Granger Causality and Equilibrium Business Cycle Theory*

Yi Wen
Department of Economics
Cornell University

Abstract

Post war US data show that consumption growth causes output and investment growth. This is puzzling if technology is the driving force of the business cycle. I ask whether general equilibrium models driven by demand shocks can rationalize the observed causal relations. My conclusion is that business cycle theory remains behind business cycle measurement.

JEL classification: E13, E32.

Keywords: Granger causality; Real business cycle; Aggregate demand.

*Correspondence: Yi WEN, department of economics, Cornell University, Ithaca, NY 14853. Tel: (607)255-6339. Email: yw57@cornell.edu.

1 Introduction

There exist a robust "causal" relationship among consumption, output, and investment. More specifically, post war US data show that consumption growth "Granger causes" GDP growth but not vis versa, and that GDP growth in turn "Granger causes" growth in business investment but not vis versa.[1] The unidirectional causal chain suggests that consumption contains more information about economic fluctuations than output does, and that output contains more information about economic fluctuations than investment does.

The information differential cannot be explained by technology shocks. Under technology shocks, output contains the best information possible for predicting future economic activities. In order to rationalize the information differential found in the US data, it seems necessary to appeal to shocks that can impact on consumption before having any influence on output and investment.

I investigate whether existing equilibrium business cycle models driven by demand shocks can explain the observed causal relationship when the following information structure is embedded: 1) Output cannot respond to demand shocks immediately; it can do so only with a lag behind consumption. 2) Investment cannot respond to demand shocks immediately; it can do so only with a lag behind output.[2]

Under these *ad hoc* assumptions, I show that standard general equilibrium business cycle models have trouble explaining the data. More specifically, the models predict the causal chain with wrong signs. Namely, consumption growth in the US economy *positively* causes output growth, it does so *negatively* in standard models. Similarly, output growth in the US economy *positively* causes investment growth, it does so *negatively* in the models. The negative causal chain emerges from standard models because of the well-known crowding out effect among components of aggregate demand in general equilibrium.

I choose to mitigate the crowding out problem by allowing for variable capacity utilization and production externalities in standard models, following Baxter and King (1991), Benhabib and Farmer (1994), Burnside and Eichenbaum (1992), Wen (1998) and Benhabib and Wen (2000). Variable capacity utilization and mild production externalities mitigate the crowding out problem by creating short run increasing returns to labor, which permit the expansion of output to meet aggregate demand

[1] The concept of causality is defined according to Granger (1969).

[2] The rational could be adjustment costs in employment and investment.

with little increase in marginal costs in the short run. These modifications, however, bring only limited success. The model is able to predict that output growth positively causes investment growth, it fails to predict that consumption growth positively causes output growth. The source of failure is still the crowding out effect: demand shocks crowd out consumption at the impact period during which neither output nor investment is able to respond.

There seems to be no simple remedies for the problems identified. More fundamental modifications of existing models are required in order to fully explain the causal aspects of the business cycle in general equilibrium. In what follows, I document in detail the causal relationships found in the US economy in section 2. Section 3 presents my attempts to rationalize these stylized facts by equilibrium business cycle models. Section 4 discusses the success and failure of my attempts. Section 5 concludes the paper.

2 The Causal Relations

To document the causal relations among aggregate consumption, output, and investment, I first estimate the following equations by ordinary least squares:[3]

$$\Delta y_t = f\left(\Delta y_{t-1}, \Delta y_{t-2}\right), \tag{1}$$

$$\Delta y_t = f\left(\Delta y_{t-1}, \Delta y_{t-2}, \Delta i_{t-1}\right); \tag{2}$$

$$\Delta y_t = f\left(\Delta y_{t-1}, \Delta y_{t-2}, \Delta c_{t-1}\right); \tag{3}$$

where Δy is growth in real GDP, Δi is growth in business fixed investment, and Δc is growth in real consumption of non-durable goods and services. A variable x is said to be "Granger causing" a variable y when a prediction of y on the basis of its past history can be improved by further taking into account the previous period's x. Estimating (1), (2) and (3) gives the following results ($t-$values are in parentheses):

$$\begin{array}{lcccc} \Delta y_t = & 0.007 & -0.00003t & +0.25\Delta y_{t-1} & +0.10\Delta y_{t-2}; \\ & (3.87)^* & (-1.59) & (3.01)^* & (1.26) \end{array} \tag{4}$$

[3] The data used are quarterly US data (1960:1 - 1996:3). Aggregate output is measured as real GDP. Aggregate consumption is measured as total consumption of nondurable goods and services. Aggregate investment is measured as business fixed investment. In terms of CITIBASE labels, these variables are named GDPQ, GCNQ+GCSQ, GINQ. The growth rates are formed as log differences.

$$\Delta y_t = \underset{(3.91)^*}{0.007} \quad \underset{(-1.63)}{-0.00003t} \quad \underset{(2.15)^*}{+0.22\Delta y_{t-1}} \quad \underset{(0.90)}{+0.08\Delta y_{t-2}} \quad \underset{(0.62)}{+0.03\Delta i_{t-1}}; \tag{5}$$

$$\Delta y_t = \underset{(0.46)}{0.001} \quad \underset{(-0.37)}{-0.00001t} \quad \underset{(0.68)}{+0.06\Delta y_{t-1}} \quad \underset{(0.63)}{+0.05\Delta y_{t-2}} \quad \underset{(4.67)^*}{+0.83\Delta c_{t-1}}; \tag{6}$$

These results lead to the following conclusions. Firstly, based on regressions (4) and (5), I cannot reject the null hypothesis that investment growth in the preceding period has no explanatory power with respect to output growth in the current period, given the past history of output growth. The past history of output growth may be a poor predictor of current output growth, but lagged investment growth does not improve the prediction.

