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Survival under Uncertainty in an Exchange Economy

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1 Introduction

The last twenty years have witnessed a significant growth of the literature on the "survival problem" ([25], p.436), primarily in the context of the causes and remedies of famines. Once a subject essentially of empirical development economics, economic survival became an issue of analytical economics and, most recently, of general equilibrium theory. Considerable progress has been achieved in the theoretical analysis and empirical investigations of the causes of famines and policy measures to combat famines (see the collection edited by Drezè [10] and the detailed list of references). There has been a recognition that a partial equilibrium model, focusing on the food market, is unable to capture the complexity of events that result in famines, and may indeed render misleading policy prescriptions. It is better to turn to general equilibrium models with an explicit treatment of survival, for a better understanding of the relevant issues.

Cast in a market economy framework, a formal analysis clearly indicates that an agent may fail to survive due to an "endowment failure" and/or "an adverse movement of the terms of trade" As Sen puts it in [25], " ... starvation is a matter of some people not *having* enough food to eat, and not a matter of there *being* not enough food to eat. While the latter can be a cause of the former, it is clearly one of many possible influences."[1] The Ethiopian famine in 1972-74 and the famine in Bangladesh in 1974 provide striking examples of the "terms of trade" effect, examples in which a particular group of agents got "decimated by the market mechanism." (Sen [26]) The famine victims often belonged to the groups of non-food producers. These individuals had to acquire food in the market in exchange for their output (or labor), and, thus, were more vulnerable to the shifts in the terms of trade affecting their food purchasing power (see also [20], p.14).

Sen's entitlement approach elaborated in [24] - [26], as well as the model of Coles and Hammond [7] are examples of static, deterministic analysis of the survival problem in a general equilibrium framework. Uncertainty was formally

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[1] As a matter of fact, "Some of the worst famines have taken place with no significant decline in food availability per head." ([26], p.17)

introduced, and the survival probability was precisely defined in a static Walrasian model in Bhattacharya and Majumdar [4]. Here again, an agent may fail to survive ("is ruined") in a particular state of the environment for two reasons: a meager endowment in this state (a direct effect on the individual) and/or an "unfavorable" equilibrium price system at which the wealth of the agent falls short of the minimum expenditure (computed at the equilibrium price system) needed for survival (an indirect terms of trade effect involving the preferences and the endowments of *all* the agents). The main results of Bhattacharya and Majumdar [4] and Hashimzade [13] (reviewed briefly in Section 2) characterize the probability of survival in a "large" Walrasian economy, under alternative assumptions on the nature of dependence among economic agents, when the endowments depend on the state of the environment.

In both these studies mentioned above the uncertainty is "intrinsic", *i.e.* affects one of the "fundamental characteristics" (endowments) of an economy. But in a dynamic world, adverse term-of-trade effects may emerge from "extrinsic" uncertainty, which may influence current prices through self-fulfilling beliefs or expectations. Static models are obviously inadequate to deal with such a role of expectations. Risk-averse agents tend to smooth consumption over time, and their intertemporal consumption decisions depend on their expectations about future endowments and prices. These decisions, in their turn, typically affect current equilibrium prices, as well as the probability of survival.

In Section 3 we explore the connection between survival and extrinsic uncertainty more formally by using the overlapping generations ("OLG") model (see [11] and [22]. A typical overlapping generations economy is an infinite horizon discrete-time economy with an infinite sequence of consumers, each living two periods. In every time period t there are "young" agents, born at t, and "old" agents, born at $t-1$. If young agents are endowed by consumption good(s), and old agents are endowed by nominal asset (fiat money), there is an opportunity for an inter-generational trade. We give an example of an overlapping generations economy in which an agent may be ruined *even when the fundamentals (endowments and preferences) of the economy are not affected by uncertainty. Self-fulfilling beliefs of the agents based on "sunspots" may generate an adverse terms of trade, i.e. may lead to an equilibrium price system at which the consumption of old agents is below the minimum subsistence level.* We note that there is already a vast literature on OLG models, following the seminal paper by Samuelson [22], and, in particular, on the role of extrinsic uncertainty (following the paper by Cass and Shell [5]), but neither this literature, nor the literature on the Arrow-Debreu model of complete markets treats the question of economic survival.

In Section 4, we turn to the question of insurance against risk, and we explore the role of markets for securities in the survival problem. Lack of insurance and financial markets and the very limited access to such markets for a vast number of agents characterize many developing countries. However, even the presence of *complete* markets for securities does not necessarily improve the chance of survival of an agent. Trade in securities allows us to achieve optimal allocation, when the set of securities is complete (an example is a complete set of Arrow securities [1]: suppose, there are two possible states of environment. Then a complete set of Arrow securities would be a set of two securities, each paying one monetary unit in one state and nothing in another). Even so, the optimal allocation can be such that the consumption of some agents falls below the survival threshold. We consider an economy where endowments of the agents are random, and the agents can trade a complete set of securities (in our example securities yield payoff denominated in a *numéraire* commodity, see [12]) to insure themselves against this type of intrinsic uncertainty. We show that trade in securities can, in fact, worsen survival prospects of the agents [2].

2 Equilibrium

In what follows, R_{++} is the set of positive real numbers, $x = (x_k) \in R^l$ is non-negative (written $x \geq 0$) if $x_k \geq 0$ for all k, and x is strictly positive (written $x \gg 0$) if $x \in R{+}{+}$.

Consider, first, a deterministic Walrasian exchange economy with two goods. Assume that an agent i has an initial endowment $e_i = (e_{i1}, e_{i2}) \gg 0$, and a Cobb-Douglas utility function

$$u(x_{i1}, x_{i2}) = x_{i1}^{\gamma} x_{i2}^{1-\gamma} \tag{1}$$

where $0 < \gamma < 1$ and the pair (x_{i1}, x_{i2}) denotes the quantities of goods 1 and 2 consumed by agent i. Thus an agent i is described by a pair $\alpha_i = (\gamma, e_i)$.

