consumption (Fin. Cons.), and transfers (TR) all as percentage of GDP for selected years in the period 1960-1994.[15] Finally, for a given value for the inverse of the elasticity of intertemporal substitution for consumption σ, the discount rate β is chosen such that savings is sufficiently high as to guarantee an annual

[15] The data source is *OECD Historical Statistics 1995.*

growth rate of 1.6%. The summary of the parameter values utilized in this section is given in Table 1.

Table 2 summarizes the effects on both the interest and the growth rates when the increase in government consumption is financed by increasing the capital income tax and Table 3 shows the results when labor income taxes are used instead. Regardless of which type of income taxes are used to raise the needed revenue, both the growth and interest rate show moderate declines. When capital taxes are used and intertemporal substitution for consumption is relatively elastic ($\sigma = 0.7$), the growth and interest rates decline from their baseline values of 1.6% and 3% to 1.34% and 2.52% per year respectively. If labor taxes are used instead, the growth and interest rates fall to 1.17% to 2.83% per year respectively. This result is robust to changes in the intertemporal elasticity of substitution for consumption. If intertemporal substitution for consumption is relatively inelastic ($\sigma = 1.2$), the growth rate drops to an annualized rate of 1.46% if the expansion is financed with capital income taxes and to 1.22% if labor income taxes are used. It is worth noting that when labor income taxes are used to finance this policy the reduction in the long run growth rate is more severe while the fall in the interest rate is less pronounced. Concerning the equilibrium growth rate, this reflects income distribution patterns: in equilibrium capital income accrues to the old while labor income is received by the young and thus labor income taxation penalizes individuals with high propensity to save in the economy.[16]

This exercise suggests at least two things. First it points out that the long run growth effect of income tax changes are closely related to the category of spending to which the

[16] I use values for σ that are commonly used in the literature. The value $\sigma = 0.7$ is taken form Hurd (1989) and the value $\sigma = 1.2$ from Hendricks (1999).

revenue is allocated. Second it suggests that for reasons already discussed, nancing higher spending on consumption goods with labor income taxes will set the economy in an equilibrium path in which economic growth is lower compared to a policy that uses capital income taxes. A caveat of this exercise is that it yields implausible large values for steady-state savings. Using the parameter values reported in Table 1 for the case in which $\sigma = 0.7$, Figures 5 and 6 plot the savings factor for different values of ϕ and τ_K and for different values of ϕ and τ_L respectively.

In the preceding analysis it has been assumed that individuals are endowed with time only in youth ($\lambda = 1$). Relaxing this assumption will not affect the results signi cantly. Other things equal, it should be clear from (14) and provided that $\gamma = 0$, that when individuals earn labor income in both periods of life ($0 < \lambda < 1$), it results in lower steady state growth. However, similar policy experiments when λ is assigned a value other than unity yields similar qualitative results to the case in which λ is set equal to one, though declines in the growth rate becomes negligible as λ becomes smaller. This is not surprising, since allowing $\lambda < 1$ modi es savings incentives. The savings motive associated with retirement diminishes. In the absence of government transfers, concentrating on $\lambda = 1$ is a mild restriction but this is not longer the case when transfers are allowed in the model.[17]

5 An Extension: No Retirement and Transfers

I proceed to study how the composition of spending affects growth when transfers are allowed ($\gamma \in (0, 1)$). To concentrate on the growth effects of transfers, it is assumed that the fraction of spending used for consumption is zero ($\phi = 0$). This assumption isolates the trade off

[17] For a full description of the growth effects of varying λ in a similar environment see Uhlig and Yanagawa (1996).

between policies aimed at redistributing income from those related to the provision of public services. As it will be pointed out below, the results are unaffected if this assumption is relaxed. As explained in the previous section, when scal policies include transfers, such transfers can affect the intergenerational distribution of income, which renders the simplifying assumption of $\lambda = 1$ less interesting. Thus it will be assumed that individual have the same endowment of time when young as when old ($\lambda = 1/2$). Finally, in order to pin down the trade off between government transfers and public services, it will be assumed that all income is taxed at a uniform at rate ($\tau_L = \tau_K = \tau$).