Secondly, regressions (4) and (6) suggest that past growth in consumption has a significant effect on current output growth even after past history of output growth is taken into account. The coefficient for consumption growth has a $t-$value of 4.67, far exceeds the 5% critical value of 1.96. In fact, consumption growth is such an important factor for determining future output growth, none of the dependent variables in regression (4) remain significant after past consumption growth is taken into account in regression (6). The R^2 of the regression is increased by 200%.

For the reversed questions, whether past output growth has an effect on current investment growth given the history of investment growth, and whether it also has an effect on current consumption growth given the history of consumption growth, I obtain the following results:

$$\begin{aligned} \Delta i_t &= \underset{(1.84)}{0.007} \quad \underset{(-0.60)}{-0.00002t} \quad \underset{(4.73)^*}{+0.39\Delta i_{t-1}} \quad \underset{(2.40)^*}{+0.20\Delta i_{t-2}}; \\ \Delta i_t &= \underset{(0.33)}{0.001} \quad \underset{(-0.08)}{-0.000003t} \quad \underset{(2.03)^*}{+0.20\Delta i_{t-1}} \quad \underset{(2.74)^*}{+0.22\Delta i_{t-2}} \quad \underset{(3.37)^*}{+0.74\Delta y_{t-1}}; \end{aligned} \tag{7}$$

$$\begin{aligned} \Delta c_t &= \underset{(5.81)^*}{0.007} \quad \underset{(-3.24)^*}{-0.00003t} \quad \underset{(3.40)^*}{+0.28\Delta c_{t-1}} \quad \underset{(0.86)}{+0.07\Delta c_{t-2}}; \\ \Delta c_t &= \underset{(5.89)^*}{0.008} \quad \underset{(-3.31)^*}{-0.00003t} \quad \underset{(2.62)^*}{+0.24\Delta c_{t-1}} \quad \underset{(0.44)}{+0.04\Delta c_{t-2}} \quad \underset{(0.99)}{+0.05\Delta y_{t-1}}; \end{aligned} \tag{8}$$

Regressions in (7) suggest that past output growth has a significant effect on current investment growth. Even though the past history of investment growth predicts current investment growth well, past growth in output improves the prediction (the R^2 is increased by 22%). For each percentage increase in output growth in the previous period, current investment grows by 0.74 percentage faster (the standard error is 0.22). On the other hand, regressions in (8) suggest that consumption growth in the preceding period is the best predictor of consumption growth in the current period. Taking into account past output growth does not improve the prediction.

These results suggest a one-way causal linkage among consumption, output, and investment growth. Namely, consumption growth in the preceding period Granger causes output growth in the current period; and output growth in the preceding period Granger causes investment growth in the current period.

To conclude that the causal chains are truly unidirectional, however, I must run two more regressions to eliminate the possibility of feedback from investment growth to consumption growth. I obtain the following results:

$$\begin{array}{cccccc} \Delta c_t = & 0.0072 & -0.0003t & +0.290\Delta c_{t-1} & +0.083\Delta c_{t-2} & -0.0087\Delta i_{t-1}; \\ & (5.55) & (-3.14) & (3.38) & (0.97) & (-0.50) \end{array} \tag{9}$$

$$\begin{array}{cccccc} \Delta i_t = & -0.0048 & +0.000018t & +0.288\Delta i_{t-1} & +0.224\Delta i_{t-2} & +1.18\Delta c_{t-1}; \\ & (-0.93) & (0.43) & (3.33) & (2.79) & (3.09) \end{array} \tag{10}$$

Regression (11) suggests that investment growth in the preceding period has no explanatory power with respect to consumption growth in the current period, given the past history of consumption growth. This establishes the one-way causal chain. Regression (12) simply confirms that the causal relations are transitive; namely, if past consumption growth causes current output growth, and past output growth causes current investment growth, then past consumption growth must also be significant in predicting current investment growth.

2.1 Robustness

The standard Granger causality concept presents a pitfall when a time series has a moving average component that is not invertible. In that case, finite history of that time series can never be sufficient for predicting its current behavior, rendering other variables significant in improving the prediction. For example, let

$$\begin{aligned} x_t &= \varepsilon_t - \varepsilon_{t-1}, \\ z_t &= 0.9 z_{t-1} + \varepsilon_t, \end{aligned}$$

where ε_t is an $i.i.d$ white noise innovation. If one defines the current information set as $\Omega_t = \{\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}...\}$, then the prediction, $P[x_t|\Omega_{t-1}]$, cannot be improved by further taking into account the history of z_t, $\{z_{t-1}, z_{t-2}, ...\}$. Strictly speaking, therefore, these two series, x_t and z_t, do not "cause" or predict one another. Past history of z_t, however, can appear to be significant in predicting the current movement of x_t in the linear regression:

$$x_t = \alpha + \sum_{j=1}^{k} \gamma_j x_{t-j} + \beta z_{t-1}, \quad 0 < k < \infty.$$

This is so because z_{t-1} contains the entire past history of innovations $\{\varepsilon_{t-1}, \varepsilon_{t-2,...,}\}$ that are useful for predicting $\{x_{t-k-1}, x_{t-k-2}, ...\}$, which are useful for predicting x_t when only the finite history, $\{x_{t-1}, ..., x_{t-k}\}$, is included in the information set of the regression.