Let p be the price of the first good. In a Walrasian model with two goods, we can normalize prices so that $(p, 1-p)$ is the vector of prices *accepted by all the agents*. The typical agent solves the following maximization problem (P):

$$\text{maximize } u(x_{i1}, x_{i2}) \tag{2}$$

subject to the "budget constraint" defined as

$$px_{i1} + (1-p)x_{i2} = w_i(p)$$

where the income or wealth w_i of the i-th agent is defined as the value of its endowment computed at $(p, 1-p)$:

$$w_i(p) = pe_{i1} + (1-p)e_{i2}. \tag{3}$$

Solving the problem (P) one obtains the *excess* demand for the first good as:

$$\zeta_{i1}(p, 1-p) = [(1-p)/p]\gamma e_{i2} - (1-\gamma)e_{i1} \tag{4}$$

One can verify that

$$p\zeta_{i1}(p, 1-p) + (1-p)\zeta_{i2}(p, 1-p) = 0 \tag{5}$$

The *total excess demand* for the first good at the prices $(p, 1-p)$ in a Walrasian exchange economy with n agents is given by:

$$\zeta_1(p, 1-p) = \sum_{i=1}^{n} \zeta_{i1}(p, 1-p) \tag{6}$$

[2]We are not addressing the issue of practical implementation of securities or insurance policies. The point of this excercise is to demonstrate that the traditional approach to the equilibrium in market economies fails to tackle the survival problem precisely because the usual concept of Pareto optimality ignores the notion of survival.

In view of (5) it also follows that

$$p\zeta_1(p, 1-p) + (1-p)\zeta_2(p, 1-p) = 0 \tag{7}$$

The "market clearing" Walrasian equilibrium price system is defined by

$$\zeta_1(p^*, 1-p^*) = \zeta_2(p^*, 1-p^*) = 0 \tag{8}$$

and direct computation gives us the equilibrium price p_n^* (we emphasize the dependence of equilibrium price on the number of agents by writing p_n^*) as:

$$p_n^* = \left[\sum_{i=1}^{n} X_i\right] / \left[\sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i\right] \tag{9}$$

where

$$X_i = \gamma e_{i2},\ Y_i = (1-\gamma)e_{i1}. \tag{10}$$

To be sure, one can verify directly that demand equals supply in the market for the second good when the excess demand for the first good is zero.

Finally, let us stress that a Walrasian economy is "informationally decentralized" in the sense that agent i has no information about (e_j) for $i \neq j$. Thus it is *not* possible for agent i to compute the equilibrium price p_n^*.

2.1 Survival

In order to provide the motivation for our formal approach, we recall the basic elements of Amartya Sen's analysis ([26], Appendix A) in our notation. Let F_i be a (nonempty) closed subset of R^2_{++}. We interpret F_i as the set of all combinations of the two goods that enable the i-th agent to survive. Now, given a price system $(p, 1-p)$, one can define a function $m_i(p)$ as

$$m_i(p) = \min_{(x_{i1}, x_{i2}) \in F_i} \{px_{i1} + (1-p)x_{i2}\} \tag{11}$$

Thus, $m_i(p)$ is readily interpreted as the minimum expenditure needed for survival at prices $(p, 1-p)$.

Example: Let $(a_{i1}, a_{i2}) \gg 0$ be a fixed element of R^2_{++}. Let

$$F_i = \{(x_{i1}, x_{i2}) \in R^2_{++} : x_{i1} \geq a_{i1}, x_{i2} \geq a_{i2}\} \tag{12}$$

Here $m_i(p) = pa_{i1} + (1-p)a_{i2}$.

In our approach we do not deal with the set F_i explicitly. Instead, let us suppose that, in addition to its utility function and endowment vector, each agent i is characterized by a continuous function $m_i(p) : [0,1] \to R_{++}$, and say that for an agent to *survive* at prices $(p, 1-p)$, its wealth $w_i(p)$ (see (3) must exceed $m_i(p)$. Hence, the i-th agent *fails to survive* (or, *is ruined*) at the Walrasian equilibrium $(p_n^*, 1-p_n^*)$ if

$$[p_n^* e_{i1} + (1-p_n^*)e_{i2}] \leq m_i(p_n^*) \tag{13}$$

or, using the definition (3)

$$w_i(p_n^*) \leq m_i(p_n^*) \tag{14}$$

From (13) and (14) one can see that an agent may face ruin due to (a) a possible endowment failure or (b) the equilibrium price system adversely affecting its wealth relative to the minimum expenditure. This issue is linked to the literature on the "price" and "welfare" effects of a change in the endowment on a deterministic Walrasian equilibrium (see the review of transfer problem by Majumdar and Mitra [17]).

Observe that in our economy even with an exact information on the total endowment ($\sum_{i=1}^{n} e_{i1}$) of the first good ("food"), it is not possible to figure out how many agents may starve in equilibrium, in the absense of detailed information on the pattern of (e_i, m_i) (and the formula (9)).

2.2 Intrinsic uncertainty: computing the probability of ruin.

Let us introduce uncertainty. Suppose that the endowments $\mathbf{e}_i$ of the agents ($i = 1, 2, \cdots n$) are random variables. In other words, each $\mathbf{e}_i$ is a (measurable) mapping from a probability space$(\Omega, \mathcal{F}, P)$ into the non-negative orthant of R^2. One interprets Ω as the set of all possible states of environment, and $\mathbf{e}_i(\omega)$ is the endowment of agent i in the particular state ω. The distribution of $\mathbf{e}_i(\cdot)$ is denoted by μ_i [formally each μ_i is a probability measure on the Borel σ field of R^2, its support being a nonempty subset of the strictly positive orthant of R^2]. From the expression (9), the "market clearing" equilibrium price $p_n^*(\omega)$ is random, i.e., depends on ω:

$$p_n^* = \left[\sum_{i=1}^{n} \gamma e_{i2}(\omega)\right] / \left[\sum_{i=1}^{n} \gamma e_{i2}(\omega) + \sum_{i=1}^{n} (1-\gamma)\, e_{i1}(\omega)\right] \tag{15}$$

The wealth $w_i(p_n^*(\omega))$ of agent i at $p_n^*(\omega)$ is simply $p_n^*(\omega)e_{i1}(\omega)+[1-p_n^*(\omega)]e_{i2}(\omega)$. The event

$$\mathcal{R}_n^i = \{\omega \in \Omega : w_i(p_n^*(\omega)) \leq m_i(p_n^*(\omega))\}$$

is the set of all states of the environment in which agent i does not survive. Again, from the definition of the event $\mathcal{R}_n^i$ it is clear that an agent may be ruined due to a meager endowment vector in a particular state of environment. In what follows, we shall refer to this situation as a "direct" effect of endowment uncertainty or as an "individual" risk of ruin. But it is also possible for ruin to occur through an unfavorable movement of the equilibrium prices (terms of trade) even when there is no change (or perhaps an increase!) in the endowment vector. A Walrasian equilibrium price system reflects the entire pattern of endowment that emerges in a particular state of the environment. Given the role of the price system in determining the wealth of an agent and the minimum expenditure needed for survival, this possibility of ruin through adverse terms of trade can be viewed as an "indirect" ("terms of trade") effect of endowment uncertainty.