With the assumptions above the model is now very similar to that of Jones and Manuelli (1992), and is not a surprise that it shares with it several futures. It differs, however, in some important respects. Growth in their model is asymptotic without specifying the institutions and forces sustaining long run growth. Since they are concerned with restoring growth in OG models, they specify a government that spends all its tax revenues in providing transfers, purposely omitting what is a key feature of this model: expenditure composition. Thus, in their model policy aimed at redistributing income through transfers do not affect the equilibrium interest rate, and from the point of view of the long run growth rate the best policy is the one that redistributes income to the young ($\eta = 1$), suggesting for example that social security transfer are detrimental for growth.

As in Jones and Manuelli (1992), given the income tax rate and a xed composition of spending, the redistributive policy that attains the highest growth rate is the one that redistributes all income to the potential savers ($\eta = 1$). However, since transfer policies affect factor productivity and thus the interest rate, the parameters governing savings behavior become pertinent in determining the impact of income distribution policies. In general, both the tax policy and government composition policies affect the growth rate directly by affecting relative incomes and indirectly through their effects on the interest rate and thus on the savings behavior of the private sector.

As in the previous subsection we need to establish parameter restrictions so that scal policies are consistent with positive steady-state growth. In this case, the assumption on parameters take the following form

$$\delta^{\alpha} < B\alpha^{2\alpha} (1-\alpha)^{1-\alpha} (1-\gamma)^{1-\alpha} \tag{20}$$

The interest rate is now given by

$$i = \alpha B^{1/\alpha} \left[(1-\gamma)\tau\right]^{1/(1-\alpha)} (1-\tau) - \delta \tag{21}$$

Given (20) and the composition of government spending, we can know study the behavior of the equilibrium interest rate associated with different government policies. Notice that increasing the fraction of spending devoted to transfers adversely affects the interest rate, while changes in the tax rate produce a Laffer effect.

Proposition 8 *Under (20) consider scal policies* (τ, γ) *with* $\gamma \in (0,1)$*. Then, given* γ*, the interest rate is a concave function of* τ*, reaching a maximum at* $\widehat{\tau} = 1-\alpha$*. Furthermore, there exist* $\underline{\tau}$ *and* $\overline{\tau}$ *such that for any* $\tau \in (\underline{\tau}, \overline{\tau})$ *the equilibrium interest rate is bounded away from zero (see Appendix for proof).*

Notice that in this section consistent scal policies are given by pairs (τ, γ) such that $\gamma \in (0,1)$ and thus $\tau \in (\underline{\tau}, \overline{\tau})$. Let us know study how policy affects growth and subsequently perform the same experiment as in the previous subsection. The growth rate is now given by

$$g = \frac{(\beta R)^{\frac{1}{\sigma}} \left[(1-\alpha)\lambda + (\eta\gamma - (1-\alpha)\lambda)\tau\right] A}{R + (\beta R)^{\frac{1}{\sigma}} + \left[(1-\alpha)(1-\lambda) + ((1-\eta)\gamma - (1-\alpha)(1-\lambda))\tau\right] A} \tag{22}$$

We need to study the effects of γ and τ. Unfortunately, no simple parametric conditions could be found that pin down the growth effects of different tax policies. Thus, we proceed directly to the numerical analysis. The parameter values used in this section are reported in Table 4.

Except for the parameters governing time endowments, the distribution of transfers across generations, and the composition of government expenditures, the rest of the parameters are identical to those used in Section 4 (see Table 1). In this section, somewhat arbitrarily, the parameter governing the distribution of income is chosen so that the latter is even across generations ($\eta = 0.5$). This is done because in this sections we are interested in studying the trade-off between different tax policies and different composition parameters.[18]

Figures 7 and 8 plot the interest and growth factors for the parameter values in Table 4 when $\sigma = 0.7$. Given the share of total outlays used as transfers, the tax rate that implements the highest growth rate is the threshold rate $\widehat{\tau}$. However, as opposed to the result in the previous section, for a given income tax rate the best policy from the point of view of long run growth is not to allocate all of the revenues to the provision of public services. There is ambiguity as to which category of spending is conducive to higher growth in the long run. These results are robust to changes in the parameter that governs intertemporal substitution. Figures 9 plots the growth rate for the case in which $\sigma = 1.2$.[19]

For the parametric case studied in this section, it is clear that when scal policy affects total factor productivity, the interest rate is positively related with the growth rate, suggesting that at least in the case in which intertemporal substitution is relatively elastic the model

[18]For a full description of the growth effects of varying η in a similar model see Jones and Manuelli (1992).