As a demonstration, a Monte Carlo experiment of the above series gives the following estimation results:

$$\begin{array}{lllllll} x_t = & 0.0003 & -0.79x_{t-1} & -0.59x_{t-2} & -0.40x_{t-3} & -0.19x_{t-4} & \\ & (0.03) & (-80.5) & (-49.0) & (-33.3) & (-19.5) & \\ x_t = & -0.0008 & -0.76x_{t-1} & -0.54x_{t-2} & -0.36x_{t-3} & -0.17x_{t-4} & -0.16z_{t-1} \\ & (-0.07) & (-82.0) & (-48.0) & (-31.5) & (-17.8) & (-34.8) \end{array} \quad (11)$$

Although $cor(x_t, x_{t-j}) = 0$ for $j \geq 2$, the first regression in (13) shows nevertheless that x_{t-j} are highly significant in predicting x_t even for $j > 2$. This happens because x_t does not have a finite autoregressive representation when its moving average component is not invertible. Failing to take into account the non-invertible moving average component can render other variables such as z_{t-1} significant in predicting x_t, although the variable z_t contains no better information than what is in x_t regarding ε_t. The second regression in (13) confirms that z_{t-1} is highly significant in predicting x_t. Even though the past history of x_t predicts x_t reasonably well ($R^2 = 0.39$), past z_t improves the prediction ($R^2 = 0.46$).

A sensible solution for the pitfall is to use a two-stage regression: Fit an optimal $ARMA(p, q)$ model to a stationary time series, and then regress the estimated residual from the $ARMA(p, q)$ model against the history of other variables that are of interest. If these other variables appear to be significant in predicting movements in the estimated residual series, then there is said to exist Granger causality between these other variables and the first time series.

Applying the idea to the above example, Monte Carlo simulations give the following results ($t-$values are in parentheses):

$$\begin{aligned} x_t &= \varepsilon_t - \underset{(-2146.9)^*}{0.999\varepsilon_{t-1}} + u_t; \\ u_t &= \underset{(0.03)}{-0.02} - \underset{(-0.48)}{0.006 z_{t-1}}; \end{aligned}$$

As expected, the results show that past z_t is not significant in predicting current x_t after the moving average component of x_t is taken into account.

The point is relevant to my analyses of the US data, since the first differences of output, consumption, and investment could contain moving average components that are not invertible when the log levels of these variables are not exactly random walk series. In such cases, consumption growth in the previous period can appear to be significant in predicting output growth in the current period even when in fact it does not contain any information superior to that in output.

With the extended notion of Granger causality in mind, I re-examine the identified causal relationship by estimating an $ARMA(4,1)$ model for the growth rate of each of the three macro variables first. I found that the moving average coefficient for all three variables are highly significant and are all close to one in absolute value. I then use the estimated residuals obtained from each $ARMA$ estimation in a second stage regression with respect to a constant, a time trend, and the lagged growth rate of another variable. For the case of output growth, I obtain the following results in the second stage estimation:

Table 1: Generalized Granger Test (Δy_t)

Variable	Coeff	Std Error	T-Stat
Constant	-0.008*	0.002	-3.63
Time trend	0.00005*	0.00002	2.77
Δc_{t-1}	0.58*	0.15	3.83
Constant	-0.002	0.002	-1.33
Time Trend	0.00003	0.00002	1.48
Δi_{t-1}	0.03	0.03	1.08

The second stage regression shows that the estimated residual of output growth obtained from the $ARMA$ model is not exogenous with respect to consumption growth in the preceding period. Namely, consumption growth in the preceding period helps predict current output growth even after past history of output growth and the moving average bias are taken into account. This is consistent with the earlier results obtained above: consumption growth causes output growth.

On the other hand, I cannot reject the null hypothesis that investment growth in the preceding period has no explanatory power with respect to output growth in the current period, given the past history of output growth and the moving average component of output growth (see the bottom row in table 12). This is also consistent with results obtained earlier: investment growth does not cause output growth.

With respect to consumption growth, I obtain the following results in the second stage estimation:

Table 2: Generalized Granger Test (Δc_t)

Variable	Coeff	Std Error	T-Stat
Constant	-0.0004	0.0009	-0.45
Time trend	0.00001	0.000009	1.28
Δy_{t-1}	0.001	0.039	-0.04
Constant	-0.00003	0.0008	-0.04
Time Trend	0.00001	0.000008	1.14
Δi_{t-1}	0.02	0.015	-1.55

Also consistent with earlier results, I cannot reject the null hypothesis that neither output growth nor investment growth in the preceding period has explanatory power with respect to consumption growth in the current period, given the past history of consumption growth and the moving average component of consumption growth.