To begin with let us make the following assumptions:

A1. $\lim_{n\to\infty} \left[\sum_{i=1}^{n} VarX_i\right] /n^2 = 0;\ \lim_{n\to\infty} \left[\sum_{i=1}^{n} VarY_i\right] /n^2 = 0.$

A2. $\{X_i\}$ are uncorrelated, $\{Y_i\}$ are uncorrelated.

A3. $\left[(1/n)\sum_i EX_i\right]$ converges to some $\pi_1 > 0$, $\left[(1/n)\sum_i EY_i\right]$ converges to some $\pi_2 > 0$ as n tends to infinity.

In the special case when the distributions of $\mathbf{e}_i$ are the same for all i (so that $1/n \sum_i EX_i = \pi_1$, where π_1 is the common expectation of all X_i; similarly for π_2), A3 is satisfied.

Under A1-A3, if the number n of agents increases to infinity, as a consequence of the weak law of large numbers (see Lamperti [15], p.22) we have the following property of equilibrium prices p_n^*:

Proposition 1. *Under A1-A3, as n tends to infinity, $p_n^*(\omega)$ converges in probability to the constant*

$$p_0 = \pi_1 / [\pi_1 + \pi_2] \tag{16}$$

Roughly, one interprets (16) as follows: for large values of n, the equilibrium price will not vary much from one state of the environment to another, and will be insensitive to the exact value of n, the number of agents.

For the constant p_0 defined by (16), we have the following characterization of the probability of ruin in a large Walrasian economy:

Proposition 2. *If $p_0 e_{i1}(\omega) + (1 - p_0)e_{i2}(\omega)$ has a continuous distribution function,*

$$\lim_{n \to \infty} \left[P(\mathcal{R}_n^i)\right] = P\{\omega : p_0 e_{i1}(\omega) + (1 - p_0)e_{i2}(\omega) \leq m_i(p_0)\} \tag{17}$$

Remark: The probability on the right side of (17) does not depend on n, and is determined by μ_i, a characteristic of agent i, and p_0.

Our first task is to characterize $P(\mathcal{R}_n^i)$ when n is large (so that the assumption that an individual agent accepts market prices as given is realistic).

One is tempted to conjecture that the convergence property of Proposition 1 will continue to hold if correlation among agents becomes 'negligible' as the size of the economy increases. We shall indicate a 'typical' result that captures such intuition.

Proposition 3. *Let the assumptions (A1) and (A3) hold. Moreover, assume*

(A.2') There exist two non-negative sequences $(\mathcal{L}_k)_{k \geq 0}$, $(\mathcal{L}'_k)_{k \geq 0}$ both converging to zero such that for all i, k

$$\begin{aligned} |Cov(X_i, X_{i+k})| &\leq \mathcal{L}_k \\ |Cov(Y_i, Y_{i+k})| &\leq \mathcal{L}'_k \end{aligned}$$

Then, as n tends to infinity, $p_n^*(\omega)$ converges in probability to the constant

$$p_0 = \pi_1 / [\pi_1 + \pi_2]$$

2.3 Some comments on Walrasian equilibria

The analysis so far is deceptively simple for one primary reason. Once one dispenses with the Cobb-Douglas functional form, one loses the formula (15) in which a unique equilibrium in every ω is conveniently computed. A more general treatment - unavoidably more technical - is in [4] which contains the proofs of Propositions 1-3 above, and Proposition 4 below.

In a more general framework with $l \geq 2$ goods (see [8] and [9] for a classical exposition of the deterministic Walrasian equilibrium theory), we begin with the price simplex $S = \left\{ p = (p_k) \in R^l : p \geq 0, \sum_{k=1}^{l} p_k = 1 \right\}$. An agent i accepts the price system $p \in S$ as given. It is described by a pair (f_i, e_i), where the endowment vector $e_i \in R^l$, $e_i \gg 0$. The wealth of the agent i at p is $w_i = p \cdot e_i \equiv \sum_{k=1}^{l} p_k e_{ik}$. The demand function f_i is a continuous function from $S \times R_{++}$ to R^l_+ such that for every $(p, w_i) \in S \times R_{++}$, $p \cdot f_i(p, w_i) = w_i$ (where $p \cdot f_i(p, w_i) \equiv \sum_{k=1}^{l} p_k f_{ik}(p, w_i)$). Usually the demand functions are derived from a utility maximization problem of type (P) indicated above. For our analysis, the key concepts are the excess demand function of agent i, defined as $\hat{\zeta}_i(p) \equiv f_i(p, w_i) - e_i$ (compare to (4)). The excess demand function for the economy is $\hat{\zeta}(p) = \sum_{i=1}^{l} \hat{\zeta}_i(p)$, a continuous function on S. Note that $\sum_{k=1}^{l} p_k \hat{\zeta}_{ik}(p) = 0$; hence, the excess demand function for the economy satisfies the "Walras Law":

$$p \cdot \hat{\zeta}(p) \equiv \sum_{k=1}^{l} p_k \hat{\zeta}_k(p) = 0. \tag{18}$$

An equilibrium price system $p^* \in S$ satisfies $\hat{\zeta}(p^*) = 0$.

By Walras Law (18), if for any $\hat{p} \in S$ $\hat{\zeta}_k(\hat{p}) = 0$ for $k = 1, \cdots, l-1$ then necessarily $\hat{\zeta}_k(\hat{p}) = 0$ for $k = l$. The Walras Law (18) can be verified directly from (3) and (4) in our example, and when the equilibrium price (9) is derived for the first market, there is also equilibrium in the second market which can be directly checked. A detailed exposition of this model with $l \geq 2$ commodities is in Debreu [9]. In [3] the Debreu model was extended to introduce random preferences and endowments, and the implications of the law of large numbers and the central limit theorem were first systematically explored. Throughout this section we shall consider $l = 2$ to see the main results in the simplest form.