[19]The results above are also robust to changes in the income distribution parameter η, although the economy evolves at a lower growth for a given scal policy as η is reduced.

complies with the observation that in the long run personal savings and income taxes are positively correlated. In addition, government size is important in determining how transfer policies affect growth in the long run. In light of the above discussion, is not surprising that when both the income tax rate and the share of expenditures in transfers are raised simultaneously by the same percentage, thus replicating the exercise carried out in the previous section, long run growth increases while the interest rate declines. However, changes in both the interest and the growth rate are negligible (see Table 5). The growth rate increases from 1.6% to 1.63% (1.66%) per year for the case in which $\sigma = 0.7$ (1.2) while the interest rate declines from the baseline 5% to 4.9% per year.

6 Conclusions

This paper presents a one-sector endogenous growth model in which a fraction of public spending (public services) affects the productivity of private factors of production, thereby distorting the investment decision of the private sector. However, the government can also employ its tax revenues in purchases of consumption goods and in transfers. This approach allows one to study the impact of tax policy and/or government spending composition on the long run rate of economic growth.

The model examined portrays total factor productivity as the conduit for income redistribution across generations. Changes in scal policy lead to changes in productivity, which in turn lead to changes in real incomes and in the interest rate. The former affects savings directly, while the latter affects savings through its impact on the interest rate. For this reason, the sensitivity of savings to changes in the interest rates is crucial in order to determine the effects of different policies on the growth rate. This paper shows that for a large class of interest elasticities of savings, government spending on services that enhance the productivity of private inputs is beneficial for growth. However, the effects of income tax policy on the long run growth rate of the economy is generally ambiguous, even in the presence of productive public spending.

Some economists have argued that private sector decisions regarding savings may be such that increasing the capital income tax rate may lead to higher growth. The preceding analysis shows that this could happen only for a highly restricted set of interest rate elasticities of savings. In particular, the elasticity of savings with respect to the interest rate must be negative and small in absolute value. In order to shed light on how tax policy affects growth, preferences and technology are restricted to functional forms widely used in the literature and a simple calibration is carried out using U.S. data. It is reported that when tax revenue is raised to finance an expansion in government purchases, the economy would evolve along a path in which growth is lower. This particular experiment also suggests that from the point of view of the long run growth rate, it is better to finance the expansion with capital rather then labor income taxes. When the revenue is used to finance transfers at the expense of public services the growth effects are ambiguous. For the particular exercise presented in Section 5, the long run growth rate is lower, but the reduction is negligible.

The analysis herein is to a certain extent too simple. This simplicity is reflected, for example, in the policy experiment which yield very high savings rates. One way to correct this will be to extend the model to an environment in which individuals live for more than two periods, but in which the time devoted to work diminishes with calendar time. Finally, it is worth noting that in order to concentrate on the positive aspects of tax and spending composition policy, this paper does not inquire into other important aspects of policy that could substantially modify its ndings, such as time consistency of policy and welfare. These and other shortcomings are intended to be overcome in future research.

7 Appendix

Since it will be sometimes useful to write everything in terms in unitless changes, the elasticity of the variable k with respect to the argument j will be denoted by $\widetilde{k}_j$, for example the elasticity of total factor productivity A with respect to the spending composition parameter ϕ is given by $\widetilde{A}_\phi$.

