Market Power and Efficiency in a Search Model*

Manolis Galenianos†, Philipp Kircher‡ and Gábor Virág§

July 2008

Abstract

We build a theoretical model to examine the welfare consequences of the introduction of a minimum wage and unemployment benefits in a frictional labor market. Our environment has two main characteristics: wages play a role in allocating labor across firms and policy is potentially beneficial because the equilibrium is inefficient. The inefficiency is due to the firms' market power which leads low productivity firms to hire too often. Minimum wages exacerbate the inefficiency because they introduce additional distortions in wage-setting. Moderate unemployment benefits increase welfare as they improve the workers' outside option, thus limiting the firms' market power.

*We thank Jan Eeckhout, Andy Postlewaite, and Randy Wright for helpful comments. Kircher thanks the National Science Foundation for financial support (grant SES-0752076).

†Pennsylvania State University. E-mail: manolis@psu.edu.

‡University of Pennsylvania and IZA. E-mail: kircher@econ.upenn.edu.

§University of Rochester. E-mail: gvirag@mail.rochester.edu.

1 Introduction

The debate on minimum wages has focused almost exclusively on whether they affect employment negatively, as predicted by the competitive model, or positively, as suggested by some frictional or monopsonistic models of the labor market.[1] However, minimum wages also distort firms' pricing decisions and prices are important for the allocation of scarce resources (labor, in this case). Therefore, even if a minimum wage impacts employment positively, it could still be welfare-reducing by distorting the allocation of labor in other ways. Since frictional models have proved a useful tool for studying labor markets, we believe that they provide a good theoretical framework for further examining this question.[2]

To examine the welfare effects of labor market interventions we consider a frictional environment where policy matters, i.e. the decentralized allocation is inefficient, and where wages have an explicit role in allocating workers across heterogeneous firms. We find that a minimum wage is always detrimental to welfare while unemployment benefits may improve welfare. This is interesting because it contrasts with the predictions of many recent frictional models where both policy instruments yield the same qualitative welfare implications (Acemoglu and Shimer (1999), Acemoglu (2001), Manning (2001)).[3] However, in these models wages have no allocative role, either because agents are assumed to be identical or because wages are decided on *after* a match has formed. When we introduce an allocative role for wages by giving firms a non-trivial wage-setting decision in an environment with heterogeneity the qualitative similarity across policy instruments disappears.

We build a directed search model of the labor market with a finite number of agents. Firms differ in their productivity and workers are homogeneous. Each firm has one vacancy to fill and it publicly posts a wage to attract workers. Frictions are introduced by assuming that workers can only apply for one job and they cannot coordinate their application decisions with each other. We fix the size of the market as we are primarily interested in the allocation of workers across existing firms.

In the presence of productivity heterogeneity and frictions, the welfare criterion concerns whether workers are allocated efficiently across firms. We prove that efficiency does not obtain: the more (less) productive firms hire less (more) often than is optimal; furthermore,

[1] There is currently no consensus about which view is favored by the data. For instance, Card and Krueger (1994) present evidence in support of the latter view which is disputed by Neumark and Wascher (2000). See Neumark and Wascher (2007) for a review of the literature.

[2] Frictional models help interpret salient features of labor markets that are difficult to reconcile with the competitive model, such as wage dispersion and involuntary unemployment to just name two.

[3] Acemoglu and Shimer (1999) do not explicitly consider a minimum wage but it is easy to show that it has the same effect as their prescribed unemployment benefits, as also remarked in Acemoglu (2001). It is worth noting that these three papers span the different classes of models of the labor search literature: they use directed search, random search with bargained wages and random search with posting, respectively.

unemployment is *too low* because workers apply to safer low-productivity jobs too frequently. The culprit of the inefficiency is that the finite nature of the market endows firms with *market power* in the sense that a single firm's action affects the equilibrium outcomes of all agents.[4] Market power reduces the elasticity of the hiring probability with respect to the wage and, as a result, all firms posts wages that are lower than at the efficient benchmark. This redistributive feature is shared with typical models of monopsony (Bhaskar, Manning and To (2002)). However, the incentive to reduce wages is stronger for high productivity (and in equilibrium high-wage) firms which leads them to reduce wages disproportionally, resulting in the misallocation of labor. The novel feature of our paper is thus the misallocation of workers towards low-productivity jobs which increases employment but reduces output (and hence welfare) by decreasing the average productivity of employed workers.

