International Capital Flows and Liquidity Crises

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Abstract

This paper develops a two-country general equilibrium model which analyzes the composition of equity flows (direct vs portfolio) across two countries in the presence of heterogeneity in liquidity risk and asymmetric information about the investment productivity. Direct investment is characterized by higher profitability and private information about investment productivity, while portfolio investment provides greater risk diversification. I demonstrate that there is a possibility of multiple equilibria due to strategic complementarities in choosing direct investment. I analyze the effect of an increase in the liquidity risk on the composition of foreign investment. If there is a unique equilibrium then higher liquidity risk leads to a higher level of foreign direct investment (FDI). If, however, there are multiple equilibria then higher liquidity risk may leads to the opposite effect: a decine of FDI. In this case, an outflow of FDI is induced by self-fulfilling expectations. The ambivalent effect of increased liquidity risk on equity flows can be related to empirically observed patterns of foreign investment during liquidity crises.

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1 Introduction

The two major types of international equity holdings are foreign direct investments (FDI) and foreign portfolio investments (FPI). Liquidity crises may be associated with an outflow foreign investment, including FDI. For example, all types of inward foreign investment into Latin America declined after the 1982 crisis.[1] Some theoretical literature argues that a liquidity crunch may induce and aggravate a real crisis, leading to an exit of foreign investors.[2] However, there is evidence that some liquidity crises have been accompanied by an outflow of FPI and a simultaneous inflow of FDI, e.g., the 1994 crisis in Mexico and the 1998-1999 crisis in South Korea.[3] This behavior reflects the *fire-sale FDI* phenomenon when domestic companies and assets are acquired by foreign investors at fire-sale prices. The following question emerges: why during some liquidity crises there is an inflow of FDI while some others are accompanied by an outflow of FDI?

In this paper, I present a model that suggests an explanation why FDI flows exhibit such divergent behavior during the crises. This paper develops a two-country general equilibrium model which analyzes the composition of investment (direct vs portfolio) across two countries in the presence of liquidity risk and asymmetric information about the investment productivity.

The characteristic feature of direct investment is concentrated ownership which provides access to private information about investment productivity[4] and results in a more efficient management. Portfolio investment is characterized by dispersed ownership which allows for risk diversification and greater liquidity. Taking advantage of the inside information, direct investors may sell low-productive investments and keep the high-productive ones under their ownership. This generates the "lemons"[5] problem: the buyers do not know whether the investment is sold because of its low productivity or due to the exogenous liquidity shock. Therefore, due to this information asymmetry, there is a discount on the prematurely sold

[1] Lipsey [22].

[2] Aghion, Bacchetta, and Banerjee [2], Chang and Velasco [9], and Caballero and Krishnamurthy [8].

[3] Krugman [21], Aguiar and Gopinath [3], Acharya, Shin, and Yorulmazer [1]

[4] Razin and Sadka [25], Klein, Peek, and Rosengren [20], Kinoshita and Mody [19], Bolton and von Thadden [7], Kahn and Winton [17]

[5] Akerlof (1970)

direct investment (relative to the portfolio investment). This assumption is consistent with the evidence that there is a negative premium associated with seller-initiated block trades.[6] The main implication of this information-based trade-off is that the choice between direct and portfolio investment will be linked to the likelihood with which investors expect to get a liquidity shock (Goldstein and Razin [13]).

In my model, the agents have the Diamond-Dybvig [10] type preferences. Agents live for 1 or 2 periods, depending on whether they receive a liquidity shock in period 1. The probability of an investor receiving a liquidity shock is country-specific. I refer to this probability as a liquidity risk. At date 0, investors choose how much to invest into risky long-term projects in each of the two countries, as well as the ownership type for each project (direct or portfolio). In period one, idiosyncratic liquidity shocks are realized and, subsequently, risky investments are traded in the financial market. All investment projects pay off in the second period.

The equilibrium prices of direct and portfolio investments depend not only on their expected payoffs but also on investors' liquidity preferences and uncertainty about the investment productivity. There are two types of equilibria. In the first type, only investors from the country with a lower liquidity risk choose to hold direct investment. In the second type, investors from both countries hold direct investments. In this case, there are strategic complementarities in choosing direct investment. This generates a possibility of multiple equilibria through the self-fulfilling expectations. If economy is in the unique equilibrium range, the country with a higher liquidity risk will have a higher level of inward foreign investment and, in particular, a higher share of FDI. Also, the country with a larger uncertainty about investment productivity will attract more FDI relative to FPI since the benefits from private information are larger.

I consider the effect of an increase in a liquidity risk on the composition of foreign investment. Such an increase results in the dry up of market liquidity as more investors have to sell their asset holdings. At the same time, it becomes more likely that if a direct investment is sold before maturity, it is sold due to the exogenous liquidity needs rather than

[6] Holthausen, Leftwich, and Mayers [16], Easley, Kiefer and O'Hara[11], Easley and O'Hara[12], Keim and Madhavan [18]

an adverse signal about investment productivity. This reduces the adverse selection problem and therefore results in a smaller discount on direct investments. This effect captures the phenomenon of fire-sale FDI during liquidity crises. If economy is in the unique equilibrium then higher liquidity risk leads to a higher level of FDI. However, if there are multiple equilibria then FDI may decline as the liquidity risk becomes higher. In this case, an outflow of FDI is induced by the self-fulfilling expectations.

There are two possible interpretations of the liquidity risk in my model. One is the probability of a liquidity crisis that is unrelated to fundamentals of the economy. In fact, recent financial crises exhibit a large liquidity run component while the underlying macro fundamentals are not necessarily weak. Another interpretation is a measure of financial market development. In more developed financial (credit) markets it is easier for agents to borrow in case of liquidity needs, and therefore the probability of investment liquidation is smaller, whereas in developing and emerging countries access to the world capital markets is limited. So a country with a low liquidity risk can be viewed as a developed economy, and a country with a high liquidity risk can be viewed as a developing or emerging economy. In addition to a lower liquidity risk, a developed country can be characterized by a higher expected payoff (adjusted for risk) and smaller benefits from private information of FDI.

In the model, the ambiguous effect of an increase in the liquidity risk on the capital flows corresponds the empirically observed pattern of FDI during liquidity crises. The positive effect of a higher liquidity risk on the inward FDI is consistent with the evidence documented by Krugman [21], Aguiar and Gopinath [3], and Acharya, Shin, and Yorulmazer [1]. Krugman [21] notes that the Asian financial crisis has been accompanied by a wave of inward direct investment. Furthermore, Aguiar and Gopinath [3] analyze data on mergers and acquisitions in East Asia between 1996 and 1998 and find that the liquidity crisis is associated with an inflow of FDI. Moreover, Acharya, Shin, and Yorulmazer [1] observe that FDI inflows during financial crises are associated with acquisitions of controlling stakes. At the same time, my model provides a possibility of a decrease in FDI through self-fulfilling expectations. This possibility is in line with the empirical evidence[7] as well as theoretical literature that associates liquidity crises with an exit of investors from the crisis economy

[7] Lipsey [22].

even if there are no shocks to fundamentals.[8]

My results are consistent with the empirical findings of Hausman and Fernandez-Arias [14] that countries that are less financially developed and have weaker financial institutions tend to attract more capital in the form of FDI. Moreover, my model can explain the phenomenon of bilateral FDI flows among developed countries, and one-way FDI flows from developed to emerging countries.[9]

The paper is organized as follows. Section 2 and 3 presents the theoretical model and its analysis. Section 4 characterizes the equilibrium. Section 5 discusses the effect of change in liquidity risk on the foreign investments. Section 6 concludes the paper. All proofs are delegated to the Appendix.

2 Related Literature.

My paper is related to several papers in the literature. My model builds on the assumption of information-based trade-off between FDI and FPI which have been introduced by Goldstein and Razin [13]. They study the choice between FDI and FPI by risk-neutral investors in the partial equilibrium setting, and show that investors with higher liquidity needs are more likely to choose FPI over FDI. Furthermore, they examine the implications of production costs, transparency and heterogeneity of foreign investors on the investment choice.

Krugman [21] points out the fire-sale FDI phenomenon and offers two possible modeling approaches. One is based on moral hazard and asset deflation. The liabilities of financial intermediaries are perceived as having an implicit government guarantee, and therefore subject to moral hazard problems. The excessive risky lending inflates the asset prices, which makes the financial intermediaries seem sounder than they actually are. During a crisis, falling asset prices make the insolvency of intermediaries visible, leading to further asset deflation. Krugman argues that this approach explains the vulnerability of Asian economies to a self-fulfilling crisis. Another explanation is based on disintermediation and liquidation, attributing the crisis to a run on financial intermediaries. Such run can be set off by self-fulfilling expectations.

[8] Aghion, Bacchetta, and Banerjee [2], Chang and Velasco [9], and Caballero and Krishnamurthy [8].

[9] Razin [26]

Acharya, Shin, and Yorulmazer [1] address the fire-sale FDI phenomenon from the firm's prospective. They provide an agency-theoretic framework in which the loss of control by domestic managers together with the lack of domestic capital results is a transfer of ownership to foreign firms during a crisis.

The following papers link financial crises and liquidity through models of self-fulfilling creditors run. Chang and Velasco [9] place international illiquidity at the center of financial crises. They argue that a small shock may result in financial distress, leading to costly asset liquidation, liquidity crunch, and large drop in asset prices. Caballero and Krishnamurthy [8] argue that during a crisis self-fulfilling fears of insufficient collateral may trigger the capital outflow.