Proof. (of Proposition 2) To see that i is concave in τ_K, let

$$H(\tau_K) = f(\tau_K) - h(\tau_K) \tag{23}$$

It is straightforward to check that $H(\widehat{\tau}_K) = 0$, where $\widehat{\tau}_K = (1-\alpha)(1-\tau_L)$. By assumption, $0 < (1-\alpha)(1-\tau_L) < 1$. Furthermore, $h(0) = 1 < \frac{1}{\tau_L} = f(0)$ with $h'(\tau_K) > 0$ and $f'(\tau_K) < 0$ in the relevant range and $\lim_{\tau_K \longrightarrow 1} h(\tau_K) = \infty$. Thus, f and h intersect only once in the relevant range precisely at $\widehat{\tau}_K$. Since $\tau_L \in (\underline{\tau}_L, 1)$, and $H(0) > 0$, $H(1) = -\delta$ existence of $\overline{\tau}_K$ is established by continuity, and the remaining claims on $\overline{\tau}_K$ trivially hold. Finally, by applying the implicit function theorem to F, one obtains

$$\frac{d\overline{\tau}_K}{d\tau_L} = -\frac{(1-\alpha)^2 (1-\overline{\tau}_K)}{\alpha (\widehat{\tau}_K - \overline{\tau}_K)} > 0 \qquad \frac{d\overline{\tau}_K}{d\phi} = \frac{(1-\alpha)(1-\overline{\tau}_K)\tau}{\alpha (1-\phi)(\widehat{\tau}_K - \overline{\tau}_K)} < 0$$

■

Proof. (of Proposition 4) Consider rst the case $\sigma = 1$. In this case (1) becomes

$$U = \ln C_t^t + \beta \ln C_{t+1}^t$$

and therefore individuals save a constant fraction of income independent of the prevailing interest rate, thus the saving rule is given by

$$S = \frac{\beta}{1+\beta}$$

Using S and (9) into (18), along with the de nitions of A and τ one can obtain

$$\widetilde{g}_\phi = \widetilde{A}_\phi = -\frac{(1-\alpha)}{\alpha\,(1-\phi)} < 0$$

Suppose now that $\sigma < 1$. Individuals saving rule is now

$$S = \frac{\beta^{1/\sigma}\,(1+i)^{1/\sigma - 1}}{1 + \beta^{1/\sigma}\,(1+i)^{1/\sigma - 1}}$$

which is increasing in i (i.e., the substitution effect dominates). In this case

$$\widetilde{g}_\phi = (1 + \varepsilon \cdot \omega)\,\widetilde{A}_\phi = -\,(1 + \varepsilon \cdot \omega)\,\frac{(1-\alpha)}{\alpha\,(1-\phi)} < 0$$

where $\epsilon \equiv \frac{\frac{1}{\sigma}-1}{1+\beta^{\frac{1}{\sigma}}(1+i)^{\frac{1}{\sigma}-1}}$ is the interest elasticity of savings. The inequality above follows because $\sigma < 1 \Longrightarrow \varepsilon > 0$ and because $\omega > 0$. ■

Proof. (of Proposition 6) From the proof of Proposition 4, $\sigma = 1$ is the case in which the momentary utility function is logarithmic and individuals save a constant fraction of income independently of the interest rate. Substituting (9) into (18), using the de nitions of A and τ, one obtains

$$\widetilde{g}_{\tau_K} = \widetilde{A}_{\tau_K} = \frac{(1-\alpha)}{\tau} > 0$$

For $\sigma < 1$ and the prevalent tax rate on capital is relatively small (lower than $\widehat{\tau}_K$),

$$\widetilde{g}_{\tau_K} = (1 + \varepsilon \cdot \omega)\,\widetilde{A}_{\tau_K} - \varepsilon \cdot \omega \frac{G_K}{I_K} = \varepsilon \cdot \omega \left(\widetilde{A}_{\tau_K} - \frac{G_K}{I_K} \right) + \widetilde{A}_{\tau_K} > 0$$

since $\tau_K < \widehat{\tau}_K \Longrightarrow \widetilde{A}_{\tau_K} > \frac{G_K}{I_K}$ and $\sigma < 1 \Longrightarrow \varepsilon > 0$. ■

Proof. (of Proposition 8) The proof of this lemma follows that of Proposition 2 with $f(\tau) \equiv \frac{1-\alpha}{\alpha\tau}$ and $h(\tau) \equiv \frac{1}{1-\tau}$. ■

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Table 1: Retirement Age and No Transfers: Parameter Values

Parameter	Value	Source
t		Equivalent to 25 years
α	0.4	Capital share (NIPA)
A	7.8125	To match K/Y=0.128 - Cooley and Prescott (1995)
B		Chosen to satisfy A for given σ
δ	0.9378	Consistent with long run interest rate of 3% per year
β		Chosen to match an annual growth rate of 1.6%
λ	1	Agents are endowed with time only in youth
τ_K	0.35	IRS data - Hendricks (1999)
τ_L	0.4	Uhlig and Yanagawa (1996)
ϕ	0.6	To match total outlays of 22% of GDP
γ	0	No transfers

Table 2: Increasing the capital income tax and the share of spending in consumption goods.