Our model shows that evaluating the welfare implications of labor market policy based on their employment effects alone can lead to misleading results: introducing a minimum wage results in higher employment but lower welfare, while unemployment benefits have exactly the opposite effect. A minimum wage forces the low productivity firms to offer higher wages than they otherwise would and hence hire even more often than in the original, already inefficient, equilibrium. Introducing unemployment benefits helps because it improves the outside option of workers, effectively reducing the firms' market power.[5] Therefore, even though both policies redistribute surplus towards the workers, they have diametrically opposing effects on aggregate welfare.

The next section describes the model. Most of the insights of our model can be conveyed in the simple setting with two workers and two firms which is examined in section 3. Section 4 generalizes our results. Section 5 concludes.

2 The Environment

We begin by presenting the environment of our model. The economy is populated with a finite number of risk-neutral workers and firms, denoted by $N = \{1, ..., n\}$ and $M = \{1, ..., m\}$ respectively, where $n \geq 2$ and $m \geq 2$. All workers are identical. Each firm j has one vacancy and is characterized by its productivity level, x_j, with $x_j > 0$ for all j. We assume without loss of generality that $x_m \leq x_{m-1} \leq ... \leq x_1 \equiv \overline{x}$. The productivity of all firms is common

[4] We distinguish this concept from *monopsony power* which allows a firm to pay its workers a wage that is strictly below their marginal product. In most frictional models of the labor market, firms enjoy (sometimes temporary) monopsony power, but typically the market consists of a continuum of agents leaving firms with no market power. Indeed, in directed search models with a continuum of agents the constrained efficient allocation obtains (Moen (1997), Shi (2001), Shimer (2005)).

[5] Of course, unemployment benefits perform additional functions that we do not consider here, such as providing insurance to workers and liquidity to the unemployed. See Shimer and Werning (2008).

knowledge.[6] The utility of a worker is equal to his wage if employed and zero otherwise. The profits of firm j are given by $x_j - w_j$ if it employs a worker at wage w_j and zero otherwise.

The hiring process has three stages:

1. Each firm j posts a wage $w_j \in [0, \overline{x}]$.

2. Workers observe the wage announcement $\mathbf{w} = \{w_1, w_2, ..., w_m\} \in [0, \overline{x}]^m$ and each worker simultaneously applies to one firm.[7]

3. A firm that receives one or more applicants hires one worker at random. A firm without applicants remains idle.

We focus our attention on equilibria with pure wage-posting strategies by the firms and we solve for the subgame perfect equilibria of the game. As is standard in the directed search literature, we restrict attention on equilibria where workers follow symmetric strategies. Symmetric strategies mean that, given any wage announcement, all workers apply to firm j with the same probability for any $j \in M$. This assumption rules out coordination among workers and it is a natural way of introducing trading frictions.[8] It is straightforward to extend this environment to introduce our two policy variables: a minimum wage puts a lower bound on the wages that firms can post; unemployment benefits increase the value of remaining unemployed. This is the standard directed search environment, for instance as in Burdett, Shi and Wright (2001, henceforth BSW).[9,10]

The environment described above delivers on the two characteristics that we discussed in the introduction. First, wages play an important role in allocating labor in this class of

[6] The case where productivity levels are private information is examined in Galenianos and Kircher (2007).

[7] Most of the directed search literature restricts attention to a single application as a way of capturing the time-consuming aspect of the job-finding process. Multiple applications were recently introduced in continuum directed search models by Albrecht, Gautier and Vroman (2006), Galenianos and Kircher (2005) and Kircher (2007). In models of finite economies, multiple applications lead to severe technical complications as shown in Albrecht, Gautier, Tan and Vroman (2005).

