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# Uniform Working Hours:

# A Culprit of Structural Unemployment


## Abstract

In this paper, we construct a simple model based on heterogeneity in workers' productivity and homogeneity in their working schedules. This simple model can generate unemployment, even if wages adjust instantaneously, firms are perfectly competitive and can perfectly observe workers' productivity and effort. Unemployment in our model falls upon low-skilled workers, because firms do not find it optimal to hire low-skilled workers when working time across heterogeneous workers must be synchronized, and low-skilled workers on the other hand do not find it attractive working for hours the same length as of high-skilled workers at competitive wages. Our model can also explain the stylized fact that both the number of employees and the number of hours comove with the business cycle. (*JEL classification*: E0, J6.)



*Keywords:* Unemployment; Structural Unemployment; Equilibrium Unemployment; Synchronized Working Hours; Uniform Working Hours; Heterogeneous Labor; Indivisible Labor.



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# 1 Introduction

Unskilled workers are a primary source of structural unemployment. Although both skilled and unskilled workers can be found in unemployment, it has been well documented that the unemployment rate of unskilled workers are much higher and more sensitive to the business cycle than that of skilled workers (e.g., Nickell and Bell, 1996; Bowlus et al., 2001).[1] Previous studies generally attribute the difference in the unemployment rates between the skilled and unskilled workers to the weaker labor demand of unskilled workers. However, this interpretation depends not only on the low elasticity of substitution between skilled and unskilled workers, but also implicitly on the assumption that wages for unskilled workers are stickier than wages for skilled workers.

Textbook explanations for why unskilled workers are more likely to contribute to structural unemployment are also very vague and lacking genuine explanation. A typical statement in this regard can be found in Abel and Bernanke (2001, p95):

> "...unskilled or low-skilled workers often are unable to obtain desirable, long-term jobs. The jobs available to them typically offer relatively low wages and little chance for training or advancement. Most directly related to the issue of structural unemployment is the fact that jobs held by low-skilled workers often don't last long. After a few months the job may end, or the worker may quit or be fired, thus entering another spell of unemployment.... Because of factors such as inadequate education, discrimination, and language barriers, some un-

[1] Also, using the 1993 U.S. Current Population Survey March Supplement, the unemployment rate for prime age male without finishing high school is 13.86%, while the unemployment rate is only 6.24% for those who graduated from high schools.

skilled workers never make the transition to long-term employment and remain chronically unemployed."

The explanation implicit in the quoted message is that low-skilled workers can find only short-term jobs since long-term jobs require skills. According to this explanation, however, unemployment due to the lack of skills should be characterized not as structural but frictional unemployment, because if it is true that short-term jobs end more quickly and more frequently than do long-term jobs, then the major reason for unskilled workers to be unemployed is that they are more frequently forced to enter the process of job search, contributing to frictional unemployment. This is obviously not the conclusion the authors intended to reach in the quoted message, as chronic unemployment due to lack of skills is different from frictional unemployment due to search. This leaves the authors with the only logical conclusion for the higher rates of unemployment of unskilled workers: there are less short-term jobs available than long-term jobs.

Thus, existing studies alike generally attribute, in one way or another, implicitly or explicitly, the difference in the unemployment rates between the skilled and unskilled workers to the weaker labor demand of unskilled workers, without offering an explicit explanation as to why a lower demand for labor leads to a higher rate of unemployment. It is true that unskilled workers are offered lower wages. But it does not explain their being unemployed. According to the standard definition of unemployment by the Bureau of Labor Statistics, "a person is unemployed only if s/he didn't work for the past week but looked for work during the past four weeks" (Able and Bernanke, 2001, p91). Unless wages are stickier for the unskilled workers, equilibrium in the labor market always equates supply and demand.

Hence a lower labor demand or a lower wage rate does not by themselves explain a higher rate of unemployment.

Why low-skilled workers are more likely to be chronically unemployed? In this paper, we develop a simple equilibrium model that conforms with the aforementioned facts. The core of our model is based on the stylized fact that working hours of both skilled and low-skilled workers are highly synchronized. For example, managers, secretaries, technicians, and workers all work during the same hours and on the same day (say from 8:00 am to 5:00 PM in a day and from Monday to Friday in a week).[2]

Costa (2000) has documented that the distribution of daily working hours are highly compressed. For men aged 25-64, the difference between the 90th percentile and the 10th percentile of the daily working hours distribution is only 2 hours in both 1973 and 1991. Moreover, she also finds that the daily working hours of median workers are the same as those of workers at the 10th percentile in the distribution. Using the most recent 1999 US. Current Population Survey (CPS) March Supplement file, we find that only 8.45% of the prime age (24-64) males worked less than 8 hours per day while more than 91% worked 8 hours per day or longer. In addition to biological reasons for such synchronization,[3] , there also exist economic rationales for adopting uniform working schedule. For example, accomplishing a task requires the coordination of many workers of different skill levels during the same period of time (think of the operation of an assembly line). Such arrangement

[2] Costa (2000, p160) claims that the most common pattern of work is to begin at 8 A.M. and end at 5 P.M. from Monday to Friday.

[3] For example, it is only natural that people sleep at night and work during daytime. Hence, working for 4 hours in the morning and 4 hours in the afternoon appears to be a good arrangement.

not only reduces coordination costs but also many other sorts of fixed production costs. This interpretation is consistent with Costa's (2000, p178) arguments. She claims that the egalitarian hours distribution is the result of coordination of work activities within and across firms. Moreover, Costa also points out that the synchronization of leisure-time activities might also be the reason for the compression of daily working hours distribution.

When workers vary in their skills (productivity), highly synchronized working time has important consequences on employment: it creates unemployment. And it turns out that it is the low-skilled workers who is more likely to end up unemployed under synchronized working schedules in a competitive labor market, regardless the elasticity of substitution between skilled and unskilled workers.

Due to the heterogeneity of skills, wage rates (reflecting a worker's marginal productivity) differ across workers. Suppose they share the same propensity to work, workers of different skill levels will opt to supply different hours, with low-skilled workers preferring shorter hours because of lower market wages. The synchronization of working schedule, however, requires that all types of workers work for the same length of time, say 8 hours a day or 40 hours per week. Low-skilled workers may therefore find the required working hours far longer than preferred at the competitive wage rates measured by their marginal productivity. Receiving wages above their marginal productivity from profit-seeking firms is not likely. As a result, unemployment will fall upon low-skilled workers as the utility they receive from working long hours could be even lower than that without working.