Experiment	Growth	Factor: g	Interest	Factor: R
Increase in τ_K	$\tau_K = 0.35$	$\tau_K = 0.3975$	$\tau_K = 0.35$	$\tau_K = 0.3975$
Increase in ϕ	$\phi = 0.6$	$\phi = 0.63$	$\phi = 0.6$	$\phi = 0.63$
$\sigma = 0.7$	1.016	1.0134	1.03	1.0252
$\sigma = 1.2$	1.016	1.0146	1.03	1.0252

Table 3: Increasing the labor income tax and the share of spending in consumption goods.

Experiment	Growth Factor: g		Interest Factor: R	
Increase in τ_L	$\tau_L = 0.4$	$\tau_L = 0.4317$	$\tau_L = 0.4$	$\tau_L = 0.4317$
Increase in ϕ	$\phi = 0.6$	$\phi = 0.63$	$\phi = 0.6$	$\phi = 0.63$
$\sigma = 0.7$	1.016	1.0117	1.03	1.0283
$\sigma = 1.2$	1.016	1.0122	1.03	1.0283

Table 4: No Retirement and Transfers: Parameter Values

Parameter	Value	Source
t		Equivalent to 25 years
α	0.4	Capital share (NIPA)
A	7.8125	To match K/Y=0.128 - Cooley and Prescott (1995)
B		Chosen to satisfy A for given σ
δ	0.9378	Consistent with long run interest rate of 5% per year
β		Chosen to match an annual growth rate of 1.6%
λ	0.5	Individuals are endowed with time in both periods
η	0.5	Even distribution of income
τ	0.4	Uhlig and Yanagawa (1996) - Hendricks (1999)
ϕ	0	No government expenditures on consumption good
γ	0.44	

Table 5: Increasing income taxes and the share of spending in transfers

Experiment	Growth	Factor: g	Interest	Factor: R
Increase in τ	$\tau = 0.4$	$\tau = 0.4317$	$\tau = 0.4$	$\tau = 0.4317$
Increase in γ	$\gamma = 0.44$	$\gamma = 0.462$	$\gamma = 0.44$	$\gamma = 0.462$
$\sigma = 0.7$	1.016	1.0163	1.05	1.049
$\sigma = 1.2$	1.016	1.0166	1.05	1.049

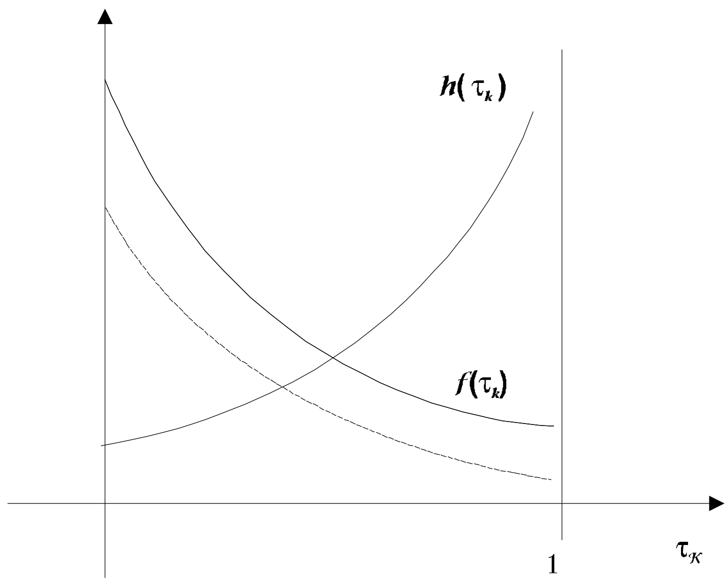

Figure 1: Determination of the threshold $\widehat{\tau}_K$ and how it changes with τ_L (the dashed line corresponds to a larger τ_L)