3 Model

I consider a model with 2 countries: A and B. There is a continuum of agents with an aggregate Lebesgue measure of unity. Let α be the proportion of investors living in country A and the rest live in country B. There are 3 time periods: $t = 0, 1, 2$. There is only one good in the economy, and in period zero, all agents are endowed with one unit of good that can be consumed and invested.

3.1 Investment technology

Agents have access to two types of constant returns technology. One is a storage technology (safe asset), which has zero net return: one unit of safe asset pays out one unit of safe asset in the next period. The safe asset is the same in both countries, and I will refer to it as "money". The other type of technology is a long-term risky investment project. In period two, the investment project (risky asset) has a random payoff of $R > 1$ per unit of investment which represents idiosyncratic investment productivity. It yields nothing at date $t = 1$. Figure 1 summarizes the payoff structure.

	payoff		
time	0	1	2
safe asset	1	1	1
investment	1	0	R

Figure 1. Payoff structure.

There is a continuum of investment projects available in each country. The investment productivity realizations are independent across projects and across countries. The investment productivity in country $k \in \{A, B\}$ is a normally distributed random variable $R_k \sim N(\overline{R}_k, \sigma_k^2)$ with mean $\overline{R}_k$ and variance σ_k^2. The productivity variance σ_k^2 is a random variable that takes a high value σ_{kh}^2 with probability π_k and a low value σ_{kl}^2 with probability $(1 - \pi_k)$. All parameters of the productivity distribution are country-specific, with $\overline{R}_k$ representing the expected profitability of investment project and σ_k^2 capturing the investment risk in country k.

Agents can invest their endowment in investment projects at home (domestic investment) and abroad (foreign investment).

3.2 Preferences

Investors are assumed to have Diamond-Dybvig type of preferences:

$$U(c_1, c_2) = \lambda u(c_1) + (1 - \lambda)u(c_2) \tag{1}$$

where λ is the probability of receiving a liquidity shock at date $t = 1$ and c_t is the consumption at dates $t = 1, 2$. In each period, investors have mean-variance utility

$$E\left[u(c_t)\right] = E\left[c_t\right] - \frac{\gamma}{2} Var\left[c_t\right] \tag{2}$$

with γ representing the degree of risk aversion[10].

[10] Maccheroni, Marinacci, and Rustichini [23] show that the mean-variance preferences is the special case of variational preferences, which is a representation of preferences for decision making under uncertainty. The mean-variance preferences have been used in the finance literature (Van Nieuwerburgh and Veldkamp (2008)).

The probability of receiving a liquidity shock in period one is country-specific: investors in each country $k \in \{A, B\}$ have the same probability λ_k. This probability (λ_k) captures the liquidity risk in a given country. Investors who receive a liquidity shock have to liquidate their risky long-term asset holdings and consume all their wealth in period one. So they are effectively early consumers who value consumption only at date $t = 1$. The rest are the late consumers who value the consumption only at date $t = 2$. Since there is no aggregate uncertainty, λ_k is also a fraction of investors that have been hit by a liquidity shock in country k.

Without loss of generality, I assume that country A has a smaller liquidity risk than country B, i.e., $\lambda_A < \lambda_B$.

Investors choose their asset holdings to maximize their expected utility.

3.3 Direct and Portfolio Investments

In period $t = 0$, agents decide how much of their endowment to invest in long-term risky investment projects. In a given country k, an agent can either invest directly in a single project, or become a portfolio investor investing in up to N_k projects[11] The expected payoff of a direct investment $\left(\overline{R}_{dk}\right)$ is higher than the expected payoff of a portfolio investment $\left(\overline{R}_{pk}\right)$. In period one, direct investors in country k observe a signal about their investment productivity: the true value of σ_k^2. Henceforth we will refer to it as the productivity signal. Portfolio investors do not observe such productivity signal. Therefore, portfolio investors use the Bayesian updating on the productivity variance in country k: $\overline{\sigma}_k^2 \equiv (1 - \pi_k)\, \sigma_{kh}^2 + \pi_k \sigma_{kl}^2$. The decision to become direct or portfolio investor is country specific, i.e., it is possible to become a direct investor in one country, and a portfolio investor in another.

The advantage of direct investment is private information about the idiosyncratic investment productivity. However, it is public knowledge which investors are informed. This generates the adverse selection problem: it is not known whether direct investors sell due to a liquidity shock or because they have observed the bad productivity signal (high variance) about the investment productivity. Therefore, there is an information discount on the price

[11] Due to the mean-variance preferences and idiosyncratic productivity, a portfolio investor will always choose to invest into the maximum number of projects allowed.

of direct investment at $t = 1$.

In this setting, the efficiency of direct over portfolio investment is reflected by higher expected productivity of the former $\left(\overline{R}_{dk}\right)$ relative to the expected productivity the latter $\left(\overline{R}_{pk}\right)$. Also, the diversification benefits from portfolio investment are captured by allowing to invest in multiple projects in one country which is effectively equivalent to reducing the investment variance by the factor of N_k. I abstract from the other gains of management control such as possibility of restructuring[12] that may lead to an increase of investment payoff from $t = 1$ to $t = 2$.

I show that the decision between direct and portfolio investment depends on the probability of getting a liquidity shock and uncertainty about the investment productivity. Agents are more likely to choose direct investment if they are less likely to receive a liquidity shock and if the benefits of private information are larger.

In period one, the liquidity shocks are realized, direct investors observe a signal about the productivity of their investment, and trading in financial market occurs. Investors who receive a liquidity shock supply their asset holdings inelastically. In addition, direct investors who have not received a liquidity shock but observe a bad productivity signal can sell their investments. The buyers are investors who have not received a liquidity shock (Allen and Gale [4] and Bhattacharya and Nicodano [6]). Figure 2 represents the time line of the model.

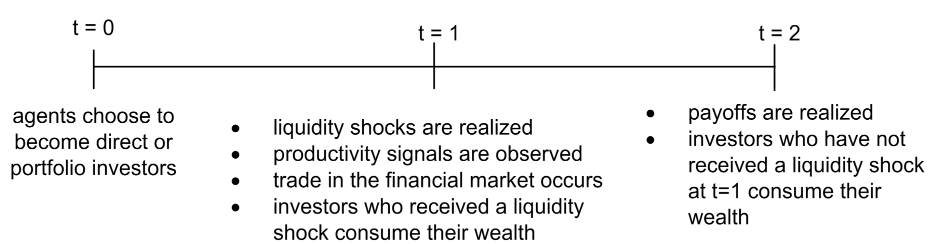

Figure 2. Time line.

[12] The trade-off between efficiency gains related to corporate control and liquidity have been addressed by Bolton and von Thadden [7], Maug [24], and Holmstrom and Tirole [15].

4 Investors' decision problem

Agents face the following two-stage decision problem. At date $t = 0$, an agent decides whether to become a direct or a portfolio investor in each country and, correspondingly, how much of their endowment to invest in the risky long-term projects. At date $t = 1$, investors who have not received a liquidity shock, decide how much of the long-term assets they would like to buy.

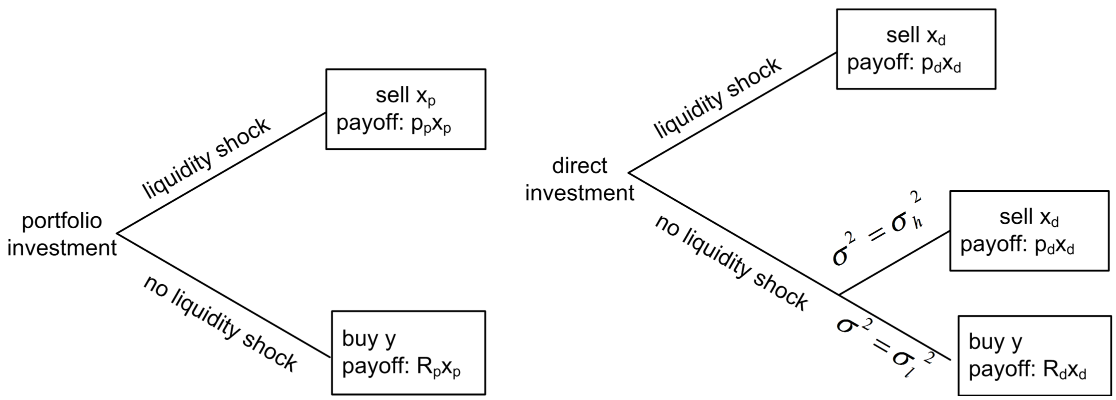

Figure 3. Investors' decision problem

In period one, investors are restricted to buying either direct or portfolio investment in each country. This assumption is imposed to prevent further risk diversification. Therefore, in the equilibrium the buyer should be indifferent between buying the direct investment or portfolio investment. Note that at period $t = 1$ there is no advantage of private information.

Let $\delta_{ik} \in [0, 1]$ be the fraction of direct investors from country i investing in country k where $i, k \in \{A, B\}$. Then the fraction of direct investors investing in country k is $\delta_k = \alpha\delta_{Ak} + (1 - \alpha)\delta_{Bk}$.