[8] Lack of coordination may seem incompatible with a finite (or, small) labor market. What we have in mind is that the labor market for some occupation may have a small number of participants while the total number of agents in the geographical vicinity is large enough to preclude coordination among them.

[9] Montgomery (1991) also considers heterogeneous firms but uses a different equilibrium concept as his firms do not take their market power into account. The environment in Peters (1991, 2000) is also similar, but he focuses on characterizing the limits of equilibria as the market becomes large.

[10] We take the trading mechanism and the associated coordination failures as given. The coordination problem would be less severe if the contracts were posted by the workers rather than the firms, as in Coles and Eeckhout (2003a, 2003b). However, in that environment the firms do not obtain any surplus and they would therefore prefer to offer the contracts themselves rather than apply for workers' services if given the choice. Analyzing the effects of competing markets is beyond the scope of our paper (see Halko, Kultti and Virrankoski (2008) for such a model). Virag (2008) considers a model of competing mechanisms with finite markets where the firms take their market power into account, but that paper assumes that firms are homogeneous.

models: the fact that workers observe the wages before applying for jobs means that they can direct their applications towards the most desirable firm. As a result, firms can compete for labor by offering higher wages which is particularly important in an environment with heterogeneity in productivity levels. Second, the finite size of the market creates strategic considerations in the inter-firm interaction which is a source of inefficiency. We discuss this feature in detail in the following section.

3 The Case of Two Workers and Two Firms

We begin our analysis by looking at the case where $n = m = 2$ and $x_1 > x_2$.[11] We find it fruitful to analyze this simple case first because the subgame can be characterized in a straightforward manner while the strategic interaction among the agents is still present. As a result, without losing on economic content, it is relatively easy to calculate the effects of a firm's action. The general case is considered in section 4.

This section's results are the following.

Proposition 3.1 *When $n = m = 2$ and $x_1 > x_2$:*

1. *Equilibrium Characterization: A unique directed search equilibrium exists. The more productive firm posts a higher wage ($w_1 > w_2$).*
2. *Efficiency Properties: Constrained efficiency does not obtain in equilibrium. The less productive firm hires too often and unemployment is too low from an efficiency viewpoint. The firms' market power is the source of the inefficiency.*
3. *Policy Implications: Introducing a binding minimum wage reduces welfare. There exists a level of unemployment benefits that leads to the constrained efficient allocation.*

3.1 Equilibrium Characterization

We show that there exists a unique equilibrium and that the high productivity firm offers a higher wage. Even though this result is not particularly surprising in itself, we find the proof to be useful for the efficiency analysis that follows.

The model is solved by backwards induction. The first step is to derive the equilibrium response of the two workers for an arbitrary wage announcement $\{w_1, w_2\}$. To facilitate exposition, the two workers are named A and B. Suppose that worker B's strategy is to visit

[11] The case where $x_1 = x_2$ and $n = m = 2$ is analyzed by BSW in great detail.

firm j with probability p_j^B. Let $G(p_j^B)$ denote the probability that worker A is hired by firm j if he applies there. It is equal to

$$G(p_j^B) = (1 - p_j^B) + \frac{p_j^B}{2} = \frac{2 - p_j^B}{2}. \tag{1}$$

With probability $1 - p_j^B$ worker B applies to firm k ($\neq j$) in which case A is hired for sure; with probability p_j^B, B applies to firm j and A is hired with probability 1/2. Therefore, A's expected utility from applying to firm j is given by $[(2 - p_j^B)\ w_j]/2$. And similarly for B.

In a symmetric subgame, $p_j^A = p_j^B = p_j$. It is straightforward to see that if $w_j \geq 2w_k$ then both workers apply to firm j for sure ($p_j = 1$ and $p_k = 0$) and in expectation they receive utility equal to $w_j/2$. Otherwise they mix ($p_l > 0$ for $l = 1, 2$) and they receive $G(p_j)\ w_j = G(w_k)\ p_k$. We define by *market utility* the utility that workers expect to receive in the equilibrium of the subgame that follows the wage announcement: $U(w_1, w_2) \equiv \max_j G(p_j)\ w_j$. Therefore, given a pair of wages $\{w_1,\ w_2\}$, if $w_j \geq 2\ w_k$ the workers' strategies and associated market utility are given by

$$p_j = 1 \ \text{ and } \ U(w_1, w_2) = \frac{w_j}{2}, \tag{2}$$

and if $w_j/w_k \in [\frac{1}{2}, 2]$ then $\{p_1, p_2\}$ and $U(w_1, w_2)$ are determined by

$$\frac{(2 - p_j)\ w_j}{2} = \frac{(2 - p_k)\ w_k}{2} = U(w_1, w_2). \tag{3}$$

In what follows we will show that only the latter case can occur in equilibrium.