Workers can also differ in their propensities to work (preferences). Similar lines of argument can show that workers with low propensity to work will find working unattractive under the scheme of synchronized working hours, hence becoming unemployed.[4] Thus, unskilled workers are more likely to be unemployed than skilled workers given similar propensity to work; and low-propensity workers are more likely to be unemployed than high propensity workers given similar skill levels. Putting together, because of synchronized working hours, unemployment is more likely to fall upon a low-skilled worker unless s/he has an extremely high propensity to work.

Synchronized working scheme thus creates a dilemma: low-skilled workers would choose to work if the wages are high enough to match their utility cost, which few firms would like to offer since they are above the workers' marginal products. Or they could work for much shorter hours at the market determined wages, which is impossible due to the synchronized working schedule. Our model thus predicts that there exists a natural rate of structural unemployment due to synchronized working hours, and that part-time jobs of various duration, if available, are more likely to be occupied by unskilled workers.[5] Also, it is only natural for our model, without resorting to the notions of sticky wage or efficiency wage, to explain that lower demand for labor due to lower productivity is associated with higher rates of unemployment. Our model thus has an important policy implication: a simple solution for reducing structural unemployment is to create more part-time jobs with flexible length

[4] Due to the heterogeneity of propensity to work, equilibrium wage rates will also differ across workers. Suppose they share the same skill level, workers of different atitudes towards working will opt to supply different hours in equilibrium, with low-propensity workers prefering shorter hours because of higher utility cost in sacrifycing leisure. Under the synchronized working schedule, however, low-prepensity workers might find that the required working hours far longer than preferred at the competetive wage rates determined by their productivity. As a result, the utility they receive when employed is lower than that when unemployed.

[5] The 1992 CPS data based on males of age 25-55 shows that in the year of 1991, among part-time workers, 24.25% are high school dropouts whereas among full-time workers that number is only 12.59%. The same data also shows that among high school dropouts, 11.64% work as part-time workers whereas among high school graduates that number is merely 5.32%.

of working hours.

The arguments presented in the paper are kin to the theory of indivisible labor (Hansen, 1985; Rogerson, 1988). According to that theory, unemployment arises because people can only choose either to work or not to work. Hence in equilibrium some individuals may be unemployed. This theory, however, requires lottery to generate equilibrium unemployment because of the homogeneity of workers assumed. Hence it cannot explain why it is the low-skilled workers who are more likely to be unemployed. In our model, equilibrium unemployment exists not because of indivisible labor *per se* – in fact working hours in our model is infinitely divisible, but because of synchronization of labor across heterogenous workers. The synchronization of labor, nevertheless, gives rise to a rigidity in the labor market similar to that of indivisible labor, hence the theory provided in this paper can be viewed as a natural extension of the indivisible labor theory.

The reminder of this paper is organized as follows. Section 2 sets up the model. Section 3 proves the existence of equilibrium and derives the equilibrium unemployment rate – the "natural rate". A calibrated numerical example is given in Section 4. The case of indivisible labor is discussed in Section 5, and section 6 concludes.

## 2 The Model

There is a continuum of agents distributed in the interval $i \in [0,1]$,working for a representative firm (say, a pin factory). They have identical preferences but differ in their skills. Let $p_i$ denote individual $i's$ skill level (productivity), which is non-negative and is decreasing in $i$:

$$\frac{dp_i}{di} < 0. \tag{1}$$

If worker $i$ supplies $n_i$ hours of work, her contribution of output (intermediate goods) is measured by a diminishing returns technology:

$$y_i = p_i f(n_i), \quad f' > 0, f'' < 0; \tag{2}$$

where, for simplicity, the labor's elasticity of output, $\alpha \equiv \frac{f'(N)N}{f(N)}$, is assumed constant. A worker's competitive real wage is determined by her marginal product

$$w_i = p_i f'(n_i). \tag{3}$$

However, we assume that the production technology is accessible to a worker *only* when the factory is open. Output is generated only as long as the factory opens. In particular, we assume that firms in the economy are identical and act competitively, and that the representative firm's final output is an aggregation of intermediate goods but is generated simultaneously with intermediate goods through collaboration of individuals, described by a Leontief technology:

$$Y = \int_{i=0}^{I} p_i f(\min\{n_j\}) di; \quad n_j \leq N \text{ and } j, I \in [0, 1]; \tag{4}$$

where the index $I$ is the rate of employment for the firm, and $N$ is the factory's operation time, both being determined by the firm. The underlying rational of the aggregate technology is that labor is productive *only* when all employees are present simultaneously during a time when the factory is open. That is, coordination of workers of different skills during the same period of time is essential for the production of final output (as in Adam Smith's pin factory). For simplicity, we have ignored the capital in the production function.

Cost minimization from the firm then implies a perfect synchronization of working hours across employees:

$$n_i = N, \quad \text{for all } i \leq I, \tag{5}$$

since spending hours longer than others do in the coordinated production process yields zero marginal product and contributes zero added values to the final output. Thus, the representative firm's profit is given by

$$\Pi = \int_{i=0}^{I} [p_i f(N) - w_i N] \, di, \tag{6}$$

and the real wage the firm pays to worker $i$ is $w_i = p_i f'(N)$.

Hence, at the given wage rate $w_i(N)$, an individual $i$ may not find the job attractive if her utility-maximizing labor supply, $n_i^s(w_i)$, is less than the required working hours, $N$. When that is the case, she would agree to work for $N$ hours if she gets paid more, at a wage rate that is higher than her marginal product, $p_i f'(N)$. Since this is not optimal for the firm, the individual will not be hired, resulting in unemployment.[6] Individuals with desired labor supply greater than $N$ are employed but can nevertheless work for $N$ hours only.