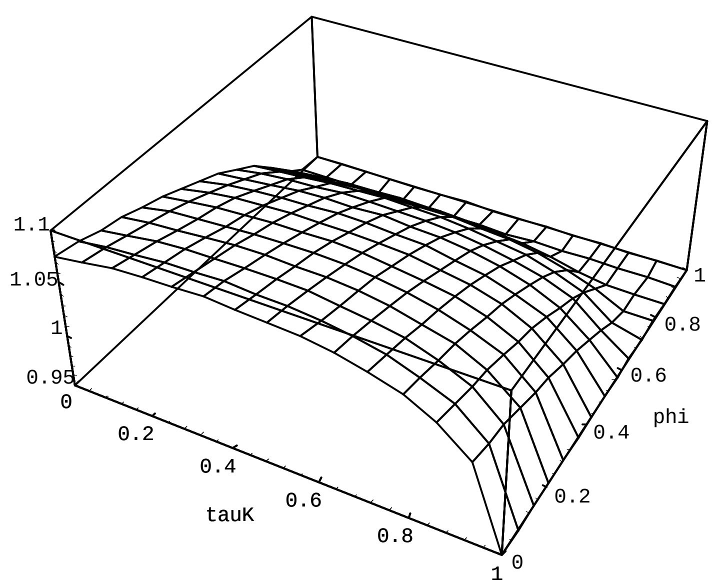

Figure 2: The Interest Factor R for different values of τ_K and ϕ for the parameter values of Table 1.

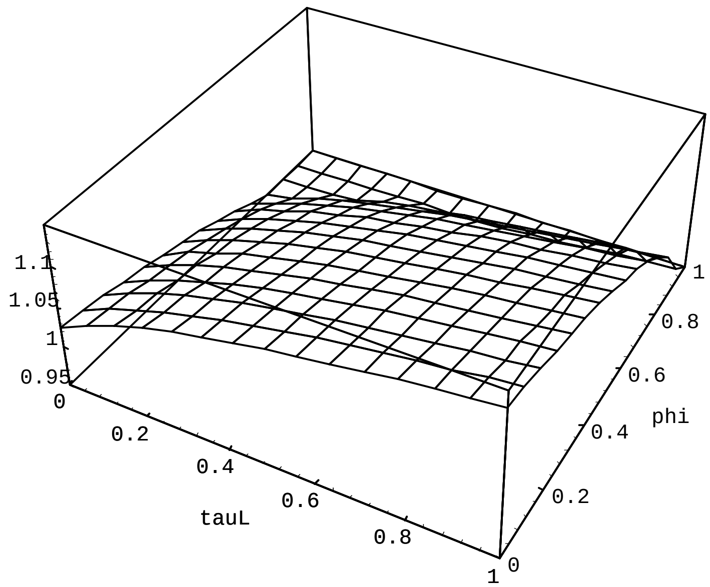

Figure 3: The Interest Factor R for different values of τ_L and ϕ for the parameter values of Table 1.