We now turn to the firms. Firm j takes w_k as given and chooses w_j to maximize profits. Equations (2) and (3) describe the interaction between the actions of the two firms and the workers' strategies. Ceteris paribus, when firm j increases its wage, it makes itself more desirable as an employer which leads workers to apply there with greater probability. Note that the maximum wage firm j would offer is $2w_k$ since that wage attracts both workers with probability 1 and there is no additional gain from offering higher pay. Therefore (3) is the only relevant constraint.

When the workers' strategy is p_j, firm j remains idle if it receives no applicants which occurs with probability $(1 - p_j)^2$. Define the probability that the firm hires a worker, and is hence able to produce, by $H(p_j) \equiv 1-(1-p_j)^2$. Firm j maximizes expected profits:

$$\pi_j(w_j, w_k) = \max_{w_j \in [0, 2w_k]} \left[1 - (1-p_j)^2\right] (x_j - w_j) \quad \text{s.t.} \quad (3) \tag{4}$$

It will prove convenient to optimize over p_j rather than w_j. Using equation (3) one can express w_j as a function of p_j and w_k (recalling that $p_k = 1 - p_j$), thus incorporating the workers' equilibrium response inside the firm's optimization problem. That substitution leads to $w_j(p_j, w_k) = (2-p_k)\ w_k/(2-p_j)$. For purposes of exposition, we use the following equivalent expression (again, derived from equation (3)), as it will facilitate the efficiency comparisons of the next section:

$$w_j(p_j, w_k) = \frac{2\ U[w_j(p_j, w_k), w_k]}{2 - p_j}. \tag{5}$$

Substituting (5) into the objective function and rearranging yields

$$\pi_j[w_j(p_j, w_k), w_k] = \max_{p_j \in [0,1]} (2-p_j)\ p_j\ x_j - 2\ p_j\ U[w_j(p_j, w_k), w_k],$$

which leads to the following first order conditions:

$$\frac{d\pi_j[w_j(p_j, w_k), w_k]}{dp_j} = 2\ (1-p_j)\ x_j - 2\ U[w_j(p_j, w_k), w_k] - 2\ p_j\ \frac{\partial U[w_j(p_j, w_k), w_k]}{\partial p_j}. \tag{6}$$

It is worth spending a moment on how profits change when attracting more workers. The first term captures the marginal benefit to the firm: the increase in the hiring probability times productivity. The second term refers to the fact that, in order to attract more applicants, the firm has to provide them with the market utility they would get elsewhere. The third term captures the fact that the firm's wage affects the workers' market utility ($\partial U[w_j(p_j, w_k), w_k]/\partial p_j \neq 0$). This third term affects the firm's equilibrium action and it turns out to have implications for efficiency which we discuss in detail in the following subsection.

To complete the characterization of the firm's problem, recall that $U[w_j(p_j, w_k), w_k] = (2-p_k)\ w_k/2 = (1+p_j)\ w_k/2$ and rewrite the first order conditions in a more tractable way:

$$\frac{d\pi_j[w_j(p_j, w_k), w_k]}{dp_j} = 2\ (1-p_j)\ x_j - (1 + 2\ p_j)\ w_k. \tag{7}$$

Equating the first order conditions of both firms to zero and combining them yields

$$\frac{x_1}{x_2} = \frac{1-p_2}{1-p_1}\frac{1+2\ p_1}{1+2\ p_2}\frac{2-p_1}{2-p_2} \tag{8}$$

using equation (3) to substitute out the wages.