The investor who buys a risky asset from a direct investor in period $t = 1$, does not know whether it is sold due to the liquidity shock or because of the high productivity variance. The buyers believe that direct investors in country k will receive a liquidity shock with probability λ_d such that

$$\lambda_d = \frac{\alpha\delta_{Ak}\lambda_A + (1 - \alpha)\delta_{Bk}\lambda_B}{\alpha\delta_{Ak} + (1 - \alpha)\delta_{Bk}} \tag{3}$$

. Therefore, the buyers believe that with probability $\frac{\lambda_d}{\lambda_d + (1-\lambda_d)\pi}$ direct investment in country k is sold due to a liquidity shock, and with probability $\frac{(1-\lambda_d)\pi}{\lambda_d + (1-\lambda_d)\pi}$ it sold because the high productivity variance. Hence, the buyers believe that the variance of the asset sold by a direct investor is σ^2_{kh} with probability $\frac{(1-\lambda_d)\pi}{\lambda_d+(1-\lambda_d)\pi}$ and $\overline{\sigma}^2_k$ with probability $\frac{\lambda_d}{\lambda_d+(1-\lambda_d)\pi}$. Using Bayesian updating, the variance of the prematurely sold direct investment in country k is $\widetilde{\sigma}^2_k \equiv \frac{(1-\lambda_d)\pi}{\lambda_d+(1-\lambda_d)\pi}\sigma^2_{kh} + \frac{\lambda_d}{\lambda_d+(1-\lambda_d)\pi}\overline{\sigma}^2_k$, and its mean is $\overline{R}_{dk}$.

Portfolio investors do not observe a productivity signal, hence they only sell their investment if they are hit by a liquidity shock. Therefore, the productivity R_{pk} of the prematurely sold portfolio investment in country k has mean $\overline{R}_{pk}$ and variance $\overline{\sigma}^2_k$. Since investment productivity is idiosyncratic, there is no updating on the productivity variance of portfolio investment based on the direct investors selling.

Several assumptions[13] are imposed on the parameters $\left(\overline{R}_{dk}, \overline{R}_{pk}, \sigma^2_{kl}, \sigma^2_{kh}, \pi_k, N_k\right)$ of the productivity distribution for each country k:

Assumption 1. At $t = 0$, all investors invest some but not all of their endowment in risky projects.

Assumption 2. At $t = 1$, investors demand for risky asset less than his money holdings.

Assumption 3. In the absence of private information benefits, investors are indifferent between holding direct and portfolio investment.

Assumption 3 implies that benefits from diversification are perfectly offset by benefits from efficiency.

The investors from country $i \in \{A, B\}$ choose their optimal investment holding x^i_k in country $k \in \{A, B\}$ at date $t = 0$ to maximize their expected utility. Denote by x^i_{dk} the demand for direct investment at $t = 0$ by an investor from country i where $i, k \in \{A, B\}$. Similarly, denote by x^i_{pk} the demand for portfolio investment at $t = 0$ by an investor from country i where $i, k \in \{A, B\}$ such that $x^i_{pk} = N_k x^i_k$.

At date $t = 1$, uncertainty about the liquidity shock is resolved and all investors observe the total proportion of early consumers. The prices of direct and portfolio investments in country $k \in \{A, B\}$ are denoted by p_{pk} and p_{dk}, respectively. In period $t = 1$, y_{pk} and y_{dk} denote the demand for direct and portfolio investment in country k. Since the liquidity shock is realized at date $t = 1$, the demands y_{pk} and y_{dk} are the same for investors from both countries (so superscript i can be omitted).[14]

[13] See Appendix A.1

[14] The demand for risky asset at $t = 1$ is independent from investment demand at $t = 0$ due to the mean-

The demand for direct and portfolio investments in period one are given by

$$y_{pk} = \frac{\overline{R}_{pk} - p_{pk}}{\gamma \overline{\sigma}_k^2 / N_k} \tag{4}$$

$$y_{dk} = \frac{\overline{R}_{dk} - p_{dk}}{\gamma \widetilde{\sigma}_k^2}$$

where $k \in \{A, B\}$[15].

Since investors are restricted to buying only either direct or portfolio investment at $t = 1$ in each country $k \in \{A, B\}$, the optimal demand for the risky asset is given by $y_k = \max\{y_{dk}, y_{pk}\}$.

The optimal demand for the portfolio investment at country k by an investor from country i in period $t = 0$ is given by

$$x_{pk}^i = N_k \frac{\left(\overline{R}_{pk} - 1\right) - \lambda_i \left(\overline{R}_{pk} - p_{pk}\right)}{\left(1 - \lambda_i\right) \gamma \overline{\sigma}_k^2} \tag{5}$$

The optimal demand for the direct investment at country k by an investor from country i in period $t = 0$ is given by

$$x_{dk}^i = \frac{\left(\overline{R}_{dk} - 1\right) - \lambda_i \left(\overline{R}_{dk} - p_{dk}\right)}{\left(1 - \lambda_i\right) \gamma \sigma_{kl}^2} \tag{6}$$

Note that the demand for risky investment (both direct and portfolio) at $t = 0$ is a decreasing function of liquidity risk (λ_i), i.e., investors from a country with a lower liquidity risk will invest more in investment project at $t = 0$. Also, the demand for risky investment is an increasing function of the price of the investment at $t = 1$., i.e., agents will invest a larger amount of their endowment into risky projects if the re-sale price in the next period is higher.

5 Equilibrium

Recall that $\delta_{ik} \in [0, 1]$ denotes the fraction of direct investors from country i investing in country k where $i, k \in \{A, B\}$.

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variance preferences and assumption 2. Since after the realization of liquidity shock, the survived investors from both countries are identical, their demands for each type of the risky asset is the same: $y_{pk}^A = y_{pk}^B$ and $y_{dk}^A = y_{dk}^B$.

[15] See Appendix A.2 for maximization problem.

Given fractions δ_{ik} of direct investors in the economy, prices (p_{pk}, p_{dk}) and demand functions $\left(x^i_{pk}, x^i_{dk}, y_k\right)$ for all $k, i \in \{A, B\}$, constitute a Rational Expectations Equilibrium (REE) if (i) $\left(x^i_{dk}, y_k\right)$ (respectively, (x^i_{pk}, y_k)) maximizes the expected utility of a direct (respectively, portfolio) investor i, given the prices (p_{dk}, p_{pk}) and (ii) the market for investments clears at $t = 1$.

The overall equilibrium in the economy is given by $\left(\delta_{ik}, (p_{dk}, p_{pk}), (x^i_{dk}, x^i_{pk}, y_k)\right)$ for $k, i \in \{A, B\}$.

5.1 Properties of Equilibrium

Property 1. *In an equilibrium, the prices satisfy* $p_{dk} \leq \overline{R}_{dk}$ *and* $p_{pk} \leq \overline{R}_{pk}$.

If the price of direct investment in country k is greater than the expected payoff then agents will invest all of their endowment in this country. So there is no money holding at $t = 1$, therefore $p_{dk} > \overline{R}_{dk}$ cannot be an equilibrium price. Similarly, for portfolio investment.

Property 2. *In an equilibrium, the optimal demands for portfolio and direct investments are equal:*

$$\frac{\overline{R}_{dk} - p_{dk}}{\gamma \widetilde{\sigma}^2_k} = \frac{\overline{R}_{pk} - p_{pk}}{\gamma \overline{\sigma}^2_k / N} \tag{7}$$

Given the assumption that investors can buy only one type of asset in each country, the expected utilities of buying direct and portfolio investments should be equal in the equilibrium. Otherwise, all investors will only buy the investment with higher expected utility.

Property 3. *In an equilibrium, a direct investor sells his investment if he observes a bad productivity signal.*

Suppose a direct investor does not sell his investment after observing a bad signal. Then by Assumption 3, ex-ante he is better off by choosing the portfolio investment at $t = 0$ since he can sell it for a higher price at $t = 1$ in case of a liquidity shock.

The equilibrium prices of direct investment (p_{dk}) and the portfolio investment (p_{pk}) are determined by equation (7) and the market clearing condition. In each country k, risky investment is supplied by the agents who received a liquidity shock or the adverse signal about investment productivity. The buyers are the agents who have not received a liquidity shock.

$$(\alpha(1-\lambda_A)+(1-\alpha)(1-\lambda_B))\,y_k = \begin{pmatrix} \alpha\delta_{Ak}(\lambda_A+(1-\lambda_A)\pi_k)\,x_{dk}^A \\ +(1-\alpha)\,\delta_{Bk}(\lambda_B+(1-\lambda_B)\pi_k)\,x_{dk}^B \\ +\alpha(1-\delta_{Ak})\,\lambda_A x_{pk}^A \\ +(1-\alpha)(1-\delta_{Bk})\,\lambda_B x_{pk}^B \end{pmatrix} \qquad (8)$$

5.2 Choice between direct and portfolio investments

In period $t = 0$ investor from country i will choose to become a direct investor in country k only if his expected utility from holding direct investment is greater than or equal to his expected utility from holding portfolio investment: $EU\left(x_{dk}^i\right) \geq EU\left(x_{pk}^i\right)$. If the two utilities are equal then an investor is indifferent between holding direct or portfolio investment.

Recall that the liquidity risk in country A is less than in country B: $\lambda_A < \lambda_B$.

Lemma 1. *For any country $k \in \{A, B\}$, if some investors from country B hold direct investment in country k, i.e., $\delta_{Bk} > 0$ then all investors from country A hold direct investment in country k, i.e., $\delta_{Ak} = 1$.*

Lemma 1 follows from the fact that from the demand for risky investment is a decreasing function in liquidity risk. It implies that if only a fraction of investors from country A (but not all) choose to hold direct investment in country k, then none of the investors from country B hold direct investment in that country. In particular, if for investors from country A the expected utility from holding direct investment is less then the expected utility from holding portfolio investment, then only portfolio investments will be held in equilibrium.