2.4 Dependence: Exchangeability

We shall now see that if dependence among agents does not "disappear" even when the economy is large, the risk of ruin due to the "indirect" terms of trade effect of uncertainty may remain significant. To capture this in a simple manner, let us say that μ and ν are two possible probability laws of $\{\mathbf{e}_i(\cdot)\}_{i \geq 1}$. Think of Nature conducting an experiment with two outcomes "H" and "T" with probabilities $(\theta, 1-\theta), 0 < \theta < 1$. Conditionally, given that "H" shows up, the sequence $\{\mathbf{e}_i(\cdot)\}_{i\geq 1}$ is independent and identically distributed with common distribution μ. On the other hand, conditionally given that "T" shows up, the sequence $\{\mathbf{e}_i(\cdot)\}_{i\geq 1}$ is independent and identically distributed with comon distribution ν. Let $\pi_{1\mu}$ and $\pi_{1\nu}$ be the expected values of X_1 under μ and ν respectively. Similarly, let $\pi_{2\mu}$ and $\pi_{2\nu}$ be the expected values of Y_1 under μ and ν. It follows that $p_n(\cdot)$ converges to $p_0(\cdot)$ almost surely, where $p_0(\cdot) = \pi_{1\mu}/[\pi_{1\mu}+\pi_{2\mu}] = p_{0\mu}$ with probability θ and $p_0(\cdot) = \pi_{1\nu}/[\pi_{1\nu}+\pi_{2\nu}] = p_{0\nu}$ with probability $1-\theta$. We now have a precise characterization of the probabilities of ruin as n tends to infinity. To state it, write

$$\begin{aligned} J &= \{(u_1,u_2)\in R_+^2 : p_{0\mu}u_1 + (1-p_{0\mu})u_2 \leq m_i(p_{0\mu})\}; \\ r_i(\mu) &= \int_J \mu(du_1, du_2). \end{aligned} \tag{19}$$

Similarly, define $r_i(\nu)$ obtained on replacing μ by ν in (19).

Proposition 4. *Assume that $p_0e_{i1}(\omega)+(1-p_0)e_{i2}(\omega)$ had a continuous distribution function under each distribution μ and ν of $\mathbf{e}_i = (e_{i1}, e_{i2})$.*

(a) *Then, as the number of agents n goes to infinity, the probability of ruin of the i-th agent converges to $r_i(\mu)$, with probability θ, when "H" occurs and to $r_i(\nu)$, with probability $1-\theta$ when "T" occurs.*

(b) *The overall, or unconditional, probability of ruin converges to*

$$\theta r_i(\mu) + (1-\theta) r_i(\nu).$$

Here, the precise limit distribution is slightly more complicated, but the important distinction from the case of independence (or, "near independence") is that the limit depends not just on the individual uncertainties captured by the distributions μ and ν of an agent's endowments, but also on θ that retains an influence on the distribution of prices even with large n.

2.5 Dependency neighborhoods

Dependency neighborhoods were introduced by Stein [28] and are defined in the following way. Consider a set of n random agents. A subset S_i of the set of integers $\{1, 2, ..., n\}$ containing an agent i is a dependency neighborhood of i if i is independent of all agents not in S_i. The sets S_i need not constitute a partition. Further, consider a dependency neighborhood of S_i - a set N_i such that $S_i \subseteq N_i$, and the collection of agents in S_i is independent of the collection of agents not in N_i. The latter can be viewed as the second-order dependency neighborhood of the agent i. In general,

$$N_i = \bigcup_{\{j\in S_i\}} S_j \tag{20}$$

need not be the case (this is related to the fact that pairwise independence does not imply mutual independence), although one might expect this relation to hold in non-exotic situations (see, for example, [21]).

Consider now an economy $\mathcal{E}_n$ with dependency neighborhoods $S_1^{(n)}, \ldots, S_n^{(n)}$ for each of n agents. As above, the i-th agent is characterized by $\alpha_i = (\gamma, \mathbf{e}_i)$, where $\mathbf{e}_i = (e_{i1}, e_{i2})$. The Walrasian equilibrium price p_n^* is given by (9)-(10). The convergence property, similar to Proposition 3, holds under modified assumptions on the distribution of random endowments and an additional assumption on the size of the dependency neighborhood.

Proposition 5. *Let the assumptions (A1) and (A3) hold. Moreover, assume*

(A.2") $B_{ni} \equiv \max_{i \neq j \in S_i^{(n)}} \mid Cov(Z_i, Z_j) \mid < B < \infty$, $Z \in \{X, Y\}$, for every $i = 1, \ldots, n$ uniformly in n for some sufficiently large positive B.

(A.4) $s_n \equiv \max_{i=1,\ldots,n} \sharp S_i^{(n)} \leq n^{1-\epsilon}$ uniformly in n for some $\epsilon \in (0,1)$.

Then, as n tends to infinity, $p_n^*(\omega)$ converges in probability to $p \lim p_n^*(\omega) = \dfrac{\pi_1}{\pi_1 + \pi_2}$.

Using the results of Majumdar and Rotar [19], we can construct approximate distribution of equilibrium price in a large Walrasian economy.

Proposition 6. *Let the assumptions (A.1), (A.2"), (A.3) and (A.4) hold. Let also assume that (20) holds for the dependency neighborhoods structure. Then the distribution of $p_n^*(\omega)$ can be approximated by normal distribution with mean p_0 and variance v_n defined as*

$$p_0 = \frac{\pi_1}{\pi_1 + \pi_2} \tag{21}$$

$$v_n = \frac{1}{(\pi_1 + \pi_2)^4} \frac{1}{n^2} Var\left[\sum_{i=1}^{n} (\pi_2 X_i - \pi_1 Y_i)\right] \tag{22}$$

(See [13] for proofs.)

3 Extrinsic uncertainty with overlapping generations: an example.

In the previous section we assumed that endowments of the agents are different in different states of environment. This type of uncertainty, that affects the so-called *fundamentals* of the economy (endowments, preferences, and technology), is called the *intrinsic* uncertainty. When the uncertainty affects the *beliefs* of the agents (for example, the agents believe that market prices depend on some "sunspots") whereas the fundamentals are the same in all states, this type of uncertainty is called *extrinsic* uncertainty. Clearly, with respect to the probability of survival, the extrinsic uncertainty has no direct effect, because it does not affect the endowments. However, it may have an indirect effect: self-fulfilling beliefs of the agents regarding market prices affect their wealth, and some agents may be ruined in one state of environment and survive in some other state, even though the fundamentals of the economy are the same in all states. To study the indirect, or the adverse term-of-trade effect of extrinsic uncertainty on survival we turn to a dynamic economy.