Figures 1 and 2 illustrate the above idea. The upward sloping line in figure 1 represents the labor supply curve that is assumed the same across agents $i \in [0, 1]$. The downward sloping lines represent labor demand curves for agents with different productivity levels. The demand for labor is obviously weaker for lower-productivity workers at any given wage rates. As long as working hours are not required to be synchronized across agents, however,

[6] Without loss of generality and to simplify the analysis, we assume that staying unemployed receives higher utility than working for hours longer than preferred. This assumption can be easily relaxed without affecting the major insights generated (see section 5). We also assume that "part-time" jobs with hours shorter than $N$ are not available in the model. In reality, although part-time jobs do exist, they are not set for arbitrary length of hours. Namely, there still exists synchronized minimum hours people are required to work for part-time jobs. Hence, the major insights of the model still apply to these situations.

competitive equilibrium implies that all agents are employed regardless of their skill levels. Consequently, in equilibrium workers differ only in their working hours and wage rates, not in their employment status. In figure 1, agent $i_0$ works for $n_0$ hours, agent $I$ works for $N$ hours, and agent $i_1$ works for $n_1$ hours, etc.

Synchronization of labor, however, implies that workers cannot work for any arbitrary hours stipulated by their desired labor supply at the competitive wage rates. They must either work for the same length of time as the others do or not to work at all. Consequently, at any given uniform working hours, there may exist unemployment. For example, figure 2 shows that agent $i_0$ is unemployed as her labor supply curve intersects with her corresponding labor demand curve at a location that is below the synchronized working hours $N$. At the required working hours ($N$), agent $i_0's$ utility cost of working ($w$) is larger than the competitive real wage she receives (i.e., $w_0$). In fact, all workers with indices nearby $i_0$ can afford working for only "part-time" jobs (with hours less than $N$) at the competitive wage rates measured by their marginal products, although they are certainly interested in (or looking for) "full-time" jobs that can pay them wages that match their disutility of working. Hence they satisfy the definition of unemployment given by the Bureau of Labor Statistics.

Only agents such as $I$ and $i_1$ are employed workers, where $I$ is also the optimal cut-off point to be determined by profit-seeking firms when choosing the synchronized hours $N$. The competitive real wages paid to the employed workers obviously differ among them due to heterogeneity in productivity. Some of them (say agent $i_1$) may find the wage rates (e.g., $w_1$) so attractive (as it is far above their marginal disutility of working) that they are willing to supply hours much longer than $N$ but can nevertheless work only for $N$ hours. In fact, all employed workers except the cut-off type ($I$) work for wage rates above their labor supply curve.

Note that the wages paid to the employed workers are also higher than their respective market-clearing levels. For example, agent $i_1$ receives real wage $w_1$ from the representative firm while her market-cleaning real wage (determined by supply and demand of labor with respect to all type $i_1$ agents in the economy) is between $w_1$ and $w$. The cut-off agent $I$ is the only exception, with her received real wage ($w$) just equal to her respective market-cleaning level ($w$). This phenominon that received wages are above market cleaning levels are reminiscent of the efficiency-wage literature (see Yellen, 1984, Katz, 1986, and Akerlof and Yellen, 1986 for surveys and references), although arising for entirely different reasons. In our model, equilibrium wage rates are higher than market clearing levels for high-skilled workers because solely of the synchronization of working hours across agents, not because of any incentive problems due to unobservable work effort or productivity. Similarly, the low-skilled workers are unemployed in our model not because of a low elasticity of substitution between high-skilled and low-skilled workers, or of a weaker labor demand for low-skilled workers, but solely because of the synchronization in working hours that results in wages paid to low-skilled workers (e.g., $w_0$) being below their least acceptable levels (i.e., $w$).

Whether unemployment in this model is "voluntary" or "involuntary" depends thus purely on the points of view. It is voluntary in the sense that low-productivity workers (such as those represented by $i_0$) refuse to take a job working for $N$ hours and being paid at the competitive wage, $w_0$, which is below their disutility of working ($w$). It is involuntary in the sense that firms refuse to hire them at the asking wage ($w$) or market prevailing wages.[7]

## 3 Equilibrium

In this section, we prove the existence of equilibrium, derive the equilibrium rate of unemployment, and conduct comparative statics with respect to changes in technology parameters.

**Proposition 1** *If the supply of labor is an increasing function of the real wage (upward sloping), then for any given uniform working hours $N > 0$, there exists a cut-off point $I(N)$, such that worker $i$ is unemployed if $i > I(N)$, and employed if $i \leq I(N)$.*

**Proof.** Let $w_i(n^s)$ be the inverse labor supply function of worker $i$. Since the cut-off worker $I's$ labor supply is exactly the same as her labor demand, we have:

$$p_I f'(N) = w_I(N), \tag{7}$$

where $w_I(N)$ is the inverse labor supply function for agent $I$. Equation (7) determines the cut-off worker's productivity as a function of the synchronized working hours $N$ :

$$p_I = w_I(N)/f'(N), \quad f'(N) > 0. \tag{8}$$

Since productivity $p_i$ is indexed by $i$, the cut-off point $I(N)$ is also determined. ■

For employed workers with $i < I(N)$, the real wage is determined by

$$p_i f'(N) > w_I(N), \tag{9}$$

and their equilibrium labor supply given by $n_i^s = N$. And for the unemployed workers with $i > I(N)$, their equilibrium labor supply is zero.

[7] By definition, a worker is said to be "involuntarily unemployed" if she is willing to work at the market-prevailing wage but cannot find a job. In our model, the market-prevailing wages are the wage rates paid to the employed workers (e.g., anywhere between $w$ and $w_1$). Since the market-prevailing wage rates are above the unemployed workers' marginal products, these workers cannot find jobs.

The cut-off point, $I(N)$, measures the rate of employment given $N$. It is a decreasing function of the synchronized working hours $N$ since equation (4) implies that the cut-off worker's productivity $p$ is increasing in $N$. The intuition is that only higher-productivity workers are willing to work for longer hours at the competitive wage rates, consequently less people are attracted to work as the working hours increase.