Proposition 1 *For any country $k \in \{A, B\}$, there exist an equilibrium. There are two types of equilibria: (1) type I: $\delta_{Ak} \in [0, 1)$ and $\delta_{Bk} = 0$, i.e., only investors from country A (but not all) hold direct investment, the equilibrium of this type is unique; or (2) type II: $\delta_{Ak} = 1$ and $\delta_{Bk} \in [0, 1]$, i.e. all investors from country A hold direct investment, there are at most three such equilibria.*

Type I equilibrium includes the (corner) equilibrium with portfolio investments only and a pooling equilibrium for investors from country A. The equilibrium of type I is unique because there is a strategic substitutability in becoming a direct investor. Therefore, there is a unique δ_{Ak} such that if the proportion of direct investors is below δ_{Ak} then $EU\left(x_{dk}^{A}\right) > EU\left(x_{pk}^{A}\right)$, and if the proportion of direct investors is above δ_{Ak} then $EU\left(x_{dk}^{A}\right) < EU\left(x_{pk}^{A}\right)$.

Type II equilibrium includes the (corner) equilibrium with direct investments only, a pooling equilibrium for investors from country B, and the separating equilibrium where direct investments are held by investors from country A and portfolio investments are held by investors in country B.

The multiplicity of type II equilibria is based on the effect of expectations on the price of direct investment. On one hand, similarly to the type I, as the fraction of direct investors increases ($\delta_{Bk} \uparrow$), the price of direct investment goes down, decreasing the benefits from direct investment. On the other hand, the information discount on the price of direct investment depends on the probability of direct investors selling due to the bad productivity signal. If there are more direct investors with a high liquidity risk then the market believes that the probability of a direct investor selling due to a liquidity shock is higher and, therefore, the price discount on the prematurely sold direct investment is smaller. So, if investors from country B expect other investors from country B to hold direct investment then more investors from country B choose to hold direct investment. This strategic complementarity generates the existence of multiple equilibria. If there are two or three equilibria then one of the equilibria is a separating equilibrium where all investors with a lower liquidity risk hold direct investment, and all investors with a higher liquidity risk hold portfolio investment.

Overall, there are five possible cases of composition of direct and portfolio investment that can occur in the equilibrium in a given country:

1. investors from both countries hold portfolio investments

2. some investors from country a hold direct investments and others hold portfolio investments

3. all investors from country a hold direct investments and all investors from country b hold portfolio investments

4. some investors from country b hold portfolio investments and others hold direct investments

5. investors from both countries hold direct investments

Figures 4 illustrates the possible equilibria regions for different values of λ_A and λ_B such that $\lambda_A \leq \lambda_B$. Each point in the (λ_A, λ_B) plane corresponds to a particular case of equilibria in the enumeration above, except for the points with multiple equilibria (when cases 3 and 4 occur simultaneously). Thus, each type corresponds to a region in the plane; these regions are colored distinctly and numbered accordingly. We consider three examples with the same values of $\overline{R}_d = \overline{R}_p = 1.1, \sigma_l^2 = 0.075, \sigma_h^2 = 0.125, \pi = 0.5, N = 1$ and different values of α (the fraction of investors in country A). Note that as α becomes larger the area with multiple equilibria disappears.

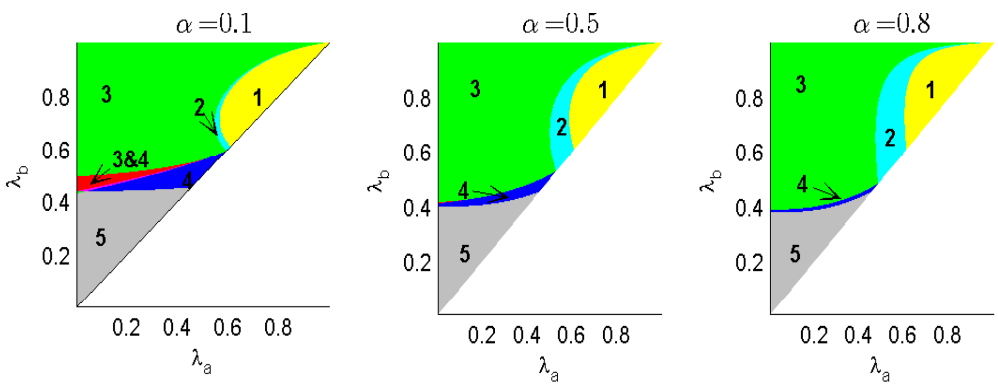

Figure 4 Possible equilibria regions for different values of λ_A and λ_B

6 Composition of Foreign Investment

Define the foreign direct investment from country A to country B as the holdings of direct investment in country B by investors from country A: $FDI_{AB} = \alpha\delta_A x_{dB}^A$. Similarly, the foreign portfolio investment from country A to country B as the holdings of portfolio investment in country B by investors from country A: $FPI_{AB} = \alpha(1-\delta_A) x_{pB}^A$. Then foreign investment from country A to country B is $FI_{AB} = \alpha\delta_A x_{dB}^A + \alpha(1-\delta_A) x_{pB}^A$. Define FDI_{BA}, FPI_{BA}, and FI_{BA} similarly.

There are two dimensions in which the two countries may differ. One is the liquidity risk (λ_k), another is the distribution parameters of investment productivity that represent the country's fundamentals $\left(\overline{R}_{dk}, \overline{R}_{pk}, \sigma^2_{kl}, \sigma^2_{kh}, \pi_k, N_k\right)$.

There are two possible interpretations of liquidity risk in my model. One is the probability of a liquidity crisis that is unrelated to fundamentals of the economy. Another is a measure of financial market development: in more developed financial markets it is easier for agents to borrow in case of liquidity needs, therefore the probability of investment liquidation is smaller. Accordingly, a country with a low liquidity risk can be viewed as a developed country, and a country with a high liquidity risk can be viewed as a developing or emerging economy.

Suppose the countries differ only in terms of liquidity risk and are identical with respect to productivity parameters. In this case, the country with a higher liquidity risk attracts less foreign investment, but a higher share of it in the form of FDI. The figures 5a, 5b,and 5c illustrate the possible compositions of bilateral investment holdings in the different types of equilibria.

5a. Type I pooling equilibrium *5b.* separating equilibrium *5c.* Type II pooling equilibrium

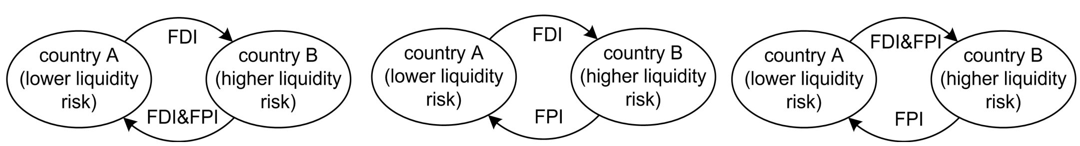

Figure 5. Bilateral investment holdings in different types of equilibria.

In addition to a lower liquidity risk, a developed country can be characterized by a higher expected payoff (adjusted for risk) and smaller benefits from private information of FDI.

Property 4. *In an equilibrium, the share of FDI from country i to country k is higher if either of the following holds: (i) efficiency gains of direct investment* $\left(\overline{R}_{dk} - \overline{R}_{pk}\right)$ *are larger, (ii) uncertainty about investment payoff* $\left(\left(\sigma^2_{kh} - \sigma^2_{kl}\right)/\overline{\sigma}^2_k\right)$ *is larger, (iii) diversification benefits* (N_k) *are smaller.*

The direct and portfolio investment holdings in each country are larger if the expected productivity is higher and the variance is lower. The larger uncertainty about investment productivity positively affects the share of direct investments relative to portfolio investments since it increases the benefits from private information. If direct investment is more efficient relative to portfolio investment, then the share of direct investments is higher, which corresponds to higher equilibrium levels of δ_a and δ_b. On the other hand, larger diversification benefits from portfolio investment result in a smaller share of FDI.

My results are consistent with the empirical findings of Hausman and Fernandez-Arias [14] that countries that are less financially developed and have weaker financial institutions tend to attract more capital in the form of FDI.This offers a liquidity-based explanation of the phenomenon of bilateral FDI flows among developed countries and one-way FDI flows from developed to emerging countries.

7 Liquidity risk

In this section, I study the effect of change in the liquidity risk (λ) on investment holdings in each country. First, I examine the effect of an unanticipated increase in liquidity risk in period one on investment prices and demands. Next, I examine the effect of an increase in liquidity risk on the composition of foreign investment in each country.

7.1 Increase in liquidity risk

Following Allen and Gale [5] approach, I perturb the model to allow for the occurrence of a state that was assigned zero probability in period $t = 0$. Denote by $S = (\lambda_A, \lambda_B)$ the state that was assigned probability one in $t = 0$. Consider a state $S' = \left(\lambda'_A, \lambda'_B\right)$ where $\lambda'_k \geq \lambda_k$ for both countries with a strict inequality for at least one country. This state is assigned probability zero in $t = 0$. If state S' is realized then the fraction of investors who receive a liquidity shock is larger than in state S. All investment decisions at $t = 0$, such as fractions of direct investors $(\delta_{Ak}, \delta_{Bk})$ and direct and portfolio investment holdings $\left(x^A_{dk}, x^A_{pk}, x^B_{dk}, x^B_{pk}\right)$, are made based on the initially anticipated state $S = (\lambda_A, \lambda_B)$. Therefore, the occurrence of state S' does not affect these investment decisions. However, it affects the prices and demands for direct and portfolio investments in period one.