Finally, the second stage regression of investment growth gives the following results:

Table 3: Generalized Granger Test (Δi_t)

Variable	Coeff	Std Error	T-Stat
Constant	-0.0050	0.0054	-0.92
Time trend	0.00002	0.00004	0.43
Δc_{t-1}	0.95*	0.37	2.59
Constant	0.0022	0.0041	0.53
Time Trend	-0.000008	0.00004	-0.20
Δy_{t-1}	0.30	0.18	1.61

The table shows that investment growth in the current period is predictable by consumption growth in the preceding period even after past history of investment growth and a moving average component are taken into account. This is consistent with the earlier result. Output growth in the preceding period, however, lost its significance in predicting current investment growth at the 5% significance level (see the bottom row in table 3). It is, however, still significant at the 1% significance level. In addition, judged by the economic significance, past output growth still helps predict current investment growth very well. The coefficient of Δy_{t-1} in the regression is 0.30 with a standard error of 0.18.

In sum, taking into account the potential bias caused by non-invertible moving average components in the growth rates does not change the conclusions I draw earlier: Post-war US aggregate data exhibit a robust causal chain among consumption, output, and investment. That causality runs in only one direction: from consumption growth to output growth, and from output growth to capital formation. Within this causal chain, the links from consumption growth to output and investment growth appear to be the most robust.

These one-way causalities suggest that there exist certain types of shocks in the US such that consumption reacts to them before output does, and that investment reacts to them after output does. These shocks are unlikely to come from total factor productivity, as output would react immediately to productivity shocks. In what follows, I try to rationalize the empirical regularity by demand shocks in general equilibrium.

3 The Model

I embed a sequential information structure into a general equilibrium framework, so as to see whether existing RBC models are capable of explaining the documented business cycle reality under the assumed information structure. I assume: 1) The source of the business cycle is from aggregate demand, and that demand shocks can impact on consumption instantaneously. 2) Output cannot respond to demand shocks immediately, but only with a lag behind consumption. 3) Firms' investment cannot respond to demand shocks immediately, but only with a lag behind output.

The model I choose to work with is based on Keydland and Prescott (1982), and King, Plosser, and Rebelo (1988). Allowing for demand shocks in the standard models, however, creates the well-known problem of negative comovement among components of aggregate demand. Such negative comovement are inconsistent with the positive causal relations documented above. I introduce modifications into standard models to mitigate the crowding out problem in a way similar to Baxter and King (1991) and Burnside and Eichenbaum (1995).[4] I assume that demand shocks are from government spending (Christiano and Eichenbaum, 1992). Output is produced according to the technology

$$y_t = (e_t k_t)^{\alpha(1+\eta)} n_t^{(1-\alpha)(1+\eta)},$$

where k is the capital stock, e is the rate of capital utilization, n is employment, and $\eta \geq 0$ measures the degree of externalities that are taken as parametric by representative households.

Similar to Greenwood *et al.* (1988), I assume that the rate of capital depreciation is linked to the rate of capital utilization in the preceding period according to:

$$\delta_t = \frac{1}{\theta} e_{t-1}\theta, \quad \theta > 1;$$

implying that capital depreciates faster when being used more intensively. Thus, the law of motion for capital accumulation is given by

$$k_{t+1} = i_t + \left(1 - \frac{1}{\theta} e_{t-1}\theta\right) k_t.$$

Under these assumptions, the representative agent's problem is to solve

$$\max_{\{k_{t+1+j}\}} E_{t-2} \left\{ \max_{\{n_{t+j}, e_{t+j}\}} E_{t-1} \left\{ \max_{\{c_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \log(c_{t+j}) - a \frac{n_{t+j}^{1+\gamma}}{1+\gamma} \right\} \right\} \right\}$$

subject to

$$c_{t+j} + g_{t+j} + k_{t+1+j} - (1 - \delta_{t+j}) k_{t+j} \leq (e_{t+j} k_{t+j})^{\alpha(1+\eta)} n_{t+j}^{(1-\alpha)(1+\eta)},$$

$$\delta_{t+j} = \frac{1}{\theta} e_{t+j-1}\theta, \quad \theta > 1;$$

and $k_0 > 0, 1 > e_{-1} > 0$ given. I also assume that government spending follows an $AR(1)$ stochastic processes in log:

$$\log g_t = 0.9 \log g_{t-1} + \varepsilon_t,$$

[4] Also see Benhabib and Farmer (1994), Wen (1998), and Benhabib and Wen (2000).

where the innovation ε_t is an $i.i.d$ white noise.

The first order conditions with respect to choices in time periods $t \geq 0$ are given by

$$\frac{1}{c_t} - \lambda_t = 0$$

$$E_{t-1}\left\{an_t\gamma - (1-\alpha)\lambda_t\left(e_tk_t\right)^{\alpha(1+\eta)}n_t^{(1-\alpha)(1+\eta)-1}\right\} = 0$$

$$E_{t-1}\left\{\alpha\lambda_t e_t^{\alpha(1+\eta)-1}k_t^{\alpha(1+\eta)}n_t^{(1-\alpha)(1+\eta)} - \beta\lambda_{t+1}\delta\theta e_t^{\theta-1}k_{t+1}\right\} = 0$$

$$E_{t-2}\left\{\lambda_t - \beta\lambda_{t+1}\left[\alpha e_{t+1}^{\alpha(1+\eta)}k_{t+1}^{\alpha(1+\eta)-1}n_{t+1}^{(1-\alpha)(1+\eta)} + 1 - \delta e_t\theta\right]\right\} = 0$$

$$c_t + g_t + k_{t+1} - (1-\delta e_{t-1}\theta)k_t = \left(e_tk_t\right)^{\alpha(1+\eta)}n_t^{(1-\alpha)(1+\eta)};$$

where the first equation equates marginal utility of consumption to its shadow price; the second equation equates the expected marginal cost and benefit of hours based on time $t-1$ information; the third equation equates the expected cost and benefit of capital utilization based on time $t-1$ information; the fourth equation equates the expected cost and benefit of savings based on time $t-2$ information; and the last equation is the period-by-period resource constraint.