Equation (8) implicitly characterizes the equilibrium. Let $R(p_1)$ denote the right hand side of equation (8) where p_2 is substituted out by $p_2 = 1 - p_1$. It is straightforward to show that $R(0) = 0$, $R'(p_1) > 0$ and $\lim_{p_1 \to 1} R(p_1) = \infty$ which imply that there is a unique $p_1^*(x_1, x_2)$ satisfying (8); therefore the equilibrium exists and it is unique. Note that p_1^* is strictly interior which means that, regardless of the level of productivity differences, the high productivity firm does not find it optimal to price its competitor out of the market.[12] The symmetry of (8) and $x_1 > x_2$ imply $p_1^* > 1/2 > p_2^*$ which means that $w_1 > w_2$. Using equation (7) the wages are given by:

$$w_k(x_1, x_2) = \frac{2\ (1 - p_j^*(x_1, x_2))\ x_j}{1 + 2\ p_j^*(x_1, x_2)}. \tag{9}$$

3.2 Efficiency Properties of Equilibrium

We now examine the efficiency properties of the equilibrium. We have two main results: first, efficiency does not obtain; second, the pattern of inefficiency is that low productivity firms hire too often and unemployment is too low. We identify the market power enjoyed by firms in a finite market as the culprit for the inefficiency.

We are concerned with the allocative efficiency of the equilibrium, i.e. whether workers direct their applications to the heterogeneous firms in a socially optimal way. We use the concept of constrained efficiency: the planner chooses the strategies of the agents to maximize his welfare function but he is still subject to the coordination frictions which we assume to be inherent in the market. This is the standard notion of constrained efficiency in a decentralized matching process as in Shi (2001) or Shimer (2005).

The planner maximizes total output in the economy which is a utilitarian welfare function given the risk neutrality of agents. The firms' strategies (wage-setting) are irrelevant for efficiency since they only affect the distribution of the surplus. Therefore, the planner chooses

[12] This result is particular to the two firm case. In the general m-firm model additional assumptions on productivity are needed to guarantee that all firms attract applications. See Galenianos and Kircher (2007).

the workers' strategies to solve

$$\begin{aligned} \max_{p_1, p_2} \quad & x_1\ H(p_1) + x_2\ H(p_2) \\ \text{s.t.}\ \ p_1 + p_2 = 1 \quad & \text{and}\ \ p_j \geq 0,\ \ j = 1, 2. \end{aligned} \tag{10}$$

Recalling that $H(p) = 1 - (1-p)^2$ and setting the first order conditions of (10) to zero yields

$$\frac{x_1}{x_2} = \frac{1 - p_2^P}{1 - p_1^P}, \tag{11}$$

Comparing equation (11) with the equilibrium condition of equation (8) it is clear that constrained efficiency does not obtain in equilibrium.

The source of the inefficiency is the market power that firms enjoy in a finite economy. Market power refers to the fact that an individual firm's action alters the workers' market utility (see below). Mathematically, it enters the first order conditions of the firm's problem in the form of the term $\partial U / \partial p_j$. Indeed, if this term were equal to zero, say either because firms are assumed to behave competitively as in Montgomery (1991) or because the market is large as in the continuum models of Shi (2001) or Shimer (2005), it is easy to show that constrained efficiency does obtain.

To understand the way in which firms' market power affects equilibrium outcomes we need to examine the pattern of inefficiency in some more detail. It is a matter of algebra to show that the product of the second and third ratios on the right hand side of (8) is larger than 1 which implies that $(1 - p_2^*)/(1 - p_1^*) < (1 - p_2^P)/(1 - p_1^P)$, and therefore

$$p_2^P < p_2^* < \frac{1}{2} < p_1^* < p_1^P. \tag{12}$$

That is, in equilibrium workers apply to the less productive firm too frequently. Furthermore, the unemployment rate is *too low* from an efficiency viewpoint. Recalling equation (1), the probability that a worker remains unemployed under strategy p_1 is

$$\text{Prob}[un|p_1] = p_1\ (1 - p_1 + \frac{p_1}{2}) + p_2\ (1 - p_2 + \frac{p_2}{2}) = \frac{1}{2}\ [1 + 2\ p_1 - 2\ p_1^2] \tag{13}$$