There are two ways in which the prices are affected, one is through the market liquidity and another is through adverse selection problem associated with direct investment. The first effect is the dry up of market liquidity as more investors have to sell their asset holdings, and less investors are buying. The resulting market clearing prices are lower.

At the same time, direct investments are more likely to be sold before maturity due to a liquidity shock rather than because of the bad productivity signal. Therefore, λ'_d, the market belief about the probability of receiving a liquidity shock, is higher than in state S : $\lambda'_d = \frac{\alpha\delta_{Ak}\lambda'_A+(1-\alpha)\delta_{Bk}\lambda'_B}{\alpha\delta_{Ak}+(1-\alpha)\delta_{Bk}} > \lambda_d$. Therefore, the variance of the pre-maturely sold direct investment in country k is lower than in state S: $\widetilde{\sigma}'^2_k < \widetilde{\sigma}^2_k$. This reduces the adverse selection problem and results in the smaller information discount on direct investment relative to portfolio investment.

The unexpected increase in liquidity risk can be interpreted as liquidity crisis. Then the depressed prices together with the reduced discount on direct investment capture the phenomenon of fire-sale FDI during the liquidity crises.

7.2 Comparative Statics

In this section, I examine the anticipated effect of an increase in liquidity risk (comparative statics) on the composition of foreign investment in each country.

Consider country A as a host country and country B as a source country. Suppose country A is in the type II pooling equilibria with respect to inward foreign investment, that is it has inflows of both FDI and FPI. In this case, an increase in the liquidity risk in the host country (λ_A) leads to a lower level of foreign investment. The effect on the composition of foreign investment is ambiguous and depends on the equilibrium. If economy is in the unique equilibrium then an increase in λ_A leads to more FDI and less FPI. However, if there are multiple equilibria then FDI may increase or decrease depending on the equilibrium.

The higher liquidity risk has two effects. One is reduced market liquidity since investors preferences for liquidity are higher. Another is the smaller information discount on the prematurely sold direct investment. The first effect leads to less FDI while the second effect results in more FDI.

If there are multiple equilibria and economy is in the equilibrium with a larger fraction of direct investors (δ_{BA}) or if the equilibrium is unique, then the second effect dominates and an increase in liquidity risk in the host country leads to a higher level of FDI. If economy is in the equilibrium with a smaller fraction of direct investors (δ_{BA}) then the first effect dominates and therefore an increase in liquidity risk in the host country leads to a lower level of FDI. In this case, the outflow of FDI is associated with self-fulfilling expectations: if an agent expects less agents to hold direct investments, then he chooses not to hold direct investment himself.

Figure 6 illustrates the effect of an increase in liquidity risk (λ_A) on foreign direct and portfolio investment.

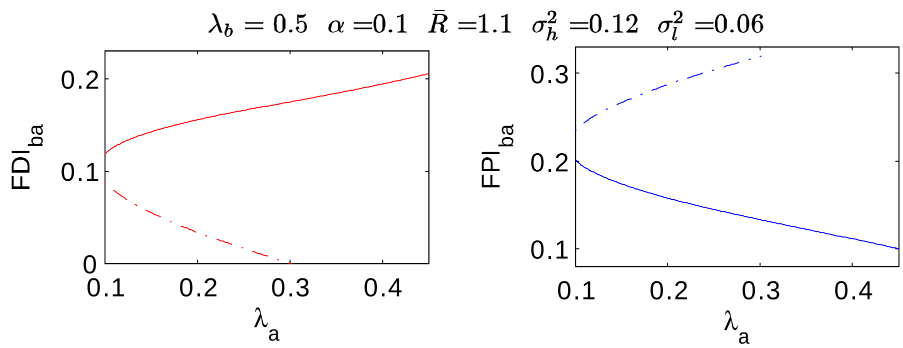

Figure 6. FDI_{BA} and FPI_{BA} as functions of λ_A

The similar argument applies to the case when country B as a host country. These results are summarized below.

Proposition 2 *Suppose country $k \in \{A, B\}$ is in type II pooling equilibrium with respect to inward foreign investment. Then (i) if there is a unique equilibrium then an increase in liquidity risk results in a higher level of FDI; (ii) if there are multiple equilibria then an increase in liquidity risk results in a higher level of FDI in one equilibrium, and a lower level of FDI in another.*

Interpreting increasing liquidity risk as a liquidity crisis, we can compare the equilibria sequentially. Then this ambivalent effect can be related to the empirically observed pattern of FDI during liquidity crises.

8 Empirical evidence

The positive effect of a higher liquidity risk on the inward FDI is consistent with the evidence of fire-sale FDI. Figure 7 shows the inward FDI and FPI flows into Korea and

Mexico. The capital flows data is from the Lane and Milesi-Ferretti (2006) dataset. They construct estimates of external assets and liabilities, distinguishing between foreign direct investment, portfolio equity investment, official reserves, and external debt for over 140 countries over the period of 1970-2004.

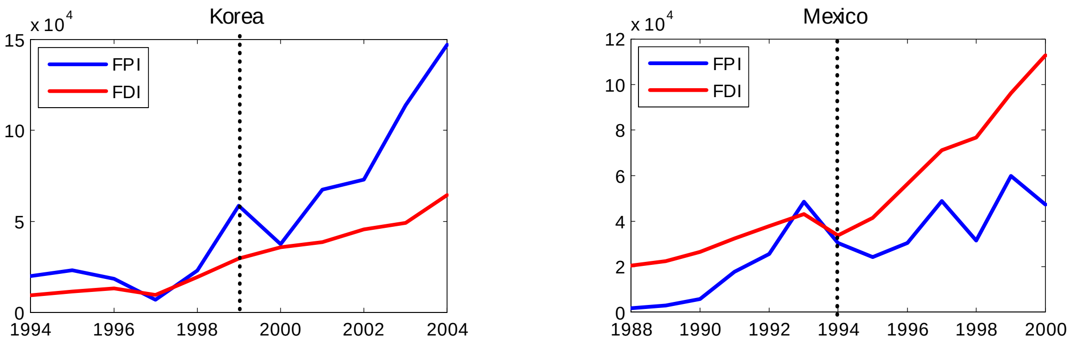

Figure 7. Crises in Korea and Mexico: inflow of FDI and outflow of FPI

As we can see from the figure, in Korea during the late 1990s crisis and in Mexico following the 1994 crisis the FDI level have been increasing while FPI level have declined.

On the other hand, my model provides a possibility of a decrease in FDI through self-fulfilling expectations. This possibility is consistent with the behavior of FDI during the early 1990s crisis in Sweden and the 2001 crisis in Argentina. As figure 8 shows, FDI declined in both cases.

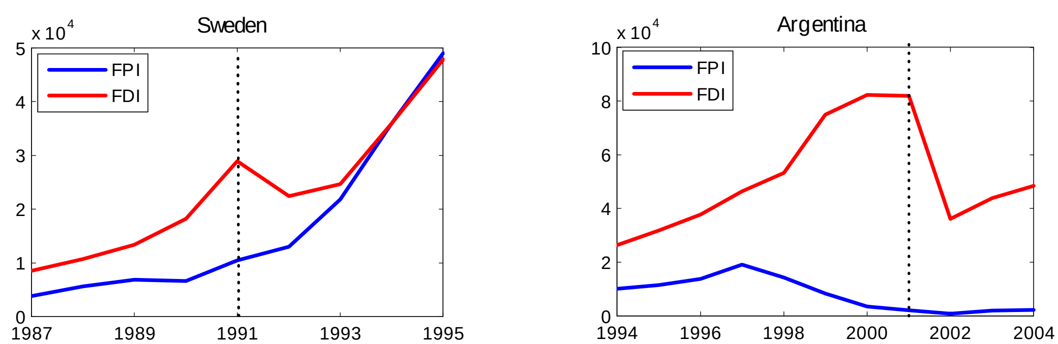

Figure 8. Crises in Sweden and Argentina: outflow of FDI

9 Conclusion

I analyze the composition of foreign investment between two countries which may differ in two dimensions: liquidity risk (probability of a liquidity crisis) and the investment productivity (fundamentals). I find that the country with a higher liquidity risk attracts less foreign investment, but a higher share of it is in the form of FDI, *ceteris paribus*. Also, a country with larger uncertainty about investment productivity attracts more FDI relative to FPI since the benefits from private information are larger. This is consistent with the empirical findings that countries that are less financially developed attract more capital in the form of FDI. This offers an explanation based on the difference in liquidity risk for the phenomenon of bilateral FDI flows among developed countries and one-way FDI flows from developed to emerging countries.

The effect on FDI of an increase in liquidity risk in the host country is ambivalent. If the economy is in the unique equilibrium then a higher liquidity risk leads to larger FDI holdings and smaller FPI holdings. This result is in line with the fire-sale FDI phenomenon. If, however, there are multiple equilibria then a higher liquidity risk may lead to the opposite effect: FDI declines. In this case, an outflow of FDI is induced by self-fulfilling expectations. This ambivalent effect of increased liquidity risk on foreign investment corresponds to the empirical evidence on capital flows during liquidity crises.

References

[1] V. Acharya, H. Shin, and T. Yorulmazer. Fire-sale FDI.

[2] P. Aghion, P. Bacchetta, and A. Banerjee. A simple model of monetary policy and currency crises. *European Economic Review* **44**(4-6) (2000).

[3] M. Aguiar and G. Gopinath. Fire-Sale Foreign Direct Investment and Liquidity Crises. *The Review of Economics and Statistics* **87**(3), 439–52 (2005).