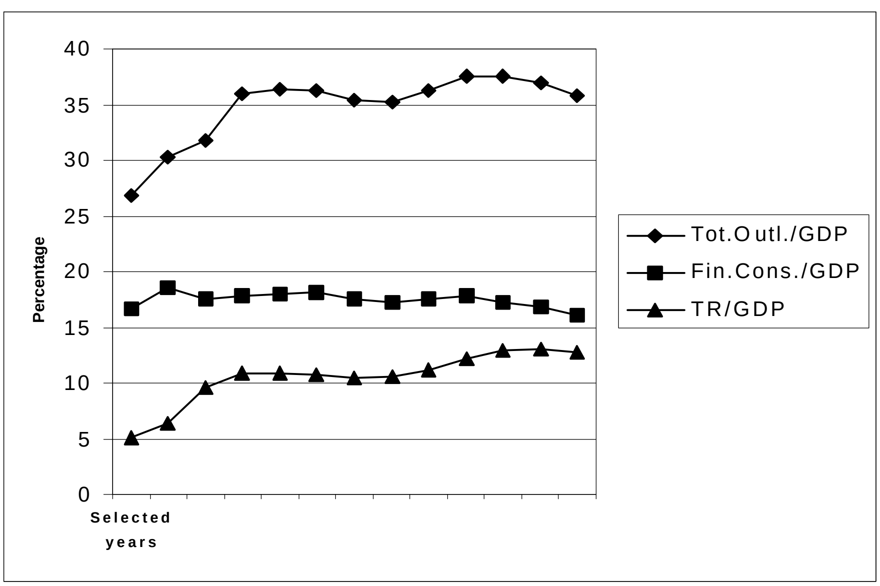

Figure 4: The Composition of Government Expenditures in the US.

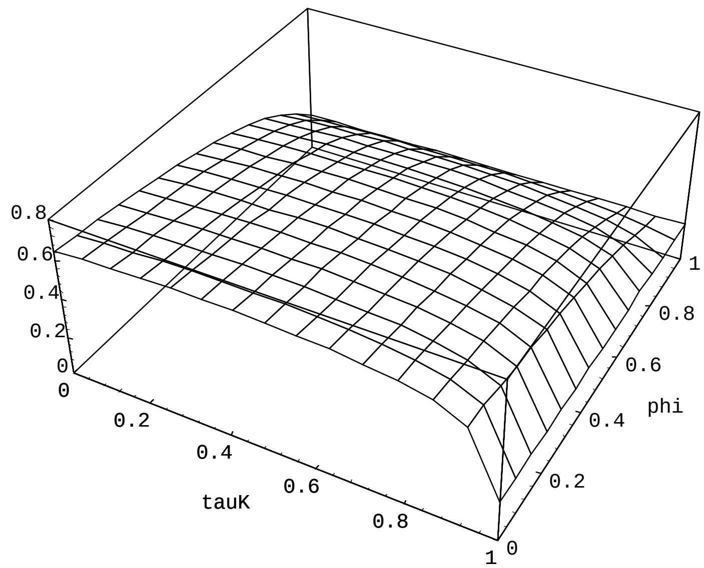

Figure 5: The Savings Factor S for different values of τ_K and ϕ (case $\sigma = 0.7$).

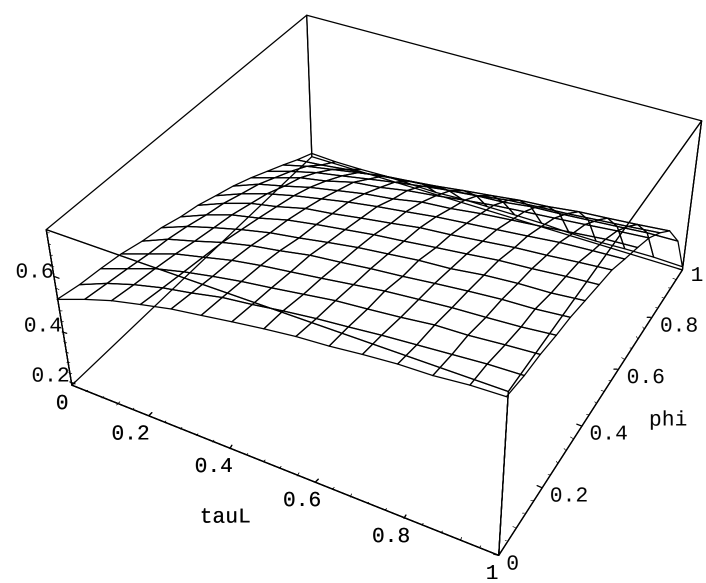

Figure 6: The Savings Factor S for different values of τ_L and ϕ (case $\sigma = 0.7$).