3.1 Solution Method

In equilibrium, consumption, output, and investment in the model should follow the following rules:

$$c_t = c\left(k_t, e_{t-1}, g_t, y_t, i_t\right)$$

$$y_t = y\left(k_t, e_{t-1}, g_{t-1}, i_t\right)$$

$$i_t = i\left(k_t, e_{t-1}, g_{t-2}\right),$$

in which current consumption contains information of current demand shock g_t that is useful for predicting the next period output y_{t+1}, and current output contains information of lagged demand shock g_{t-1} that is useful for predicting the next period investment i_{t+1}.

Since exact solutions for equilibrium rules are not obtainable,I solve the model by log linearization around the steady state. The linearized equations are given by (after re-arrangement):

$$\hat{c}_t + \hat{\lambda}_t = 0 \tag{12}$$

$$E_{t-1}\left\{\left(1+\gamma-(1-\alpha)(1+\eta)\right)\hat{n}_t - \alpha(1+\eta)\hat{e}_t - \alpha(1+\eta)\hat{k}_t - \hat{\lambda}_t\right\} = 0 \tag{13}$$

$$E_{t-1}\left\{\hat{\lambda}_t + (\alpha(1+\eta)-\theta)\,\hat{e}_t + \alpha(1+\eta)\hat{k}_t + (1-\alpha)(1+\eta)\hat{n}_t - \hat{\lambda}_{t+1} - \hat{k}_{t+1}\right\} = 0 \tag{14}$$

$$E_{t-2}\left\{\hat{\lambda}_{t+1} - \hat{\lambda}_t - \beta\delta\theta\hat{e}_t + \eta_k\hat{k}_{t+1} + \eta_e\hat{e}_{t+1} + \eta_n\hat{n}_{t+1}\right\} = 0 \tag{15}$$

$$s_c\hat{c}_t + s_g\hat{g}_t + \frac{s_i}{\delta}\hat{k}_{t+1} - \frac{s_i(1-\delta)}{\delta}\hat{k}_t = \alpha(1+\eta)\hat{k}_t + \alpha(1+\eta)\hat{e}_t + (1-\alpha)(1+\eta)\hat{n}_t - s_i\theta\hat{e}_{t-1}, \tag{16}$$

where variables with hats denote percentage deviations from their steady state values, and where s_c, s_g, s_i in equation (16) denote the steady state shares of consumption, government expenditure, and investment with respect to output. In addition, the notations in equation (15) denote:

$$\begin{aligned}
\eta_k &\equiv (1-\beta(1-\delta))\,(\alpha(1+\eta)-1)\,, \\
\eta_e &\equiv (1-\beta(1-\delta))\,\alpha(1+\eta), \\
\eta_n &\equiv (1-\beta(1-\delta))\,(1-\alpha)(1+\eta).
\end{aligned}$$

I follow 3 steps to solve the model:

Step 1: Solving for $E_{t-2}\hat{\lambda}_t$, $E_{t-2}\hat{e}_t$ and $\hat{k}_{t+1}$ as functions of the state, $\left(\hat{k}_t, \hat{e}_{t-1}, E_{t-2}g_t\right)$. Taking expectations with respect to information in period $t-2$ on each equation above, it can be shown that state and co-state variables have the following reduced form:

$$E_{t-2}\begin{bmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \\ \hat{e}_{t+1} \\ \hat{e}_t \\ g_{t+1} \end{bmatrix} = ME_{t-2}\begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \\ \hat{e}_t \\ \hat{e}_{t-1} \\ g_t \end{bmatrix},$$

where M is a 5×5 matrix. The equilibrium is unique if there exist exactly two eigenvalues in the matrix M that exceed one in absolute value. In that case, the expected co-state variables, $E_{t-2}\hat{\lambda}_t$ and $E_{t-2}\hat{e}_t$, can be solved forward as functions of the states. The solutions take the form

$$\begin{aligned}
E_{t-2}\hat{\lambda}_t &= \lambda\left(\hat{k}_t, \hat{e}_{t-1}, E_{t-2}g_t\right) \\
E_{t-2}\hat{e}_t &= \lambda\left(\hat{k}_t, \hat{e}_{t-1}, E_{t-2}g_t\right) \\
\hat{k}_{t+1} &= k\left(\hat{k}_t, \hat{e}_{t-1}, E_{t-2}g_t\right),
\end{aligned}$$

where I have used the fact that $E_{t-2}\hat{k}_{t+j} = \hat{k}_{t+j}$ for $j = \{-2, -1, 0, 1\}$, since $\hat{k}_{t+j}$ is known at $t-2$ for $j = \{-2, -1, 0, 1\}$, and the fact that $E_{t-j-1}\hat{e}_{t-j} = \hat{e}_{t-j}$, since the decision for $\hat{e}_{t-j}$ is made in period $t - 1 - j$.