[4] F. Allen and D. Gale. Limited Market Participation and Volatility of Asset Prices. *The American Economic Review* **84**(4), 933–955 (1994).

[5] F. Allen and D. Gale. Financial Contagion. *Journal of Political Economy* **108**(1), 1–33 (2000).

[6] S. Bhattacharya and G. Nicodano. Insider Trading, Investment, and Liquidity: A Welfare Analysis. *The Journal of Finance* **56**(3), 1141–1156 (2001).

[7] P. Bolton and E. von Thadden. Blocks, Liquidity, and Corporate Control. *The Journal of Finance* **53**(1), 1–25 (1998).

[8] R. Caballero and A. Krishnamurthy. International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics* **48**(3) (2001).

[9] R. Chang and A. Velasco. A Model of Financial Crises in Emerging Markets*. *Quarterly Journal of Economics* **116**(2) (2001).

[10] D. Diamond and P. Dybvig. Bank Runs, Deposit Insurance, and Liquidity. *The Journal of Political Economy* **91**(3), 401–419 (1983).

[11] D. Easley, N. Kiefer, and M. O'Hara. The information content of the trading process. *Journal of Empirical Finance* **4**(2-3), 159–186 (1997).

[12] D. Easley and M. O'Hara. Price, trade size, and information in securities markets. *Journal of Financial Economics* **19**(1), 69–90 (1987).

[13] I. Goldstein and A. Razin. An information-based trade off between foreign direct investment and foreign portfolio investment. *Journal of International Economics* **70**(1), 271–295 (2006).

[14] R. Hausmann and E. Fernandez-Arias. Foreign Direct Investment: Good Cholesterol?

[15] B. Holmstrom and J. Tirole. Market Liquidity and Performance Monitoring. *The Journal of Political Economy* **101**(4), 678–709 (1993).

[16] R. Holthausen, R. Leftwich, and D. Mayers. The Effect of Large Block Transactions on Security Prices: A Cross-sectional Analysis. *Journal of Financial Economics* **19**(2), 237–267 (1998).

[17] C. Kahn and A. Winton. Ownership Structure, Speculation, and Shareholder Intervention. *The Journal of Finance* **53**(1), 99–129 (1998).

[18] D. Keim and A. Madhavan. The upstairs market for large-block transactions: analysis and measurement of price effects. *Review of Financial Studies* **9**(1), 1–36 (1996).

[19] Y. Kinoshita and A. Mody. "Private and Public Information for Foreign Investment Decisions". (1999).

[20] M. Klein, J. Peek, and E. Rosengren. Troubled Banks, Impaired Foreign Direct Investment: The Role of Relative Access to Credit. *The American Economic Review* **92**(3), 664–682 (2002).

[21] P. Krugman. Fire-Sale FDI. (1998).

[22] R. Lipsey. Foreign Direct Investors in Three Financial Crises. *NBER Working Paper* (2001).

[23] F. Maccheroni, M. Marinacci, and A. Rustichini. Ambiguity Aversion, Robustness, and the Variational Representation of Preferences. *Econometrica* **74**(6), 1447–1498 (2006).

[24] E. Maug. Large Shareholders as Monitors: Is There a Trade-Off between Liquidity and Control? *The Journal of Finance* **53**(1), 65–98 (1998).

[25] A. Razin and E. Sadka. Gains from FDI inflows with incomplete information. *Economics Letters* **78**(1), 71–77 (2003).

[26] A. Razin and E. Sadka. "Foreign Direct Investment: Analysis of Aggregate Flows". Princeton University Press (2007).

10 Appendix

A1. Assumptions

For each country $k \in \{A, B\}$ the parameters of payoff distribution have to satisfy the following assumptions:

Assumption 1a. At $t = 0$, the demand for risky asset in each country k is non-negative, i.e., $x_k^i \geq 0$ and $x_{dk}^i \geq 0$ if

$$\frac{\left(\overline{R}_{pk} - 1\right)}{\overline{\sigma}_k^2 / N} \geq \frac{\left(\overline{R}_{dk} - 1\right)}{\widetilde{\sigma}_k^2}$$

Assumption 1b. At $t = 0$, the demand for risky asset in both countries is less than or equal to one, i.e., $\sum_{k \in \{A,B\}} x_k^i < 1$

$$\frac{\left(\overline{R}_{pk} - 1\right)}{\gamma \overline{\sigma}_k^2 / N} < \frac{\left(\overline{R}_{dk} - 1\right)}{\gamma \widetilde{\sigma}_k^2} + 0.5 \left(1 - \lambda_A\right) \gamma \frac{\overline{\sigma}_k^2 / N}{\widetilde{\sigma}^2}$$

Assumption 2. At $t = 1$, investor's demand for risky asset in both countries is less than his money holdings.

$$\sum_{k \in \{A,B\}} \max \left\{ \frac{\overline{R}_{dk} - p_{dk}}{\gamma \widetilde{\sigma}_k^2}, \frac{\overline{R}_{pk} - p_{pk}}{\gamma \left(\overline{\sigma}_k^2 / N\right)} \right\} < \min \left\{ \frac{1 - x_{pk}^i}{p_{pk}}, \frac{1 - x_{dk}^i}{p_{dk}} \right\}$$

where

$$\begin{aligned}
p_{pk} &= \overline{R}_{pk} - \frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2 / N}\right)^{0.5} \left(\overline{R}_{pk} - 1\right)}{\lambda_B \left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2 / N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2 / N}\right)^{0.5}\right)} \\
p_{dk} &= \overline{R}_{dk} - \frac{\widetilde{\sigma}_k^2}{\overline{\sigma}_k^2 / N} \frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2 / N}\right)^{0.5} \left(\overline{R}_{pk} - 1\right)}{\lambda_B \left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2 / N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2 / N}\right)^{0.5}\right)} \\
x_{pk}^i &= \frac{\left(\overline{R}_{pk} - 1\right) - \lambda_A \left(\overline{R}_{pk} - p_{pk}\right)}{\left(1 - \lambda_A\right) \gamma \overline{\sigma}_k^2 / N} \\
x_{dk}^i &= \frac{\left(\overline{R}_{dk} - 1\right) - \lambda_A \left(\overline{R}_{dk} - p_{dk}\right)}{\left(1 - \lambda_A\right) \gamma \sigma_{kl}^2}
\end{aligned}$$

Assumption 3.

$$N_k = \frac{\overline{R}_{dk} - 1}{\overline{R}_{pk} - 1}$$

A.2a. Decision problem at t=1.

Without loss of generality, consider the decision problem of a portfolio investor in period one. Due to the mean-variance utility and assumption 2, the demand for risky asset in period one is independent from the demand in period $t = 0$, so that direct and portfolio investors who have not received a liquidity shock have the same demands for risky asset at $t = 1$.

If at $t = 1$ a portfolio investor i chooses to buy a portfolio investment y^i_{pk} given his investment $x^i_{pk} = Nx^i_k$ at date $t = 0$:

$$\begin{aligned} \max_{y_k} \quad & \sum_{k=a,b} \left\{ 1 - x^i_{pk} - p_{pk} y_{pk} + \left(x^i_{pk} + y^i_{pk} \right) \overline{R}_{pk} - \tfrac{1}{2} \left(x^i_{pk} \right)^2 \gamma \left(\overline{\sigma}^2_k / N \right) - \tfrac{1}{2} \left(y^i_{pk} \right)^2 \gamma \left(\overline{\sigma}^2_k / N \right) \right\} \\ \text{s.t.} \quad & p_{pk} y_{pk} \leq 1 - x^i_{pk} \\ & y_{pk} \geq 0 \end{aligned} \tag{9}$$

The optimal demand y_{pk} for portfolio investment by a portfolio investor i at country $k \in \{A, B\}$ in period $t = 1$ is given by

$$y^i_{pk} = \begin{cases} 0 & \text{if} \quad (i) : p_{pk} > \overline{R}_{pk} \\ \frac{\overline{R}_{pk} - p_{pk}}{\gamma \left(\overline{\sigma}^2_k / N \right)} & \text{if} \quad (ii) : p_{pk} \leq \overline{R}_{pk} \\ \frac{1 - x^i_{pk}}{p_{pk}} & \text{if} \quad (iii) : \frac{\overline{R}_{pk} - p_{pk}}{\gamma \left(\overline{\sigma}^2_k / N \right)} > \frac{1 - x^i_{pk}}{p_{pk}} \end{cases} \tag{10}$$

The case (iii) is ruled out by assumption 2 and case (i) can not occur in the equilibrium (Property 1). Therefore, the solution is interior $y_{pk} = \frac{\overline{R}_k - p_{pk}}{\gamma \overline{\sigma}^2_k}$ and it does not depend on the probability of receiving a liquidity shock, so superscript i can be omitted.