Consider a discrete time, infinite horizon OLG economy with constant population. We use Gale's terminology [11] wherever appropriate. For expository simplicity, and without loss of generality we assume that at the beginning of every time period $t = 1, 2, \cdots$ there are two agents: one "young" born in t, and one "old" born in $t-1$. In period $t = 1$ there is one old agent of generation 0. There is one (perishable) consumption good in every period. The agent born in t (generation t) receives an endowment vector $e_t = (e_t^y, e_t^o)$ and consumes a vector $c_t = (c_t^y, c_t^o)$. We consider the Samuelson case [3] and assume, without loss of generality, $e_t = (1, 0)$. We assume that the preferences of the agent of generation t can be represented by expected utility function $\mathcal{U}_t(\cdot) = E\left[U^t(c_t)\right]$ with Bernoulli utility $U^t(c_t)$, continuously differentiable and almost everywhere twice continuously differentiable, strictly concave and strictly monotone in $\mathcal{D}$, compact, convex subset of R^2_{++}. The old agent of generation 0 is endowed with one unit of fiat money, the only nominal asset in the economy. In every period the market for the perishable consumption good is open and accessible to all agents. Denote the nominal price of the consumption good at time t by p_t. Define a *price system* to be a sequence of positive numbers, $\mathbf{p} = \{p_t\}_{t=0}^{\infty}$, a *consumption program* to be a sequence of pairs of positive numbers $\mathbf{c} = \{c_t\}_{t=0}^{\infty}$, a *feasible program* to be a consumption program that satisfies $c_t^y + c_{t-1}^o \leq e_t^y + e_{t-1}^o = 1$. The agent of generation t maximizes his lifetime expected utility in the beginning of period t. In period 1, the young agent gives its saving (s_1^y) of the consumption good, to the old agent in exchange for one unit of money (the exchange rate is determined by p_1). Thus, $p_1 s_1 = 1$. This unit of money is carried into period 2 (the old age of agent born in period 1) and is exchanged (at the rate determined by p_2) for the consumption food saved by the young agent born in period 2 (s_2^y). The process is repeated.

3.1 Perfect Foresight Equilibrium

If there is no uncertainty, with perfect foresight the price-taking young agent's optimization problem is the following:

$$\max U(c_t^y, c_t^o)$$

subject to

$$\begin{aligned} c_t^y &= 1 - s_t^y \\ c_t^o &= p_t s_t^y / p_{t+1} \end{aligned}$$

$(0 \leq s_t^y \leq 1,\ t = 1, 2, \cdots)$.

Here, $s_t^y \equiv e_t^y - c_t^y$ is savings of the young agent (this is the Samuelson case, in Gale's definitions [11]). A *perfect foresight competitive equilibrium* is defined as a feasible program and a price system such that

(i) the consumption program $\bar{\mathbf{c}} = \{\bar{\mathbf{c}}_\mathbf{t}\}$ solves optimization problem of each agent given $\bar{\mathbf{p}} = \{\bar{p}_t\}$: $(\bar{c}_t^y, \bar{c}_t^o) \in \mathcal{D}$, $\bar{c}_t^y = 1 - s_t$ and $\bar{c}_t^o = \bar{p}_t s_t / \bar{p}_{t+1}$ with

$$s_t = arg \max_{0 \leq s_t^y \leq 1} U\left((1 - s_t^y), s_t^y \frac{\bar{p}_t}{\bar{p}_{t+1}}\right)$$

and

[3] If a population grows geometrically at the rate γ, so that γ^t agents is born in period t, and there is only one good in each period, the Samuelson case corresponds to marginal rate of intertemporal substitution of consumption under autarky, $U_1(e^y, e^o)/U_2(e^y, e^o)$, being less than γ. In our case $\gamma = 1$.

(ii) the market for consumption good clears in every period:

$$\begin{array}{rclcll} \bar{c}_t^y + \bar{c}_{t-1}^o & = & 1 & (\text{demand} & = & \text{supply for the consumption good}) \\ \bar{p}_t s_t & = & 1 & (\text{demand} & = & \text{supply for money}) \end{array}$$

for $t = 1, 2, \cdots$.

By strict concavity of the utility function $U(c_t^y, c_t^o)$, the young agent's optimization problem has a unique solution. Hence, we can express s_t as a single-valued function of p_t/p_{t+1}, *i.e.* we write $s_t = s_t(p_t/p_{t+1})$. This function (called savings function) generates an offer curve in the space of net trades, as price ratios vary. In the perfect foresight equilibrium

$$s_t(p_t/p_{t+1}) = 1/p_t. \tag{23}$$

The stationary perfect foresight monetary equilibrium is a sequence of constant prices p and constant consumption programs $(1 - \bar{s}, \bar{s})$, where $\bar{s} = s(1)$.[4]

3.2 Sunspot equilibrium

Now consider an extrinsic uncertainty in this economy. There is no uncertainty in fundamentals, such as endowments and preferences, but the agents believe that market prices depend on realization of an extrinsic random variable (sunspot). We assume that there is one-to-one mapping from the sunspot variable to price of the consumption good. Because the agents cannot observe future sunspots, they maximize expected utility over all possible future realization of the states of nature. We examine the situation with two states of nature, $\sigma \in \{\alpha, \beta\}$, that follow a first-order Markov process with stationary transition probabilities,

$$\Pi = \begin{bmatrix} \pi^{\alpha\alpha} & \pi^{\alpha\beta} \\ \pi^{\beta\alpha} & \pi^{\beta\beta} \end{bmatrix} \tag{24}$$

where $\pi^{\sigma\sigma'} > 0$ is the probability of being in state σ' in the next period given that current state is σ, and $\pi^{\sigma\alpha} + \pi^{\sigma\beta} = 1$. A young agent born in t observes price p_t^σ and solves the following optimization problem:

$$\max \left[\pi^{\sigma\alpha} U(c_t^{y,\sigma}, c_t^{o,\alpha}) + \pi^{\sigma\beta} U(c_t^{y,\sigma}, c_t^{o,\beta})\right]$$

subject to

$$\begin{array}{rcl} c_t^{y,\sigma} & = & 1 - s_t^\sigma \\ c_t^{o,\sigma'} & = & p_t^\sigma s_t^\sigma / p_{t+1}^{\sigma'} \end{array}$$

$(0 \leq s_t^\sigma \leq 1,\ s_t^{\sigma'} \geq 0,\ \sigma, \sigma' \in \{\alpha, \beta\})$.

We restrict our attention to stationary equilibria, in which prices depend on the current realization of the state of nature σ, and do not depend on the calendar time nor the history of σ. A *stationary sunspot equilibrium*, SSE, is a pair of feasible programs and nominal prices, such that for every $\sigma \in \{\alpha, \beta\}$

[4] Given our assumptions on preferences and endowments, the stationary perfect foresight monetary equilibrium exists and is optimal (see, for example, [16], Ch. 8).