**Proposition 2** *Define the cumulative product index as* $P(N) \equiv \int_{i=0}^{I(N)} p_i di \geq 0$, *the elasticity of the cumulative product with respect to the factory operation time* $N$ *as* $\varepsilon(N) \equiv \frac{P'(N)N}{P(N)}$, *and the elasticity of a worker's output with respect to hours as* $\alpha \equiv \frac{f'(N)N}{f(N)}$. *Assuming that* $\alpha$ *is constant. An optimal synchronization time* $N^*$ *exists and is determined by the condition:*

$$-\varepsilon(N) = \alpha. \tag{10}$$

**Proof.** The firm's optimization program is to choose an uniform working hours $N$ to solve:

$$\max_N \Pi = \int_{i=0}^{I(N)} [p_i f(N) - w_i N] \, di = (1-\alpha) f(N) \int_{i=0}^{I(N)} p_i di, \tag{11}$$

where the cut-off point $I(N)$ is determined by (4), and $\alpha \in (0, 1]$ is the constant output elasticity of hours.[8] Using the definition for the cumulative productivity index, $P$, the profit maximization program can then be rewritten as

$$\max_N \Pi = (1-\alpha) f(N) P(N). \tag{12}$$

Without loss of generality, assuming $f(N) = 0$ for $N = 0$ and $I(N) = 0$ for $N \geq M < \infty$. Since the profit function is non-negative over the domain $N \in R^+$ and it takes zero values at the two points, $N = \{0, M\}$, a maximum therefore exists in the open interval $N \in (0, M)$. This proves the existence of $N^*$. The necessary condition for optima is given by

$$f'(N)P(N) + f(N)P'(N) = 0, \tag{13}$$

[8] When $\alpha = 1$, the optimal condition (2.4) can be obtained by maximizing total revenue rather than total profit.

which implies

$$\frac{f'(N)N}{f(N)} = -\frac{P'(N)N}{P(N)}, \tag{14}$$

or $\alpha = -\varepsilon(N)$.■

This optimal condition says that, given that the firm must choose an uniform working hours ($N$) across all types of agents with different skill levels, $N$ should be chosen at the point where the elasticity of cumulative product with respect to hours (the percentage loss of aggregate output due to the loss of the number of employees as working hours increase) is equal to the elasticity of individual output (the percentage gain in individual's production as working hours increase).

To understand this condition, notice that the profit function, $\Pi = (1-\alpha)f(N)P(N)$, is a combination of output due to per-worker quantity ($f(N)$) and an index of aggregate quantity ($P$) of all employees. The quantity per worker increases with hours worked per person ($f'(N) > 0$). The aggregate quantity of all employees ($P$), however, decreases with hours worked per person because longer uniform working hours imply that fewer workers are employed under competitive real wages, hence the aggregate product index, $P = \int_{i=0}^{I} p_i di$, decreases. (Note $P'(N) = P'(I)I'(N) < 0$ since $I'(N) < 0$). Increasing working hours in the factory thus has two opposite effects on total profit:

$$\frac{d\Pi}{dN} = f'(N)P(N) + f(N)P'(N), \tag{15}$$

where the first terms measures the marginal gain given the number of employees, and the second term measures the marginal loss due to a reduction in the rate of employment as working hours increase.

**Proposition 3** *If* $\frac{P''N}{P'} > -(1+\alpha)$, *then the equilibrium is unique.*

**Proof.** Differentiating equation (13) again gives

$$\begin{aligned}\frac{d^2\Pi}{dNdN} &= f''P + f'P' + f'P' + fP'' \\ &= f'P'\left[\frac{f''N}{f'}\frac{P}{P'N} + 2 + \frac{f}{f'N}\frac{P''N}{P'}\right].\end{aligned} \tag{16}$$

Note that $\frac{f'N}{f} = \alpha$, $\frac{f''N}{f'} = \alpha - 1$, $\frac{P'N}{P} = \varepsilon = -\alpha$, and $P' < 0$. Hence,

$$\frac{d^2\Pi}{dNdN} = f'P'\left[\frac{1-\alpha}{\alpha} + 2 + \frac{1}{\alpha}\frac{P''N}{P'}\right] < 0 \tag{17}$$

if and only if $\frac{P''N}{P'} > -(1+\alpha)$. ■

The intuition for the condition, $\frac{P''N}{P'} > -(1+\alpha)$, is that we require that the loss of cumulative product due to the loss of low-skilled workers caused by an increase in the uniform working hours do not accelerate too fast when $N$ increases, meaning that the cut-off function $I$ does not decrease too fast as $N$ increases, or that the inverse labor supply curve in figure 1 is not too steep. Suppose that the condition fails to hold, e.g., the inverse labor supply curve in figure 1 is vertical, then we can imagine multiple or even a continuum of equilibria for the cut-off function $I$.

The optimal rate of employment is then given by $I(N^*)$, in which $N^*$ solve equation (10). A "natural" rate of unemployment in the economy can then be defined as

$$U^{NR} = 1 - I(N^*), \tag{18}$$

which depends on both preferences and technology parameters.

The following two propositions establish the direction of changes in both $I$ and $N$ when technology parameters change. For that purpose, we introduce an aggregate technology shifter $A$ into the workers' production function:

$$y_i = Ap_i f(n_i), \tag{19}$$

so that the cut-off condition becomes:

$$p_I = \frac{w(N)}{Af'(N)}. \tag{20}$$

Note that the cut-off condition implies that $I$ is decreasing in $N$ and that $\frac{\partial I}{\partial A} > 0$ holding $N$ constant (since $\frac{dp_i}{di} < 0$).

**Proposition 4** $\frac{dN}{dA} > 0$ *if* $\frac{\partial^2 P}{\partial I \partial I} \leq 0$. *Namely, the response of* $N$ *to changes in the aggregate technology level is positive if the cumulative product index* $P$ *is non-convex in* $I$.

**Proof.** Rewrite the first-order condition (10) as

$$\alpha P(N, A) = -P_N(N, A)N(A). \tag{21}$$

Totally differentiatin bosth sides of the equation with respect to $A$ gives

$$\alpha \left[ P_N \frac{dN}{dA} + P_A \right] = - \left[ P_{NN} N \frac{dN}{dA} + P_{NA} N + P_N \frac{dN}{dA} \right]. \tag{22}$$

Collecting terms gives

$$\alpha P_A + P_{NA} N = -P_N \left[ \frac{P_{NN} N}{P_N} + (1 + \alpha) \right] \frac{dN}{dA}. \tag{23}$$