Similarly, if at $t = 1$ portfolio investor i chooses to buy direct investment y^i_{dk} given his investment Nx^i_{pk} at date $t = 0$:

$$\begin{aligned} \max_{y_k} \quad & \sum_{k=a,b} \left\{ 1 - x^i_{pk} - p_{dk} y_{dk} + x^i_{pk} \overline{R}_{pk} + y^i_{dk} \overline{R}_{dk} - \tfrac{1}{2} \left(x^i_{pk} \right)^2 \gamma \left(\overline{\sigma}^2_k / N \right) - \tfrac{1}{2} \left(y^i_{dk} \right)^2 \gamma \widetilde{\sigma}^2_k \right\} \\ \text{s.t.} \quad & p_{dk} y_{dk} \leq 1 - x^i_{pk} \\ & y_{dk} \geq 0 \end{aligned} \tag{11}$$

The optimal demand y_{dk} for portfolio investment by a portfolio investor i at country $k \in \{A, B\}$ in period $t = 1$ is given by

$$y_{dk}^{i}=\begin{cases} 0 & \text{if} \quad (i): p_{dk}\geq \overline{R}_{k} \\ \frac{\overline{R}_{dk}-p_{dk}}{\gamma\tilde{\sigma}_{k}^{2}} & \text{if} \quad (ii): p_{dk}<\overline{R}_{k} \\ \frac{1-x_{pk}^{i}}{p_{dk}} & \text{if} \quad (iii): \frac{\overline{R}_{dk}-p_{dk}}{\gamma\tilde{\sigma}_{k}^{2}}>\frac{1-x_{pk}^{i}}{p_{dk}} \end{cases} \tag{12}$$

The case (iii) is ruled out by assumption 2 and case (i) can not occur in the equilibrium. Therefore, $y_{dk}=\frac{\overline{R}_{dk}-p_{dk}}{\gamma\tilde{\sigma}_{k}^{2}}$.

A.2b. Decision problem at t=0.

The decision problem of a portfolio investor from country $i\in\{A,B\}$ at $t=0$ becomes

$$\max_{x_{k}^{i}} \sum_{k=a,b}\left\{\begin{array}{l} \lambda_{i}\left(1-Nx_{k}^{i}+p_{pk}Nx_{k}^{i}\right)+ \\ \left(1-\lambda_{i}\right)\left(1+Nx_{k}^{i}\left(\overline{R}_{pk}-1\right)-\frac{1}{2}N\left(x_{k}^{i}\right)^{2}\gamma\overline{\sigma}_{k}^{2}+\frac{1}{2}\frac{\left(\overline{R}_{pk}-p_{pk}\right)^{2}}{\gamma\overline{\sigma}_{k}^{2}}\right) \end{array}\right\} \tag{13}$$

s.t. $\quad 0\leq x_{k}^{i}\leq 1/N$

The optimal demand for the investment at country k by an investor from country i in period $t=0$ is given by

$$x_{k}^{i}=\frac{\left(\overline{R}_{pk}-1\right)-\lambda_{i}\left(\overline{R}_{pk}-p_{pk}\right)}{\left(1-\lambda_{i}\right)\gamma\overline{\sigma}_{k}^{2}} \tag{14}$$

Then the portfolio investment is $x_{pk}^{i}=N_{k}x_{k}^{i}$.

The decision problem of a direct investor from country $i\in\{A,B\}$ at $t=0$ becomes

$$\max_{x_{dk}^{i}} \sum_{k=a,b}\left\{\begin{array}{l} \lambda_{i}\left(1-x_{dk}^{i}+p_{dk}x_{dk}^{i}\right)+ \\ \left(1-\lambda_{i}\right)\left(1+x_{dk}^{i}\left(\overline{R}_{dk}-1\right)-\frac{1}{2}\left(x_{dk}^{i}\right)^{2}\gamma\sigma_{kl}^{2}+\frac{1}{2}\frac{\left(\overline{R}_{pk}-p_{pk}\right)^{2}}{\gamma\overline{\sigma}_{k}^{2}}\right) \end{array}\right\} \tag{15}$$

s.t. $\quad 0\leq x_{dk}^{i}\leq 1$

The optimal demand for the investment at country k by an investor from country i in period $t=0$ is given by

$$x_{dk}^{i}=\frac{\left(\overline{R}_{dk}-1\right)-\lambda_{i}\left(\overline{R}_{dk}-p_{dk}\right)}{\left(1-\lambda_{i}\right)\gamma\sigma_{kl}^{2}} \tag{16}$$

B. Proof of Lemma 1.

Proof. The optimal demand for the investment at country $k = a, b$ in period $t = 0$ is given by

$$x^i_{pk} = \frac{\left(\overline{R}_{pk} - 1\right) - \lambda_i \left(\overline{R}_{pk} - p_{pk}\right)}{\left(1 - \lambda_i\right) \gamma \overline{\sigma}^2_k / N_k}$$
$$x^i_{dk} = \frac{\left(\overline{R}_{dk} - 1\right) - \lambda_i \left(\overline{R}_{dk} - p_{dk}\right)}{\left(1 - \lambda_i\right) \gamma \sigma^2_{kl}}$$

First, let's show that $x^i_{dk} \geq x^i_{pk}$ for any $\lambda_i \in [\lambda_A, \lambda_B]$

$$x^i_{dk} = \frac{\left(\overline{R}_{dk} - 1\right) - \lambda_i \left(\overline{R}_{dk} - p_{dk}\right)}{\left(1 - \lambda_i\right) \gamma \sigma^2_{kl}} > \frac{\left(\left(\overline{R}_{dk} - 1\right) - \lambda_i \left(\overline{R}_{dk} - p_{dk}\right)\right)}{\left(1 - \lambda_i\right) \gamma \widetilde{\sigma}^2_k} =$$
$$= \frac{\left(\overline{R}_{pk} - 1\right) - \lambda_i \left(\overline{R}_{pk} - p_{pk}\right) \frac{\overline{\sigma}^2_k}{\widetilde{\sigma}^2_k}}{\left(1 - \lambda_i\right) \gamma \overline{\sigma}^2_k / N_k} > \frac{\left(\overline{R}_{pk} - 1\right) - \lambda_i \left(\overline{R}_{pk} - p_{pk}\right)}{\left(1 - \lambda_i\right) \gamma \overline{\sigma}^2_k / N_k} = x^i_{pk}$$

The expected utilities from holding direct and portfolio investments in country k are given by

$$EU\left(x^A_{dk}\left(\lambda_i\right)\right) = 1 + 0.5\left(1 - \lambda_i\right) x^2_{dk}\left(\lambda_i\right) \gamma \sigma^2_{kl} + 0.5 y^2_k \gamma \overline{\sigma}^2_k / N_k$$
$$EU\left(x^A_{dk}\left(\lambda_i\right)\right) = 1 + 0.5\left(1 - \lambda_i\right) x^2_{pk}\left(\lambda_i\right) \gamma \overline{\sigma}^2_k / N_k + 0.5 y^2_k \gamma \overline{\sigma}^2_k / N_k$$

Suppose $\delta_b > 0$, this implies that $EU\left(x^A_{dk}\left(\lambda_B\right)\right) \geq EU\left(x_{pk}\left(\lambda_B\right)\right) \Longleftrightarrow x^2_{dk}\left(\lambda_B\right) \gamma \sigma^2_{kl} \geq x^2_{pk}\left(\lambda_B\right) \gamma \overline{\sigma}^2_k$

To show that $\delta_a = 1$ we need $EU\left(x_{dk}\left(\lambda_A\right)\right) \geq EU\left(x_{pk}\left(\lambda_A\right)\right) \Longleftrightarrow x^2_{dk}\left(\lambda_A\right) \gamma \sigma^2_{kl} \geq x^2_{pk}\left(\lambda_A\right) \gamma \overline{\sigma}^2_k$

Taking derivative of $x^2_{dk}\left(\lambda_i\right) \gamma \sigma^2_{kl}$ and $x^2_{pk}\left(\lambda_i\right) \gamma \overline{\sigma}^2_k$ with respect to λ, we get

$$\frac{\left(1 - p_{dk}\right)}{\left(1 - \lambda_i\right)^2 \gamma \sigma^2_{kl}} > \frac{\left(1 - p_{pk}\right)}{\left(1 - \lambda_i\right)^2 \gamma \overline{\sigma}^2_k / N_k}$$

The above inequality follows from

$$\frac{\overline{R}_{dk} - p_{dk}}{\gamma \widetilde{\sigma}^2_k} = \frac{\overline{R}_{pk} - p_{pk}}{\gamma \overline{\sigma}^2_k / N_k} \Longrightarrow \frac{1 - p_{dk}}{\gamma \sigma^2_{kl}} > \frac{1 - p_{dk}}{\gamma \widetilde{\sigma}^2_k} > \frac{1 - p_{pk}}{\gamma \overline{\sigma}^2_k / N_k}$$

Therefore, for $\lambda_A < \lambda_B$ such that $x^2_{dk}\left(\lambda_B\right) \gamma \sigma^2_{kl} \geq x^2_{pk}\left(\lambda_B\right) \gamma \overline{\sigma}^2_k$, we have $x^2_{dk}\left(\lambda_A\right) \gamma \sigma^2_{kl} > x^2_{pk}\left(\lambda_A\right) \gamma \overline{\sigma}^2_k$. This implies that all investors from country a obtain a higher utility by holding direct investment rather than portfolio, hence, $\delta_a = 1$.

Next, suppose $\delta_a < 1$, this this implies that $EU\left(x_{dk}\left(\lambda_A\right)\right) = EU\left(x_{pk}\left(\lambda_A\right)\right) \Longleftrightarrow x^2_{dk}\left(\lambda_A\right) \gamma \sigma^2_{kl} = x^2_{pk}\left(\lambda_A\right) \gamma \overline{\sigma}^2_k$

$\Longrightarrow x_{dk}^2(\lambda_B)\gamma\sigma_{kl}^2 < x_{pk}^2(\lambda_B)\gamma\overline{\sigma}_k^2 \Longleftrightarrow EU(x_{dk}(\lambda_B)) < EU(x_{pk}(\lambda_B))$. Hence, $\delta_b = 0$.