(i) the consumption programs solve the agents' optimization problem:

$$\begin{aligned} & s^{\sigma}(p^{\sigma}/p^{\sigma'}) = \\ arg \max_{0\le s^{\sigma}\le 1} & \left[\pi^{\sigma\alpha}U\left((1-s^{\sigma}), s^{\sigma}p^{\sigma}/p^{\alpha}\right) + \right. \\ & \left. +\pi^{\sigma\beta}U\left((1-s^{\sigma}), s^{\sigma}p^{\sigma}/p^{\beta}\right)\right] \end{aligned} \tag{25}$$

and

(ii) markets clear in every period, in every state.

$$\begin{aligned} c^{y,\sigma} + c^{o,\sigma} &= 1 \\ p^{\sigma}s^{\sigma} &= 1 \end{aligned}$$

It is easy to see that a stationary sunspot equilibrium exists when the equation

$$\frac{p^{\alpha}}{p^{\beta}}s^{\alpha}\left(\frac{p^{\alpha}}{p^{\beta}}\right) - s^{\beta}\left(\frac{p^{\beta}}{p^{\alpha}}\right) = 0 \tag{26}$$

has positive solutions for p^{α}/p^{β} other than 1 . Solution $p^{\alpha}/p^{\beta} = 1$ corresponds to the equilibrium in which uncertainty does not matter. It can be shown that, if sunspot equlibria exist in this economy, there is at least two of them, with $p^{\alpha}/p^{\beta} > 1$ and $p^{\alpha}/p^{\beta} < 1$ (see, for example, [6], [27]). This means that in the sunspot equilibrium consumption of old agents is above the certainty equilibrium consumption of olds in one state of nature and below in the other. Suppose, we introduce an exogenous minimal subsistence level of consumption (independent of $\sigma \in \{\alpha, \beta\}$). It may be the case that in one of the states of nature consumption of old agents falls short of minimal subsistence level: old agents are ruined. Note that the endowments are not affected by the uncertainty, and, therefore, there is no *direct* effect of uncertainty on ruin. The event of ruin is caused purely by an *indirect*, or term-of-trade effect: the equilibrium price system is such that the wealth of old agents does not allow them to survive. The following numerical example illustrates this possibility for the case of quadratic utility.

3.3 Ruin in equilibrium.

Let the preferences of the agents be represented by expected utility function with

$$\begin{aligned} U(\mathbf{c}) &= u(c^y, c^o) - v(c^o) \\ u(c^y, c^o) &= 2a\sqrt{c^y c^o} + q\,c^y + r\,c^o - \frac{1}{2}b(c^y)^2 - \frac{1}{2}d(c^o)^2 \\ v(c^o) &= \begin{cases} \frac{\theta}{2}\left(A - c^o\right)^2, & 0 < c^o \le A \\ 0, & c^o > A \end{cases} \end{aligned}$$

where a, b, c, q, r, θ, A are positive constants such that the utility function is increasing and jointly concave in its arguments in $\mathcal{D}$. $v(\cdot)$ is the disutility of consuming less than A, the minimal subsistence level.[5] As above, agents in each generation receive identical positive endowments $e = 1$ of consumption good when young and zero endowments when old; the initial olds are endowed with one unit of money.

[5]It may seem odd that the disutility from starvation is finite, but this can be justified by the willingness of the agents to take a risk. Consider the following. In the continuous time,

3.3.1 Benchmark case: perfect foresight

For the above preferences, savings function $s_t(p_t/p_{t+1})$ is implicitly defined by

$$\rho_t = \frac{a\sqrt{\rho_t s_t/(1-s_t))} + q - b\,(1-s_t)}{a\sqrt{(1-s_t)/(\rho_t s_t)} + r - d\,\rho_t s_t - v'(\rho_t s_t)}, \tag{27}$$

where $\rho_t \equiv p_t/p_{t+1}$. The offer curve is described by

$$(1-x)\left(a\sqrt{\frac{y}{x}} + q - b\,x\right) - y\left(a\sqrt{\frac{x}{y}} + r - d\,y - v'(y)\right) = 0 \tag{28}$$

In the stationary (deterministic) perfect foresight monetary equilibrium consumption plan of an agent is $(x, 1-x)$, where x solves

$$a\left(\sqrt{\frac{x}{1-x}} - \sqrt{\frac{1-x}{x}}\right) + x\,(b+d) + v'(1-x) + q - r - b = 0 \tag{29}$$

3.3.2 Stationary sunspot equilibria

Two states of nature, α and β evolve according to a stationary first-order Markov process. The states of nature do not affect the endowments. Agents can trade their real and nominal assets. In a stationary sunspot equilibrium with trade s^α, s^β solve the following system of equations:

$$\pi^{\alpha\alpha} a\sqrt{\tfrac{s^\alpha}{1-s^\alpha}} + (1-\pi^{\alpha\alpha})\,a\sqrt{\tfrac{s^\beta}{1-s^\alpha}} + q - b\,(1-s^\alpha) = \tag{30}$$

$$= \quad \pi^{\alpha\alpha} a\left(\sqrt{\tfrac{1-s^\alpha}{s^\alpha}} + r - d\,s^\alpha - v'(s^\alpha)\right) + (1-\pi^{\alpha\alpha})\left(a\sqrt{\tfrac{1-s^\alpha}{s^\beta}} + r - d\,s^\beta - v'(s^\beta)\right)\tfrac{s^\beta}{s^\alpha}$$

and

$$\pi^{\beta\beta} a\sqrt{\tfrac{s^\beta}{1-s^\beta}} + \left(1-\pi^{\beta\beta}\right) a\sqrt{\tfrac{s^\alpha}{1-s^\beta}} + q - b\left(1-s^\beta\right) = \tag{31}$$

$$= \quad \pi^{\beta\beta} a\left(\sqrt{\tfrac{1-s^\beta}{s^\beta}} + r - d\,s^\beta - v'(s^\beta)\right) + \left(1-\pi^{\beta\beta}\right)\left(a\sqrt{\tfrac{1-s^\beta}{s^\alpha}} + r - d\,s^\alpha - v'(s^\alpha)\right)\tfrac{s^\alpha}{s^\beta}$$

It is easy to see that one solution is $s^\alpha = s^\beta = 1-x$, where x solves the equation for the perfect foresight above. This solution does not depend on the transition probabilities, prices and consumption are not affected by the uncertainty: sunspots *do not matter* in this equilibrium. However, there may be more solutions. For example, for $a = 2$, $b = 0.5$, $d = 7$, $q = 0.02$, $r = 0.6\,\theta = 0.05\,A = 0.3$ and $\pi^{\alpha\alpha} = \pi^{\beta\beta} = 0.15$ there are three stationary monetary equilibria in the economy: one coinciding with the perfect foresight equilibrium and two sunspot equilibria. Prices and consumption programs for these equilibria are given in the following table.

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if the consumption of an old agent is above A, he lives to the end of the second period. If his consumption is below A, perhaps, he does not die immediately. Albeit low, the amount consumed allows him to live some time in the second period, and his lifespan in the second period is the longer, the closer is his consumption to A. In the discrete time this translates into probability of survival in the second period as a function of consumption. Thus, the old agent survives with probability 1 if $c^o \geq A$ and with probability less than 1 if $c^o < A$. Suppose, the objective of the agent is to maximize the probability of survival (or maximize his expected lifespan). Then it can be presented equivalently as the objective to minimize the disutility from consumption at the level below A. Clearly, this disutility can be finite, at least in the vicinity of A, if the agent is willing to take a risk. The authors are indebted to David Easley for this argument.