Note that uniqueness of equilibrium requires $\left[ \frac{P_{NN} N}{P_N} + (1 + \alpha) \right] > 0$ (Proposition 3). We also know that $P_N < 0$ since $P_N = -\alpha \frac{P}{N}$ as in (21). Hence,

$$\frac{dN}{dA} > 0 \quad \text{if} \quad \alpha P_A + P_{NA} N > 0. \tag{24}$$

But we know that $P_A = \frac{\partial P}{\partial I}\frac{\partial I}{\partial A} > 0$, where $\frac{\partial P}{\partial I} > 0$ and $\frac{\partial I}{\partial A} > 0$ since the cut-off worker's poductivity $p$ in (20) is decreasing in $A$ holding $N$ constant. Therefore, we require only

$$P_{NA} > 0, \tag{25}$$

where $P_{NA}$ satisfies

$$P_{NA} = \frac{\partial}{\partial A}\left(\frac{\partial P}{\partial N}\right) = \frac{\partial}{\partial A}\left(\frac{\partial P}{\partial I}\frac{\partial I}{\partial N}\right) = \frac{\partial^2 P}{\partial I^2}\frac{\partial I}{\partial A}\frac{\partial I}{\partial N} + \frac{\partial P}{\partial I}\frac{\partial^2 I}{\partial N \partial A}, \tag{26}$$

denote it as $P_{II}I_AI_N + P_II_{NA}$, in which we know $I_AI_N < 0$ since $\frac{\partial I}{\partial A} > 0$ and $\frac{\partial I}{\partial N} < 0$; and we also know $P_II_{NA} > 0$ since $\frac{\partial P}{\partial I} > 0$ and $\text{sign}\left(\frac{\partial^2 I}{\partial N \partial A}\right) = -\text{sing}\left(\frac{\partial^2 p}{\partial N \partial A}\right) = +$, where $p$ is the cut-off worker's productivity satisfying $\frac{\partial^2 p}{\partial N \partial A} < 0$ (see equation 20). Therefore, $P_{NA} = P_{II}I_AI_N + P_II_{NA} > 0$ if $P_{II} \leq 0$. ■

This proposition is intuitive since a higher $A$ raises each worker's productivity. However, if the second-order condition, $P_{II} \leq 0$, is not satisfied, then it is possible for $N$ to decrease in response to an increase in $A$, because in that case firm opt to increase the number of workers ($I$) by so much even to reduce hours worked.

**Proposition 5** $\frac{dI}{dA} > 0$ *if the elasticity of equilibrium hours* ($N$) *with respect to* $A$ *satisfies*

$$\frac{dN}{dA}\frac{A}{N} < \frac{1}{\varepsilon_w + 1 - \alpha}, \tag{27}$$

*where* $\varepsilon_w > 0$ *is the wage elasticity of labor supply.*

**Proof.** Totally differentiating the cut-off condition (20) with respect to $A$ yields

$$\begin{aligned} \frac{dp}{dI}\frac{dI}{dA} &= \frac{w'_N A f'_n \frac{dN}{dA} - w\left(f'_N + A f''_{NN}\frac{dN}{dA}\right)}{\left(Af'_N\right)^2} \\ &= \frac{wAf'_N\left[\frac{w'_N}{w} - \frac{f''_{NN}}{f'_N}\right]\frac{dN}{dA} - wf'_N}{\left(Af_N\right)^2}. \end{aligned} \tag{28}$$

Since $\frac{dp}{dI} < 0$, the requirement $\frac{dI}{dA} > 0$ implies that the righ-hand side of the equation must be negative:

$$wAf'_N \left[ \frac{w'_N}{w} - \frac{f''_{NN}}{f'_N} \right] \frac{dN}{dA} - wf'_N < 0, \tag{29}$$

which implies

$$\left[ \frac{w'_N N}{w} - \frac{f''_{NN} N}{f'_N} \right] \frac{dN}{dA} \frac{A}{N} < 1, \tag{30}$$

or $\frac{dN}{dA}\frac{A}{N} < \frac{1}{\varepsilon_w + 1 - \alpha}$ since $\frac{f''N}{f'} = \alpha - 1$. ■

This proposition says that the rate of employment can also positively respond to the aggregate technology shock $A$ simultaneously with $N$ if the supply of hours is sufficiently elastic ($\varepsilon_w$ small). The intuition is that a higher aggregate technology raises the low-skilled workers' productivity, resulting in a higher rate of employment for the low-skilled workers, provided that the corresponding increase in hours is not too big to curtail the positive technology effect on employment rate. This would be the case if the inverse labor supply curve is sufficiently flat or $\alpha$ is sufficiently large so that the cut-off function $I$ is less sensitive to changes in hours.

# 4 A Specific Example

Consider a parameterized model economy. Let $N$ be the uniform working hours, and let the per-worker production function be given by $y_i = Ap_i N^\alpha$, where $A$ represents an aggregate productivity shifter. In addition, let the productivity parameter of individual $i$ follow $p_i = 1 - i, i \in [0, 1]$, and the inverse labor supply function be given by

$$w = \gamma_0 + \gamma_1 N. \tag{31}$$

Suppose the cut-off point for worker's type is $I$, then workers with $i \leq I$ will be employed at wage rates $w_i = \alpha A p_i N^{\alpha-1}$, and workers with $i > I$ will be unemployed. Given $N$, the cut-off point $I$ is determined by condition (20):

$$\alpha A(1-I)N^{\alpha-1} = \gamma_0 + \gamma_1 N, \tag{32}$$

or

$$I = 1 - \frac{(\gamma_0 + \gamma_1 N)}{\alpha A} N^{1-\alpha}. \tag{33}$$

Each employed worker ($i \leq I$) receives the real wage $w_i = \alpha A(1-i)N^{\alpha-1}$, which is greater than the cut-off worker's real wage by the factor $\frac{1-i}{1-I} \geq 1$.

The firm chooses a synchronized working hours $N$ to solve

$$\max_N \int_{i=0}^{i=I(N)} (1-\alpha)AN^{\alpha}(1-i)di = (1-\alpha)AN^{\alpha}(I - \frac{1}{2}I^2), \tag{34}$$

where $I$ is given in (33). The first order condition (equation 10) is

$$\alpha = (\varepsilon_w(N) + 1 - \alpha)\, \frac{(1-I)^2}{I(1-0.5I)}, \tag{35}$$

where $\varepsilon_w > 0$ is the elasticity of wage with respect to hours supply: $\frac{w'N}{w}$.