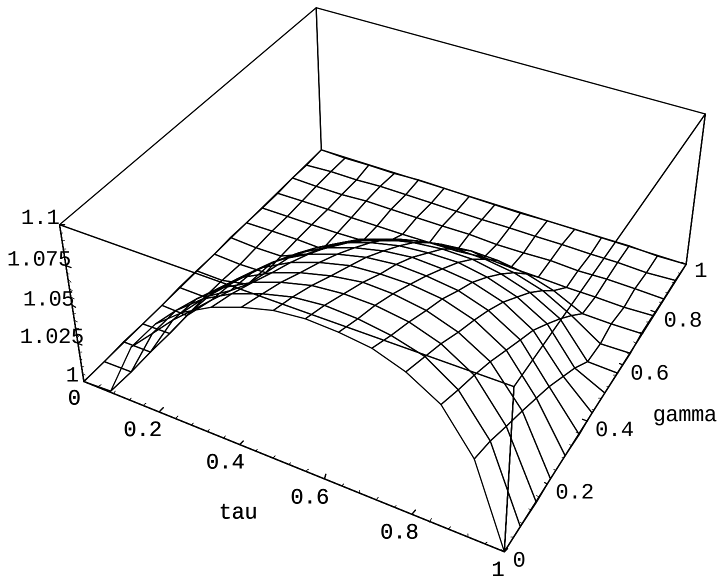

Figure 7: The Interest Factor R for different values of τ and γ.

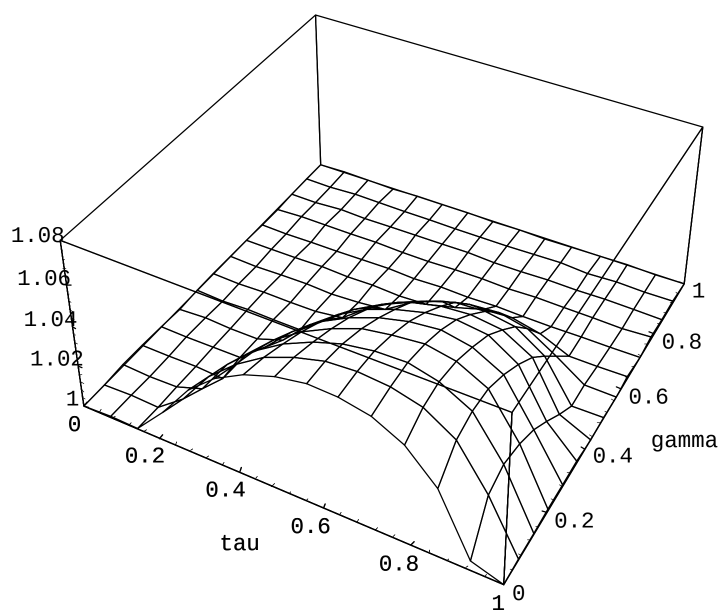

Figure 8: The Growth Factor g for different values of τ and γ (case $\sigma = 0.7$)

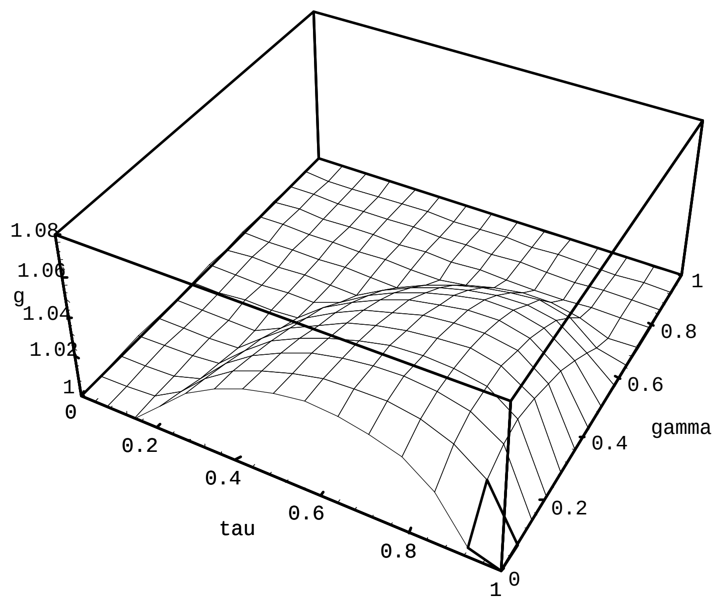

Figure 9: The Growth Factor g for different values of τ and γ (case $\sigma = 1.2$)