Step 2: Solving for $\hat{n}_t$, $\hat{e}_t$, and $E_{t-1}\hat{c}_t$. Taking expectation against time $t-1$ information for equation (#), combining with equations (#) and (#) gives (after substitution using $\hat{c} = -\hat{\lambda}$)

$$E_{t-1}\begin{bmatrix} \hat{c}_t \\ \hat{n}_t \\ \hat{e}_t \end{bmatrix} = B_1 E_{t-1}\hat{k}_{t+1} + B_2 E_{t-1}\begin{bmatrix} \hat{k}_t \\ \hat{e}_{t-1} \\ g_t \end{bmatrix},$$

where I have used the fact that $E_{t-1}\lambda_{t+1} = \lambda\left(\hat{k}_{t+1}, E_{t-1}\hat{e}_t, E_{t-1}g_{t+1}\right)$ from step 1. Since the decisions for n and e are made with respect to $t-1$ information, the solutions for $\{n, e\}$ take the form (using results from step 1):

$$\hat{n}_t = E_{t-1}\hat{n}_t = n(\hat{k}_t, \hat{e}_{t-1}, E_{t-1}g_t, E_{t-2}g_t)$$

$$\hat{e}_t = E_{t-1}\hat{e}_t = e(\hat{k}_t, \hat{e}_{t-1}, E_{t-1}g_t, E_{t-2}g_t).$$

Step 3: Solving for $\hat{c}_t$. Using the budget constraint and results from step 1 and step 2, I can solve for $\hat{c}_t$ as

$$\hat{c}_t = c\left(\hat{k}_t, \hat{e}_{t-1}, g_t, E_{t-1}g_t, E_{t-2}g_t\right).$$

Giving the law of motion for the shock processes, and the production function and the investment function, One can derive equilibrium policy rules for consumption, output, and investment. They have the following reduced functional form:

$$c_t = c\left(k_t, \hat{e}_{t-1}, g_t, g_{t-1}, g_{t-2}\right)$$

$$y_t = y\left(k_t, \hat{e}_{t-1}, g_{t-1}, g_{t-2}\right)$$

$$i_t = i\left(k_t, \hat{e}_{t-1}, g_{t-2}\right).$$

These equilibrium policy rules imply that consumption in the preceding period ($\hat{c}_{t-1}$) help predicting output in the current period ($\hat{y}_t$) even after the history of past output, $\{\hat{y}_{t-1}, \hat{y}_{t-2}, ...\}$, is taken into account. This is so because $\hat{c}_{t-1}$ has information for demand shock g_{t-1} that is useful for predicting $\hat{y}_t$ but is missing in the past history of $\hat{y}_t$. They also imply that output in the preceding period ($\hat{y}_{t-1}$) help predicting investment in the current period ($\hat{\imath}_t$) even after the past history of investment, $\{\hat{\imath}_{t-1}, \hat{\imath}_{t-2}, ...\}$, is taken into account, since $\hat{y}_{t-1}$ has information for demand shock g_{t-2} that is useful for predicting $\hat{\imath}_t$ but is missing in the past history of $\hat{\imath}_t$.

3.2 Calibration and Impulse Responses

I set t as the number of quarters and calibrate the model's parameters as follows: the time discounting factor $\beta = 0.99$, the capital's share $\alpha = 0.3$, the steady state rate of capital depreciation $\delta = 0.025$, the steady state government expenditure to output ratio $s_g = 0.15$. These parameter values imply that the steady state capital-output ratio is 10, the steady state interest rate is 4% per year, the steady state investment to output ratio is 0.2, the steady state consumption to output ratio is 0.65, and the utilization elasticity of depreciation $\theta = 1.4$. Following Wen (1998) and Benhabib and Wen (2000), I choose the externality parameter $\eta = 0.15$.

The impulse responses of the model to government spending shocks are shown in figure 1. At the impact of the shock, consumption decreases, exhibiting a perfect crowding out since output and investment cannot adjust at the impact period. In the second period, capacity utilization and employment start to respond to the shock positively, resulting in higher output. This reduces the crowding out pressure from government spending on consumption, so consumption rises. Two periods after the shock, investment starts to respond positively. The increase in aggregate demand stimulates further increases in capacity utilization and employment, resulting in higher output. Thanks to increasing returns to scale, consumption starts to rise above the steady state, resulting in a positive comovement with output and investment.

The most notable features of figure 1 are the hump-shaped impulse responses of output and investment to the demand shock. Hump-shaped impulse responses at the levels imply positive autocorrelations in growth rates. Hence the model succeeds in resolving the puzzle of lack of positive autocorrelations in output growth identified by Cogley and Nason (1995).[5]

The model also conform to the aspects of actual fluctuations that have been identified in the RBC literature as defining features of the business cycle: the positive comovement among consumption, output, and investment; and the smoothness of consumption relative to investment. The statistics are summarized in table (4) (where y, c, i denote output, consumption and investment respectively):

Table 4

σ_c/σ_y	σ_i/σ_y	$Cor(y,c)$	$Cor(y,i)$	$Cor(c,i)$
0.22	3.70	0.26	0.96	0.19

[5] The dynamic multiplier effect of demand shocks in the model has also been studied by Benhabib and Wen (2000) in a slightly different context.