An implicit solution for the equilibrium rate of employment, $I$, is given by

$$I = 1 - \sqrt{\frac{\alpha}{2\varepsilon_w(N) + 2 - \alpha}}. \tag{36}$$

The solution is implicit because the wage elasticity, $\varepsilon_w(N)$, still depends on equilibrium hours worked $N$:

$$\varepsilon_w = \frac{\gamma_1 N}{\gamma_0 + \gamma_1 N}. \tag{37}$$

It is easy to see that the rate of employment ($I$) and hours worked ($N$) comove together in response to aggregate technology shocks $A$, regardless of $\alpha$. Differentiate both sides of equation (36) with respect to the aggregate technology shifter $A$, we get:

$$\frac{dI}{dA} = \eta \varepsilon_w'(N) \frac{dN}{dA}, \tag{38}$$

where $\eta \equiv \sqrt{\frac{\alpha}{(2\varepsilon_w + 2 - \alpha)^3}} > 0$ and $\varepsilon_w'(N) = \frac{\gamma_0}{(\gamma_0 + \gamma_1 N)^2} > 0$. Hence the direction of changes in $I$ is the same as the direction of changes in $N$ regardless of $\alpha$. For this reason, we can assume $\alpha = 1$ without loss of generality, so as to gain further insight on the comovement of $I$ and $N$. When $\alpha = 1$, the solutions for $I$ and $N$ are simple and explicit:[9]

$$I = 1 - \frac{\gamma_0 + \sqrt{\gamma_0^2 + 3A^2}}{3A}, \quad \gamma_0 \leq A \tag{39}$$

$$N = \frac{-2\gamma_0 + \sqrt{\gamma_0^2 + 3A^2}}{3\gamma_1}, \quad \gamma_0 \leq A. \tag{40}$$

Differentiating both equations with respect to $A$ gives

$$\frac{\partial I}{\partial A} = \frac{\gamma_0 \sqrt{\gamma_0^2 + 3A^2} + \gamma_0^2}{3A^2 \sqrt{\gamma_0^2 + 3A^2}} > 0, \tag{41}$$

and

$$\frac{\partial N}{\partial A} = \frac{A}{\gamma_1 \sqrt{\gamma_0^2 + 3A^2}} > 0. \tag{42}$$

It is a well documented stylized fact in the business cycle literature that both the rate of employment and hours worked are procyclical (e.g., see Cho and Cooley, 1994). This is consistent with the predictions of our model. A lower period of aggregate productivity induces profit-seeking firms to adjust downward both the number of employees and the number of

[9] The firm's profit is zero when the technology is linear. In that case, the total revenue rather than the total profit is being maximized.

hours worked per person. Since it is the low-skilled workers who are exposed to the layoff risk when employment rate decreases, the unemployment rate of low skilled workers is therefore more sensitive to the business cycle than that of skilled workers.

The relative magnitude of adjustment in the two different margins (number of workers and number of hours) in response to aggregate disturbances depend crucially on the slope of the labor supply curve and on the magnitude of the disturbance itself:

$$\frac{\partial I/\partial A}{\partial N/\partial A} = \gamma_1 \left( \frac{\gamma_0 \sqrt{\gamma_0^2 + 3A^2} + \gamma_0^2}{3A^3} \right). \tag{43}$$

A flatter labor supply curve (a larger $\gamma_1$) or a lower propensity to work (a larger $\gamma_0$) implies more volatile employment rate relative to hours worked during the business cycle. Given the status quo of the labor supply curve, however, a lower level of aggregate productivity implies relatively smaller reactions from hours worked to business cycle shocks than that from employment rate. This prediction is interesting as it indicates that developed economies would have a higher volatility in hours worked but a lower volatility in employment rate than underdeveloped economies, as the impact of technology shocks being mostly absorbed by hours worked in economies where aggregate productivity level is high.

Another interesting case to consider is the relationship between $I$ and $\alpha$. To facilitate the discussion, assume that $\gamma_0 = 0$. Hence $\varepsilon_w = 1$ and equation (36) becomes

$$I = 1 - \sqrt{\frac{\alpha}{4 - \alpha}}. \tag{44}$$

The equilibrium employment rate is thus a decreasing function of the technology parameter $\alpha$. Notice that $\alpha$ measures not only the labor's elasticity of output for an individual worker, but also the elasticity of substitution between skilled and low-skilled workers as is clear from the aggregate production function (4).

Previous studies attribute the higher rate of unemployment of low-skilled workers to a low elasticity of substitution between high-skilled and low-skilled workers. Such conventional interpretations make sense only if hours worked are not synchronized so that workers with different skill levels can work for different hours. Profit maximization would then induce firms to replace high-skilled workers with low-skilled workers so as to cut down wage costs, if the elasticity of substitution between these different types of workers is high.

Equation (44) indicates, however, that such intuitions are not necessarily correct when working time is synchronized across workers of all skills. We see here that unemployment of low-skilled workers increases as the elasticity of substitution between high-skilled and low-skilled workers increases. This is so because when low-productivity workers are no longer essential in the synchronized production process such that their work can be substituted out (replaced) by high-productivity workers, profit maximization would induce the firm to extend working hours to further utilize the productivity of high-skilled workers, resulting in higher rate of unemployment (as the low-skilled workers cannot afford working for hours the same length as the high-skilled workers).

*The Case of Indivisible Labor* – In the above discussions, we have considered synchronized working schedule due solely to cost minimization from the firm side. But synchronization of working hours can also be due to biological reasons (e.g., it is only natural for human body to sleep during night and work during day). Suppose that for biological reasons only two discrete choices of hours exist: either working for $\bar{N}$ hours not to work at all. What are the consequences of indivisible labor on employment when workers are heterogenous in their skill levels?

Assuming $\alpha = 1$ for simplicity, the firm's profit-maximization program then becomes

$$\max_{N} \ \Pi = AN \int_{i=0}^{i=I(N)} p_i di \tag{45}$$

subject to

$$N = \{0, \hat{N}\}. \tag{46}$$

The solution is obviously $N = \hat{N}$, since the profit is zero when $N = 0$. Hence equation (20) or (33) suffice for determining the equilibrium level of employment in the model, which is

$$I = \max\left(0, 1 - \frac{\gamma_0}{A} - \frac{\gamma_1}{A}\hat{N}\right), \quad \gamma_0 < A. \tag{47}$$

Note that both the aggregate technology ($A$) and the length of working hours ($\hat{N}$) affect the equilibrium rate of employment. Since hours are indivisible, adjustment of output in response to aggregate technology shock ($A$) falls entirely upon the rate of employment $I$. In particular, the employment rate decreases as $A$ decreases. Again, in this model, unemployment falls upon the low-skilled workers, not because of a low elasticity of substitution between skilled and unskilled labor, but because of synchronization of labor. Obviously, the model cannot explain why hours also respond to business cycle disturbances. It is hence more likely that both biological factors, institutional factors, and production coordination all play a role in synchronizing people's working schedules. For example, biological or institutional factors allows people to work for 8 hours per day and 40 hours per week on average, but for reasons of production coordination and profit maximization, firms can adjust the actual working hours up or down (say between 35 and 45 hours per week) in response to business conditions.