State	PFE	1st SSE	2nd SSE
α	(0.6670; 0.3330; 3.00)	(0.5973; 0.4027; 2.48)	(0.7518; 0.2482; 4.03)
β	(0.6670; 0.3330; 3.00)	(0.7518; 0.2482; 4.03)	(0.5973; 0.4027; 2.48)

(In every entry, the first number is consumption of young, the second is consumption of old, and the third is nominal price of consumption good.)

The consumption programs in sunspot equilibria are Pareto inferior to the program in the perfect foresight equilibrium. Furthermore, in two sunspot equilibria old agents survive in one state of nature and fail to survive in another *with the same amount of resources*, because equilibrium price is too high. (We intentionally considered the case where agents survive in the certainty equilibrium to demonstrate that survival is always feasible. Also, in this model young agents always survive, - otherwise, the overlapping generations structure collapses.)

4 Insurance and survival

The purpose of the following examples is to demonstrate that trade in securities does not guarantee survival of all agents. Furthermore, trade in securities can even deteriorate the survival chances of some agents. For expositionary simplicity, we consider a static Cobb-Douglas-Sen economy, similar to the one described in Section 2.

4.1 Static economy with two states: definitions

Let us first restate the definitions of a stochastic general equilibrium concept in a Cobb-Douglas-Sen economy with logarithmic preferences for a particular case of two possible states of environment. Consider a pure exchange economy with two goods, $l \in \{1,2\}$, with good 1 being a *numéraire*. There are two states of nature, $s \in \Omega = \{\alpha, \beta\}$, with $\pi = P[s = \alpha] = 1 - P[s = \beta]$. Two consumers, $i \in \{1,2\}$, receive endowments $\mathbf{e}_i(s) = (e_{i1}(s), e_{i2}(s)) \in R_2^+$. Each consumer is characterized by the Cobb-Douglas logarithmic utility function:

$$u(x_{i1}, x_{i2}) = \gamma_i \ln x_{i1} + (1 - \gamma_i) \ln x_{i2}. \tag{32}$$

In addition, each consumer is characterized by the minimum expenditure function, $m_i(p^*(\cdot))$, the level of wealth at and below which consumer i fails to survive in the equlibrium with (random, normalized) equilibrium price vector $(1, p^*(\cdot))$. Consumers maximize utility in every state, taking price as given. A *random equilibrium* is defined as a set of vectors of allocations, $\{\mathbf{x}_i(s)\}$, and prices, $p^*(s)$ for each state of nature, such that

- Given normalized price vector $(1, p^*(s))$ in state s, consumption vector $\mathbf{x}_i(s) = (x_{i1}(s), x_{i2}(s))$ maximizes utility of consumer i in state s subject to his budget constraint, $x_{i1}(s) + p^*(s)x_{i2}(s) \leq e_{i1}(s) + p^*(s)e_{i2}(s)$, for every i and s;

- Markets for consumtpion goods clear in every state.

If we allow γ (the parameter in Cobb-Douglas preferences) vary across the consumers, the equilibrium price in state s will be

$$p^*(s) = \frac{\sum_i (1-\gamma_i)\, e_{i1}(s)}{\sum_i \gamma_i e_{i2}(s)}.$$

Hence, wealth of consumer i in state s is

$$w_i(s) = e_{i1}(s) + p^*(s)e_{i2}(s) = e_{i1}(s) + \frac{\sum_i (1-\gamma_i)\, e_{i1}(s)}{\sum_i \gamma_i e_{i2}(s)} e_{i2}(s).$$

Assume, for simplicity, that the minimum expenditure function is the same for all agents and has linear form:

$$m(p^*(s)) = a_0 + p^*(s)a_1$$

for some positive constants a_0 and a_1. Then, consumer i is ruined in state s if

$$e_{i1}(s) + p^*(s)e_{i2}(s) \leq a_0 + p^*(s)a_1.$$

If this inequality holds for consumer i for $s = \alpha$ only, then consumer i is ruined with probability π. If it holds for $s = \beta$ only, then i is ruined with probability $(1-\pi)$. If it holds for consumer i in both states, then i is ruined with probability 1.

Suppose, consumers know π. The question is, if consumers could trade securities before s is realized, would this improve their chances to survive?

4.2 Arrow-type securities in a two-period economy

Assume now, that in the economy described in Section 4.1 there are two time periods, $t = 0, 1$. Let the preferences of the consumers be described by von Neumann-Morgenstern expected utility function, with Bernoulli utility in the log Cobb-Douglas form (32), with γ varying across consumers.

At $t = 0$ consumers can issue and trade contracts in real Arrow-type securities. At $t = 1$ consumers receive their endowments, execute the contracts and trade consumption goods. Markets for securities are complete: for every state of nature there is a security that promises to deliver at $t = 1$ one unit of *numéraire* good if this particular state occurs, and nothing in other states (see [23] and [12] for a more general exposition). Denote the holdings of security that pays in state s by y_i^s for consumer i; $y_i^s \in R$. Consumers know probability distribution of the states of nature. In time period $t = 0$ they choose holdings of securities, or portfolios, (y_i^α, y_i^β) to maximize expected utility of consumption in time period $t = 1$. We normalize price of the asset that pays in state α to unity and denote price of the asset that pays in state β by q. A *random equilibrium with complete asset markets* is a set of vectors of portfolios $\{(\bar{y}_i^\alpha, \bar{y}_i^\beta)\}$, allocations $\{\bar{\mathbf{x}}_i(s)\}$, security prices $(1, \bar{q})$ and consumption good prices $(1, \bar{p}(s))$ for each state of nature, such that

- Given asset prices $(1, \bar{q})$ and normalized consumption good price vector $(1, \bar{p}(s))$ in state s, portfolio (y_i^α, y_i^β) and consumption vector $\mathbf{x}_i(s) = (x_{i1}(s), x_{i2}(s))$ maximize expected utility of consumer i at $t = 0$ subject to his budget constraints at $t = 0$, $y_i^\alpha + qy_i^\beta \leq 0$, at $t = 1$, $x_{i1}(s) + p^*(s)x_{i2}(s) \leq e_{i1}(s) + p^*(s)e_{i2}(s) + y_i^s$, for every i and s;

- Asset markets clear at $t = 0$;
- Markets for consumption goods clear at $t = 1$ in every state.