4 Causal Relations Predicted by Theory

4.1 Predictions of a Standard RBC Model

To better appreciate the scale economy model, it is useful to review the salient failure of standard RBC models in this regard. I simulate the model of King, Plosser and Rebelo (KPR, 1988)[6] to obtain artificial data series for consumption, output and investment (sample size = 10,000), and then apply the two-stage regression procedure discussed above to estimate causal relations among the growth rates of the three variables. I apply an $ARMA(2,1)$ model to each variable in the first stage of the regression.[7] The estimated residuals from the $ARMA(2,1)$ model are then used in the second stage regression using as independent variables a constant, a time trend, and the lagged growth rate of a second variable. I obtain the following results in the second stage regression for the three variables respectively:

Table 5: KPR Model ($t-$values in parentheses)

Residual of Δy_t			Residual of Δi_t			Residual of Δc_t		
Δc_{t-1}	0.04	(0.73)	Δc_{t-1}	0.24	(1.13)	Δy_{t-1}	0.00	(0.09)
Δi_{t-1}	0.00	(0.03)	Δy_{t-1}	0.01	(0.30)	Δi_{t-1}	0.00	(0.10)

The table shows that non of the variables Granger causes another in the KPR model. The coefficients are all statistically insignificant from zero. This is expected since all variables in the model share the same information set about technology shocks. Hence, adding other variables into the regressions does not improve prediction power.

4.2 Predictions of KPR Model with Sequential Information Structure

I change the source of shocks from technology to government spending and embed the sequential information structure into the KPR model. Using similar estimation procedures above, I obtain the following results in the second stage regressions:

Table 6: KPR Model with Sequential Information

Residual of Δy_t			Residual of Δi_t			Residual of Δc_t		
Δc_{t-1}	-0.27*	(-125)	Δc_{t-1}	-0.15*	(-79.0)	Δy_{t-1}	0.03	(1.42)
Δi_{t-1}	-0.00	(-0.03)	Δy_{t-1}	-0.66*	(-378)	Δi_{t-1}	0.07*	(2.31)

[6] The results are similar for the Kydland-Prescott model (1982).

[7] All growth series generated from the model contain a non-invertible moving average component.

The left column of the table shows that consumption growth in the preceding period has significant explanatory power on the residual of output growth in the current period, even after past history of output growth and the moving average (first differencing) component are taken into account. The middle column of the table shows that output growth (as well as consumption growth) in the preceding period has significant explanatory power on the residual of investment growth in the current period, even after past history of investment growth and the moving average component are taken into account. The last column of the table shows that neither output growth nor investment growth in the preceding period has significant effects on the residual of consumption growth in the current period (although the coefficient on Δi_{t-1} is statistically significant, it is economically insignificant).

Hence, introducing the sequential information structure brings the standard RBC model into closer conformity with the data's causal structure. But, the model fails on two grounds: 1) The causal relationships among consumption, output, and investment are of the wrong sign – they are all negative in the model; 2) The order of the relative volatilities of consumption, output, and investment are exactly the opposite of the data – in the model consumption is more volatile than output, which is in turn more volatile than investment. Both failures are due to the crowding out effect, which renders consumption and output to be negatively correlated, and prevents consumption from smoothing when government expenditure fluctuates.

4.3 Predictions of the Scale-Economy Model

Under variable capacity utilization and mild externalities, the scale economy model is able to mitigate the crowding out effect, as is shown by Benhabib and Wen (2000). The scale economy model therefore improves the previous models substantially in explaining the observed Granger causalities. Applying the two-stage estimation procedures to the model gives the following results:

Table 7: Scale Economy Model with Sequential Information

Residual of Δy_t			Residual of Δi_t			Residual of Δc_t		
Δc_{t-1}	-0.02*	(-2.74)	Δc_{t-1}	3.08*	(100)	Δy_{t-1}	0.01	(1.29)
Δi_{t-1}	-0.03*	(-21.1)	Δy_{t-1}	1.15*	(35.5)	Δi_{t-1}	-0.00	(-0.70)

The scale economy model improves the performance of the previous models along several dimensions. First, the middle column of table (7) shows that both consumption growth and output growth in the scale economy model positively cause investment growth. Secondly, the first column of table (7) shows that the negative causal relation found between consumption growth and output growth in the previous models is no longer economically significant in the scale economy model, although it is still non-positive. Another significant improvement of the current model is that the relative volatilities among consumption, output, and investment is restored to the right order; namely, consumption is now the least volatile and investment the most volatile in the scale economy model. This smoothing effect is explained by Wen (1998). Capacity utilization and production externalities help smooth consumption because they render the real wage relatively smooth compared to employment.

What have prevented the model from generating a positive causal relation between consumption growth and output growth? The following autoregressions from the model may help understand:

$$\begin{aligned} \Delta y_t &= 0.72\Delta y_{t-1} + u_{yt} \\ \Delta c_t &= -0.47\Delta c_{t-1} + u_{ct} \\ \Delta i_t &= 0.52\Delta i_{t-1} + u_{it} \end{aligned}$$

Output growth and investment growth in the model are positively autocorrelated, as they are in the data. But consumption growth in the model is negatively autocorrelated, unlike what is in the data. This negative autocorrelation of consumption growth is caused mainly by the crowding out effect of government shocks at the impact period, during which output and investment are both fixed (see the impulse responses of consumption in figure 1). As long as output is not allowed to respond to shocks at the impact period, such crowding out effect is unavoidable.