## 5 Robustness

In this section, we prove that explicitly taking into account workers' utility function in determining their labor supply behavior does not alter our conclusions reached in the paper, as long as the utility function is consistent with an upward-sloping labor supply curve (i.e., the substitution effect dominates the income effect). The crucial thing to check is that such consideration does not affect the main features of the cut-off function ($I$) which was determined previously by the condition:

$$p_I f'(N) = w_I(N), \tag{48}$$

where the right hand side is the real wage determined by the marginal product, and the left-hand side is the worker's inverse labor supply function. There are two major properties implied by this condition and were used in the paper to prove propositions (2)-(5). The first property is that $\frac{dp_I}{dN} > 0$, which also implies $\frac{dI}{dN} < 0$ since $p_I$ is decreasing in $I$. The second property is that the cumulative productivity index, $P \equiv \int_{i=0}^{I(N)} p_i di$, is decreasing in $N : \frac{dP}{dN} > 0$. This is a natural consequence of $\frac{dI}{dN} < 0$.

Let the index number that solves condition (48) be $I_1$. The assumption behind condition (48) for being a marginal condition is that workers are better off by not working than working for any hours longer than desired. We show here that relaxing this assumption does not change anything qualitatively except that the newly determine cut-off point ( call it $I_2$) lies above the original cut-off point determined from (48), $I_1$. The intuition for $I_2 > I_1$ is that workers with indices immediately above $I_1$ (i.e., their labor demand curves lie immediately below worker $I_1$) may still find working for $N$ hours more attractive than not working at all, although $N$ exceeds their desired labor supply. Workers with indices $i > I_2$, however, definitely find working not attractive as the utility received from working for $N$ hours is less than that from not working at all.

Consider the utility function for worker $i \in [0, 1]$ :

$$u(c_i, 1 - n_i) = u(\gamma + w_i n_i, 1 - n_i), \quad u_1', u_2' > 0, u_1'', u_2'' < 0; \tag{49}$$

where $c_i = \gamma + w_i n_i$ is consumption, $w_i$ is the real wage, $n_i$ is the hours worked, 1 is the time endowment, and $\gamma > 0$ is the non-human wealth. The optimal labor supply is determined by

$$u_c(c_i, n_i)\frac{dc}{dn} = u_{1-n}(c_i, n_i), \tag{50}$$

or

$$u_c\left(\gamma + w_i n_i, 1 - n\right) w = u_{1-n}\left(\gamma + w_i n_i, 1 - n\right). \tag{51}$$

Let $w_i = p_i f'(n_i)$, $i = I_1$, and $n_i = N$, (51) becomes exactly the condition (48) which was used to determine the cut-off point $I_1$ in the paper, provided that the inverse labor supply function $w(n_i)$ as an implicit solution to (51) exists and is unique. But (51) is no longer the right condition for determining the cut-off point in the current case, as workers with $i > I_1$ may still prefer working to not working. The right condition is given by

$$u\left(\gamma + p_i f'(N)N, 1 - N\right) \geq u(\gamma, 1). \tag{52}$$

Namely, facing the synchronized working hours $N$, the individual will choose to work if and only if the utility received from working for $N$ hours exceeds the utility of not working at all. Hence, a cut-off point $I_2(N)$ exists and is determined implicitly by the equation,

$$u\left(\gamma + p_{I_2} f'(N)N, 1 - N\right) = u(\gamma, 1). \tag{53}$$

Note that the following inequalities hold:

$$u_c(c_i, N)\frac{dc}{dN} > u_{1-n}(c_i, N), \quad \text{if } i \le I_1, \tag{54}$$

$$u_c(c_i, N)\frac{dc}{dN} < u_{1-n}(c, N), \quad \text{if } i > I_1; \tag{55}$$

namely, for a worker $i$ ($\le I_1$) whose desired labor supply is greater than that of the marginal worker $I_1$ which is determined by (51), increasing her working hours beyond $N$ increases her utility; and for a worker ($i > I_1$) whose desired labor supply is less than that of the marginal worker $I_1$, increasing her working hours beyond $N$ decreases her utility.

Given that $I_2 > I_1$, we thus have

$$u_c(c_{I_2}, N)\frac{dc}{dN} < u_{1-N}(c_{I_2}, N). \tag{56}$$

Now totally differentiating the cut-off equation (53) with respect to $N$, realizing that $I = I(N)$ and $f'(N)N = \alpha f(N)$, gives

$$u_c\left(c_{I_2}, N\right)\left(\frac{dp_{I_2}}{dN}\alpha f(N) + \frac{dc}{dN}\right) = u_{1-N}\left(c_{I_2}, N\right). \tag{57}$$

Comparing (57) with (56) immediately gives

$$\frac{dp_{I_2}}{dN} > 0. \tag{58}$$

Since $p_I$ is decreasing in $I$, we also have

$$\frac{dI_2}{dN} < 0. \tag{59}$$

This completes the proof.[10]

[10] The other property implied by (21) and was used to prove propositions (4) and (5), $\frac{\partial I}{\partial A} > 0$, is easy to check from (23).

Figure 3 illustrates the idea fully, where agent $I_1$ has desired labor supply just equal to her labor demand at $N$, but no longer determines the cut-off employment rate. The cut-off employment rate is determined instead by agent $I_2$ who is just indifferent between working for $N$ hours and not working at all.

## 6 Conclusion

In this paper, we proposed a model to explain the structural unemployment, without resorting to conventional labor-market frictions such as sticky wages and imperfect information on workers' productivity. The model is built on two commonly observed facts. First, working schedules are highly synchronized across labor. For example, in the 1890s, about 47% of male workers in the US. worked 10 hours per day and "the most common pattern was for work to begin at 7:00 A.M, and end at 5:30 P.M. with a 30-minute break for lunch (Costa 2000, p159)." One hundred years later, the degree of synchronization had become even stronger. In 1991, 57% of male workers in the US. reported that they worked 8 hours per day, and the most common pattern was from 8:00 A.M. to 5:00 P.M. (Costa 2000, p160). The other observation is that workers are heterogeneous in their skill levels, some are more productive than others even in the same department working on similar tasks. Difference in productivity implies difference in wages, which in turn implies difference in hours supply. Synchronization of labor, however, requires the same working hours.