Routine calculations give the following expressions for equilibrium prices:

$$\begin{aligned}\bar{q} &= \frac{1-\pi}{\pi}\frac{E_1(\alpha)}{E_1(\beta)} \\ \bar{p}(\beta) &= \bar{p}(\alpha)\frac{E_2(\alpha)E_1(\beta)}{E_1(\alpha)E_2(\beta)}\end{aligned}$$

and

$$\bar{p}(\alpha) = \frac{\sum_i (1-\pi\gamma_i)e_{i1}(\alpha) - (1-\pi)\sum_i \gamma_i e_{i1}(\beta)E_1(\alpha)/E_1(\beta)}{\pi\sum_i \gamma_i e_{i2}(\alpha) + (1-\pi)\sum_i \gamma_i e_{i2}(\beta)E_2(\alpha)/E_2(\beta)}.$$

Here, $E_l(s) \equiv \sum_i e_{li}(s)$ is aggregate endowment of good l in state s. Wealth (in terms of the *numéraire*) of consumer i at $t = 1$ is then

$$\begin{aligned}\bar{W}_i(\alpha) &= \pi\bar{w}_i(\alpha) + (1-\pi)\frac{E_1(\alpha)}{E_1(\beta)}\bar{w}_i(\beta) \\ \bar{W}_i(\beta) &= (1-\pi)\bar{w}_i(\beta) + \pi\frac{E_1(\beta)}{E_1(\alpha)}\bar{w}_i(\alpha)\end{aligned}$$

where $\bar{w}_i(s) = e_{i1}(s) + \bar{p}(s)e_{i2}(s)$, $s = \alpha, \beta$.

Note, that $\bar{W}_i(\beta) = \dfrac{E_1(\beta)}{E_1(\alpha)}\bar{W}_i(\alpha)$, which means that if there is no aggregate uncertainty in the endowment of *numéraire*, wealth is equalized across states. If there is no aggregate uncertainty in the endowments of both goods, relative price of consumption goods is also equalized across states. Then $\bar{p} = \bar{p}(\alpha) = \bar{p}(\beta)$ will be between $p^*(\alpha)$ and $p^*(\beta)$ and $\bar{W}_i = \bar{W}_i(\alpha) = \bar{W}_i(\beta)$ will be between $w_i(\alpha)$ and $w_i(\beta)$. For the minimum expenditure function in the above form, we will also have that $m_i(\bar{p}) = m_i(\bar{p}(\alpha)) = m_i(\bar{p}(\beta))$ will be between $m_i(p^*(\alpha))$ and $m_i(p^*(\beta))$. Could it happen that wealth of a consumer in a particular state falls below the minimum subsistence level in an economy with securities, whereas without securities his wealth in the same state is above the minimum subsistence level?

The following simple numerical examples demonstrate this possibility for the case with no aggregate uncertainty and for the case with aggregate uncertainty in endowments.

4.2.1 Example A: No Aggregate Uncertainty

Consider an economy with two consumers, $i \in \{1, 2\}$. Let the preferences of these two consumers and their endowments in two states be the following:

Consumer i	γ_i	$\mathbf{e}_i(\alpha)$	$\mathbf{e}_i(\beta)$
$i = 1$	1/2	$(1, 0)$	$(0, 2)$
$i = 2$	1/3	$(1, 4)$	$(2, 2)$

Let $P[s=\alpha] = 1 - P[s=\beta] = \pi = 1/4$. Then in the equilibrium without securities

$$\begin{aligned} p^*(\alpha) &= \frac{7}{8} \\ p^*(\beta) &= \frac{4}{5} \end{aligned}$$

and in the equilibrium with securities

$$\bar{p}(\alpha) = \bar{p}(\beta) = \frac{31}{38}.$$

Suppose, both consumers have minimal expenditure function in the linear form, with the same parameters $a_0 = 3/4$ and $a_1 = 1$. Then the survival threshold in the economy without securities is 1.625 in state α and 1.55 in state β. It is easy to see that agent $i = 1$ is ruined in state $s = \alpha$ and survives in state $s = \beta$; agent $i = 2$ survives in both states. With securities, the survival threshold in both states is ≈ 1.5658, and agent $i = 2$ still survives in both states, but agent $i = 1$ is now ruined in both states.

4.2.2 Example B: Aggregate Uncertainty

Consider the same economy, now with aggregate uncertainty in the endowments:

Consumer i	γ_i	$\mathbf{e}_i(\alpha)$	$\mathbf{e}_i(\beta)$
$i = 1$	$1/2$	$(1, 0)$	$(0, 2)$
$i = 2$	$1/3$	$(0, 2)$	$(2, 2)$

With $\pi = 1/4$ the equilibrium price without securities is

$$\begin{aligned} p^*(\alpha) &= \frac{3}{4} \\ p^*(\beta) &= \frac{4}{5} \end{aligned}$$

and with securities

$$\begin{aligned} \bar{p}(\alpha) &= \frac{15}{19} \\ \bar{p}(\beta) &= \frac{30}{19} \end{aligned}$$

Let the minimal expenditure function for both consumers be linear, with $a_0 = 1/5$ and $a_1 = 1$. The survival threshold in an economy without securities is, then, 0.95 in state α and 1 in state β. Both agents survive in both states. With securities, the survival threshold is ≈ 0.990 in state α and ≈ 1.779 in state β. In that case, agent 2 still survives in both states, but agent 1 survives only in β and is ruined in α.

These two examples demonstrate how trade in securities may worsen survival prospects of the agents with random endowments even when markets for securities are complete.

5 Concluding remarks

In this paper we introduced a formal general equilibrium approach to the problem of survival under uncertainty. The question of obvious practical importance is "how does one improve the chance of survival of an agent"? Clearly, when ruin is caused by market forces, the intervention of the government is desirable. The choice of the optimal policy is determined by the policy tools available to the government, and the sensitivity of the survival probability to the changes in policy variables. For the case of static economy with intrinsic uncertainty this problem was touched upon in [4]. In particular, under certain assumptions on the joint distribution of the endowments and linearity of the minimum expenditure function, the probability of survival of an agent increases as the limiting averages of the endowments increase. For the OLG economy with extrinsic uncertainty we showed elsewhere [14] that a lump-sum tax and transfer policy, with the amounts of taxes and transfers depending on equilibrium market price, can stabilize consumption at certainty equilibrium level (without affecting prices), thus eliminating the possiblity of ruin of the agents. In any case, the general equlibrium framework has to be used in order to accurately predict the outcomes of various policy measures.

Another issue should be mentioned. Throughout this paper we assumed that the objective of an agent is to maximize his expected utility (as the traditional economic theory postulates). In a model with a single agent Majumdar and Radner [18] explored the implications for maximization of the probability of survival. A systematic extension of this analysis to a framework with many interacting agents remains an important direction of research.

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