4.4 Remarks

It is important to point out that adding technology shocks into the model does not help resolve the problem, because the causal relations found in the data are conditional predictions. What matters is the information differential between consumption and output. Technology shocks or any other shocks can have no effect on the causal chain unless they can change the information differential. This is why other types of shocks were not examined in the paper, such as taste shocks and sunspots shocks, because given our framework these shocks can not affect consumption at the impact period when no other components in the budget equation is able to react to shocks. If consumption does not move at the impact period, then its information set will be the same as that of output.

5 Conclusion

The empirical causal chain identified in the paper may not surprise a businessman. According to a businessman's intuition, production would not rise until consumption demand rises; and investment would not rise until profit rises along with the rise in production. The key elements missing in the business man's intuition, however, are the aggregate resource constraint and the price mechanism. Without changes in production possibilities or prices, what would enable consumption to rise at the first place without crowding out? General equilibrium business cycle models embodying the resource constraint and price mechanism, nevertheless, has trouble conforming to the data. There must be something fundamental missing in the models too.

One possible missing element is inventory investment. Inventories provide a perfect buffer for consumption when output cannot react immediately to demand shocks. Like money, however, inventories are not held in equilibrium because they are dominated in return by other assets such as fixed capital. To model inventory behavior in general equilibrium is itself a challenge to equilibrium business cycle theory and is beyond the scope of the paper.

References

[1] M. Baxter and R. King, 1991, Productive externality and cyclical volatility. Working paper 245, University of Rochester.

[2] J. Benhabib and R. Farmer, Indeterminacy and increasing returns, *Journal of Economic Theory* **63** (1994), 19-41.

[3] J. Benhabib and Y. Wen, 2000, Indeterminacy, aggregate demand, and the real business cycle, New York University Working Paper.

[4] C. Burnside and M. Eichenbaum, 1996, Factor hoarding and the propagation of business cycle shocks, *American Economic Review* 86 (December), 1154-1174.

[5] L. Christiano and M. Eichenbaum, 1992, Current real-business-cycle theories and aggregate labor-market fluctuations, American Economic Review 82, 430-50.

[6] T. Cogley and J. Nason, 1995, Output dynamics in real-business-cycle models, *American Economic Review* **85**, 492-511.

[7] R. Farmer and J. T. Guo, 1994, Real business Cycles and the animal spirits hypothesis, *Journal of Economic Theory* **63**, 42-72.

[8] C. Granger, 1969, Investigating causal relations by economic models and cross-spectral methods, Econometrica 37, No. 3 (July), 424-438.

[9] J. Greenwood, Z. Hercowitz and G. Huffman, 1988, Investment, capacity utilization, and the real business cycle, *American Economic Review* **78**, 402-417.

[10] King, R., C. Plosser and S. Rebelo, 1988, Production, growth and business cycles: I. The basic neoclassical model, *Journal of Monetary Economics* **21**, 195-232.

[11] J. Rotemberg and M. Woodford (1996), Real-business-cycle models and the forecastable movements in output, hours, and consumption, *American Economic Review* 86 (March), 71-89.

[12] S. Schmitt-Grohe, 2000, Endogenous business cycles and the dynamics of output, hours, and consumption, *American Economic Review,* forthcoming.

[13] Y. Wen, 1998, Capacity utilization under increasing returns to scale, *Journal of Economic Theory* **81***,* 7-36.

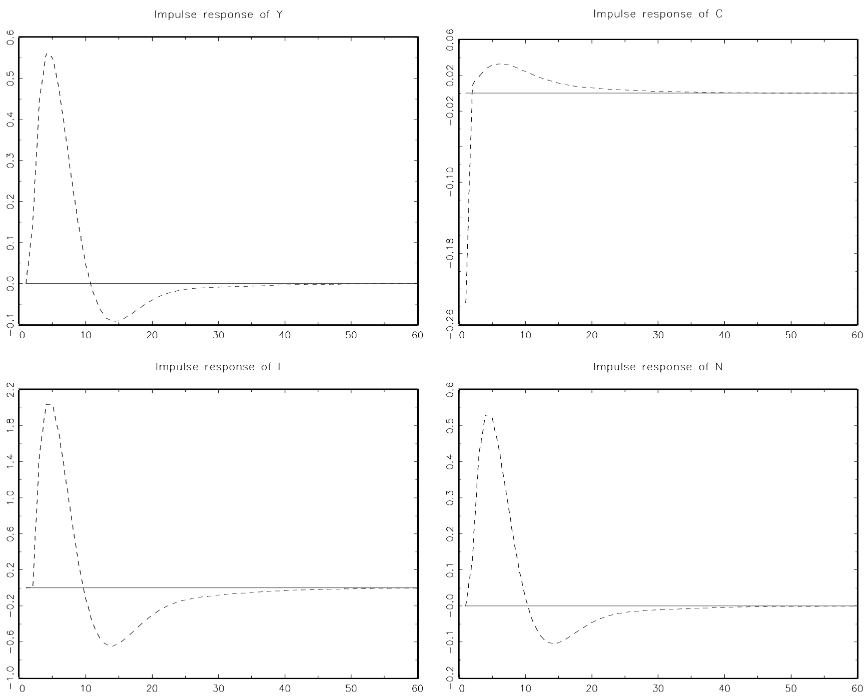

Figure 1. Impulse Responses of the Scale Economy Model to a Demand Shock.