As a result, unskilled workers are unemployed and the unemployment rate of low-skilled workers is expected to be higher than that of the skilled workers. The intuition is that the 8 hours per day schedule is too long for the unskilled workers at a wage rate measured by marginal productivity. Consequently, unskilled workers find such jobs unattractive on the one hand (even though they might find the wage rate attractive at shorter hours), and firms do not find it optimal on the other hand to hire them (at wages sufficing for compensating their disutility of working).

Our analyses showed that that the rate of employment and the average hours worked per worker can both respond to business cycle shocks and can do so in the same direction. During economic booms due to positive aggregate technology shocks, not only the average working hours (synchronized across workers) are longer, but more people also are absorbed into the work force from the low tail of the skill spectrum. The converse is also true during economic recessions. As a result, low-skilled workers are more sensitive to the business cycle than skilled workers, as is observed in the US economy.

Since the rate of employment negatively depends on the length of working hours, regardless labor being indivisible or not, an obvious policy implication of the models examined in this paper is that reducing working hours can boost employment, other things equal. The reason is that more low-skilled workers are able to find jobs when the working time shortens. French government, for example, has been pushing for a 35-hour work week against the traditional 40-hour work week in an attempt to reduce unemployment. The welfare gain of such policy, however, is not clear, since a shorter working time could also cause loss of aggregate output, especially with respect to the high productivity workers. To carefully evaluate the welfare consequence of adopting shorter working week in an environment like ours from a social point of view is a task we want to push in future works.

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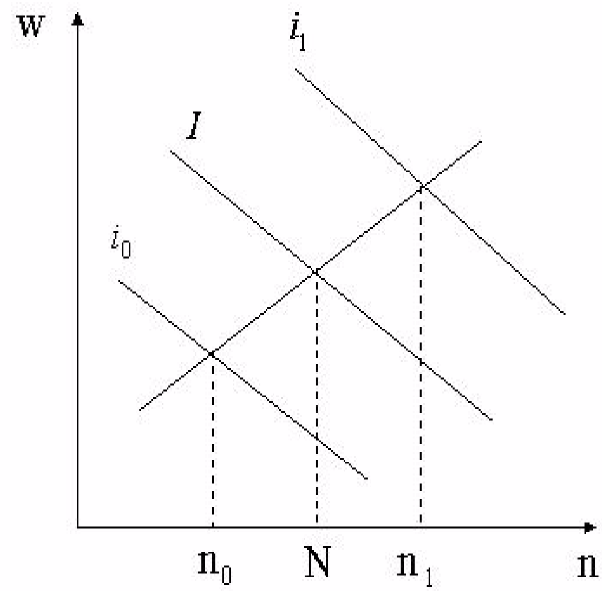


Figure 1. Labor market equilibrium when hours are not synchronized, where agent $i_0$ works for $n_0$ hours, agent $I$ works for $N$ hours, and agent $i_1$ works for $n_1$ hours.

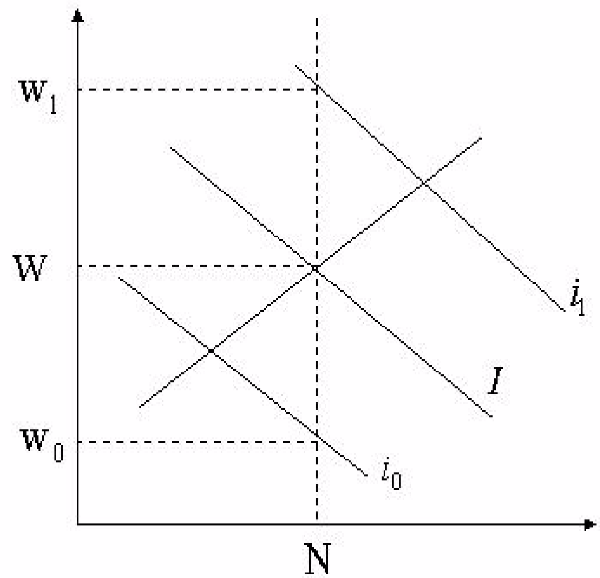


Figure 2. Labor market equilibrium when hours are synchronized at $N$, where worker $i_0$ is unemployed, worker $i_1$ is employed, and worker $I$ is the cut-off type.

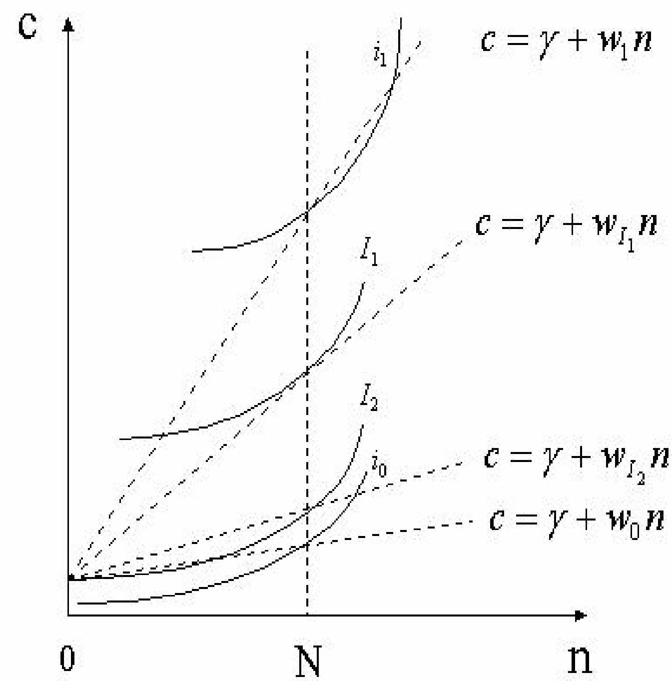


Figure 3. Labor market equilibrium when hours are synchronized, where worker $i_0$ is unemployed, workers $I_1$ and $i_1$ are employed, and worker $I_2$ is the cut-off type.