■

C. Proof of Proposition 1.

Proof. *Part 1* Define $\Delta EU\left(x_k^i\right) = EU\left(x_{dk}^i\right) - EU\left(x_{pk}^i\right)$

$$\Delta EU\left(x_k^i\right) = x_{dk}^2(\lambda_i)\gamma\sigma_{kl}^2 - x_{pk}^2(\lambda_i)\gamma\overline{\sigma}_k^2/N_k$$

If $\Delta EU\left(x_k^a\right) < 0$ then by Lemma1 $\Delta EU\left(x_k^b\right) < 0$. Therefore, there is no direct investment in the equilibrium, i.e., $\delta_a^* = \delta_b^* = 0$.

Next $\Delta EU\left(x_k^a\right) = 0$ then together with Property 2., we can derive the equilibrium prices:

$$\begin{aligned} p_{pk} &= \overline{R}_{pk} - \frac{\left(\overline{R}_{dk}-1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk}-1\right)}{\lambda_A\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)} \\ p_{dk} &= \overline{R}_{dk} - \frac{\widetilde{\sigma}_k^2}{\overline{\sigma}_k^2/N}\frac{\left(\overline{R}_{dk}-1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk}-1\right)}{\lambda_A\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)} \end{aligned}$$

Then δ_{Ak} is determined by market clearing condition:

$$\delta_{Ak} = \frac{\left(\alpha(1-\lambda_A) + (1-\alpha)(1-\lambda_A)\right)y_k - \alpha\lambda_A x_{pk}(\lambda_A) + (1-\alpha)\lambda_B x_{pk}(\lambda_B)}{\alpha\left(\lambda_A + (1-\lambda_A)\pi_k\right)x_{dk}(\lambda_A) - \alpha\lambda_A x_{pk}(\lambda_A)}$$

If $\Delta EU\left(x_k^a\right) \geq 0$ then $\delta_{Ak} \geq 0$. If $\Delta EU\left(x_k^a\right) = 0$ and $\delta_{Ak} \leq 1$ then by Lemma 1 $\Delta EU\left(x_k^b\right) > 0$. This implies that $\delta_{Bk} = 0$, this constitute an equilibrium of type I. Note, if type I equilibrium exist, it is unique.

Part 2. Next consider $\Delta EU\left(x_k^a\right) > 0$ and $\delta_{Ak} \geq 1$ then $\Delta EU\left(x_k^b\right)$ can be less then, equal to, or greater than zero.

(i) Consider $\Delta EU\left(x_k^a\right) > 0, \delta_{Ak} \geq 1$ and $\Delta EU\left(x_k^b\right) < 0$. Then $\delta_{Bk} = 0$. This is a separating equilibrium with $\delta_{Ak} = 1$ and $\delta_{Bk} = 0$. Prices are determined by Property 2 and market clearing condition.

(ii) Consider $\Delta EU\left(x_k^a\right) > 0, \delta_{Ak} \geq 1$ and $\Delta EU\left(x_k^b\right) = 0$. Then prices are given by

$$\begin{aligned}
p_{pk} &= \overline{R}_{pk} - \frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk} - 1\right)}{\lambda_B\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)} \\
p_{dk} &= \overline{R}_{dk} - \frac{\widetilde{\sigma}_k^2}{\overline{\sigma}_k^2/N}\frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk} - 1\right)}{\lambda_B\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)}
\end{aligned}$$

The equilibrium fraction of direct investors from country b is determined by market clearing condition. Contrary to the Part 1, the market clearing condition is no longer linear in δ_{Bk} since market beliefs about the probability of direct investor receiving a liquidity shock (λ_d) depends on δ_{Bk} $\left(\lambda_d = \frac{\alpha\delta_{Ak}\lambda_A + (1-\alpha)\delta_{Bk}\lambda_B}{\alpha\delta_{Ak} + (1-\alpha)\delta_{Bk}}\right)$, and therefore, variance $\widetilde{\sigma}_k^2$ also depends on δ_{Bk}. We can write the market clearing condition

$$\begin{aligned}
& \alpha\left(\lambda_A + (1-\lambda_A)\pi_k\right)x_{dk}\left(\lambda_A\right) + (1-\alpha)\left(\lambda_B + (1-\lambda_B)\pi\right)\delta_{Bk}x_{dk}\left(\lambda_B\right) + (1-\alpha)\lambda_B\left(1-\delta_{Bk}\right)x_{pk}\left(\lambda_B\right) \\
= \; & \left[\alpha\left(1-\lambda_A\right) + (1-\alpha)\left(1-\lambda_B\right)\right]y
\end{aligned}$$

as a quadratic equation in $\delta_{Bk} : c_1\delta_{Bk} + c_2\delta_{Bk} + c_3 = 0$ where $c_1 < 0$ and $c_2 < 0$. If $c_3 < 0$ and then there are 2 interior $\delta_{Bk} \in (0,1)$

If there are two equilibria with $\delta_a^{**} = 1, \delta_b^{**} \in (0,1]$ such that $\Delta EU\left(x_k^b\right) = 0$ and $\Delta EU\left(x_k^a\right) > 0$ then $\delta_b^{**} = 0, \delta_a^{**} = 1$ (separating equilibrium) is also an equilibrium.

(iii) Consider $\Delta EU\left(x_k^a\right) > 0, \delta_{Ak} \geq 1$ and $\Delta EU\left(x_k^b\right) > 0$. In this case $\delta_b^{**} = 1, \delta_a^{**} = 1$ is a unique equilibrium.

All three cases are captured by type II equilibria and can be summarized in the following way: If $\Delta EU\left(x_k^a\right) > 0$ and $\delta_{Ak} \geq 1$ and

∘ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ then there is at least one equilibrium $\delta_b^{**} = 0, \delta_a^{**} = 1$:

∘ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) > 0$ then there is 2 equilibria

∘ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) < 0$ and $\max_{\delta_b}\{\Delta EU\left(x_k^b\right)\} > 0$ then there is 3 equilibria

∘ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) < 0$ and $\max_{\delta_b}\{\Delta EU\left(x_k^b\right)\} = 0$ then there is 2 equilibria

∘ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) > 0$ then there is 2 equilibria

$\circ$ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) < 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) < 0$ and $\max_{\delta_b}\{\Delta EU\left(x_k^b\right)\} < 0$ then there is 1 equilibrium

$\circ$ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) > 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) < 0$ then there is 1 equilibrium

$\circ$ at $\delta_b = 0 : \Delta EU\left(x_k^b\right) > 0$ and at $\delta_b = 1 : \Delta EU\left(x_k^b\right) > 0$ then there is no equilibrium

■

D. Proof of Proposition 2

Proof. (1) consider λ_A as a host country. The type II pooling equilibrium should satisfy the following conditions:

$$\begin{aligned}
p_{pk} &= \overline{R}_{pk} - \frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk} - 1\right)}{\lambda_B\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)} \\
p_{dk} &= \overline{R}_{dk} - \frac{\widetilde{\sigma}_k^2}{\overline{\sigma}_k^2/N}\frac{\left(\overline{R}_{dk} - 1\right) - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\left(\overline{R}_{pk} - 1\right)}{\lambda_B\left(\frac{\widetilde{\sigma}^2}{\overline{\sigma}^2/N} - \left(\frac{\sigma_l^2}{\overline{\sigma}^2/N}\right)^{0.5}\right)}
\end{aligned}$$

And δ_{Bk} is determined from

$$\begin{aligned}
&\alpha\left(\lambda_A + (1-\lambda_A)\pi_k\right)x_{dk}(\lambda_A) + (1-\alpha)\left(\lambda_B + (1-\lambda_B)\pi\right)\delta_{Bk}x_{dk}(\lambda_B) + (1-\alpha)\lambda_B(1-\delta_{Bk})x_{pk}(\lambda_B) \\
= \; &\left[\alpha(1-\lambda_A) + (1-\alpha)(1-\lambda_B)\right]y_k
\end{aligned}$$

Define excess demand by ED. We can write the market clearing condition as a quadratic equation in $\delta_{Bk} : ED = c_1\delta_{Bk} + c_2\delta_{Bk} + c_3 = 0$

where $c_1 < 0$ and $c_2 < 0$. There are 2 possibilities: either unique equilibrium or two equilibria.

If there two equilibria than $c_3 < 0$. If λ_A increases to λ_A then the $\max_{\delta_{Bk}} ED$ increases and $\arg\max_{\delta_{Bk}} ED$ decreases. Denote δ_{Bk} and δ_{Bk} the two solutions to $ED = 0$.

So that $\delta_{Bk}(\lambda_A) > \delta_{Bk}(\lambda_A)$ and $\delta_{Bk}(\lambda_A) < \delta_{Bk}(\lambda_A)$. If there is a unique equilibrium then $c_3 > 0$ so that only the solution δ_{Bk} remains. Therefore, if equilibrium is unique then the increase in λ_A leads to a higher fraction of direct investors in equilibrium. If there are multiple equilibria, then the effect is ambivalent.

(2) consider λ_B as a host country. If λ_B increases to λ_B then the $\max_{\delta_{Bk}} ED$ decreases and $\arg\max_{\delta_{Bk}} ED$ increases. In this case $\delta_{Bk}(\lambda_B) < \delta_{Bk}(\lambda_B)$ and $\delta_{Bk}(\lambda_B) > \delta_{Bk}(\lambda_B)$. If

there is a unique equilibrium then $c_3 > 0$ so that only the solution δ_{Bk} remains. Therefore, if equilibrium is unique then the increase in λ_A leads to a higher fraction of direct investors in equilibrium. If there are multiple equilibria, then the effect is ambivalent. ■