Equation (13) is minimized at $p_1 = 1/2$ which, together with equation (12), implies that

$$\text{Prob}[un|p_1^*] < \text{Prob}[un|p_1^P].$$

Reallocating workers toward the more productive firm has two effects in our model: first, the average productivity of an employed worker increases as the productive firm is left idle less often; second, there are fewer employed workers since they crowd each other out more often at the productive firm. The decentralized equilibrium does not strike the right balance between these two forces because the more productive firms use their market power to depress wages and increase profits.

The reasoning is as follows: a firm that contemplates reducing its wage anticipates that it will receive fewer applicants on average. The magnitude of the reduction, however, depends on whether the firm has market power or not. When the firm enjoys market power, it reduces the workers' market utility by lowering its wage; therefore, the workers become less picky about where they apply to and hence the firm suffers a modest reduction in its applicant pool. In contrast, when the firm has no market power, its actions do not affect the workers' market utility and, therefore, the firm suffers a larger drop in applications in response to a lower wage offer. In other words, the elasticity of the hiring probability with respect to the wage is lower when the firm has market power. Since it is the more productive (and hence high-wage) firms that affect the workers' market utility to a larger extent, they reduce their wages disproportionally and hence hire with suboptimal frequency.

The equilibrium behavior of firms creates inefficiencies, in addition to the redistribution of surplus from workers to firms which does not enter our welfare criterion. Furthermore, the inefficiencies result from a misallocation of labor *across* firms rather than the more common underutilization of labor suggested by frictionless models of monopsony (e.g. Bhaskar, Manning and To (2002)). The result that in equilibrium workers under-apply to the more productive firms and face lower unemployment than is optimal is similar in flavor to Acemoglu and Shimer (1999). The driving force in that paper, however, is the workers' risk aversion, while in this paper it is due to the firms' market power. More importantly, the focus of the two papers is quite different: we focus on the interaction between policy and firms' pricing decisions while Acemoglu and Shimer (1999) concentrate on how to counter the effects of workers' risk aversion.

3.3 Policy implications

The next step is to examine whether policy can improve on the decentralized allocation. We consider two policy interventions: a minimum wage and unemployment benefits. Our findings are that the introduction of a minimum wage exacerbates the misallocation resources while an unemployment benefits scheme can achieve constrained efficiency. We provide a discussion of these results at the end of the section.

First, consider the minimum wage. The goal is to show that the more productive firm hires less frequently when a minimum wage is imposed. Fix our original economy $\{x_1, x_2\}$, label the equilibrium before the introduction of a (binding) minimum wage as *unconstrained* and denote the equilibrium wages and probabilities by $\{w_1^U(x_1, x_2), w_2^U(x_1, x_2)\}$ and $\{p_1^*, p_2^*\}$. Consider a minimum wage in the interval $\underline{w} \in (w_2^U(x_1, x_2), x_2)$.[13] The equilibrium that follows the introduction of the minimum wage $\underline{w}$ is labeled *constrained* and the associated wages and probabilities are $\{w_1^C(x_1, x_2, \underline{w}), \underline{w}\}$ and $\{p_1^C, p_2^C\}$.

The constrained equilibrium is equivalent to the unconstrained equilibrium of an alternative economy $\{x_1, \tilde{x}_2\}$ where $\underline{w}$ is the low productivity firm's profit maximizing wage, i.e. the alternative economy is such that $w_2^U(x_1, \tilde{x}_2) = \underline{w}$ and $w_1^U(x_1, \tilde{x}_2) = w_1^C(x_1, x_2, \underline{w})$. It is straightforward to show that $\tilde{x}_2 > x_2$. Comparing the unconstrained equilibria of the two economies, the low productivity firm of the alternative economy hires more often than its counterpart in the original economy since $\tilde{x}_2 > x_2$. Therefore, when the minimum wage is imposed on the original economy, the low productivity firm starts hiring more often, pushing the economy further away from efficiency: $p_1^P > p_1^* > p_1^C$. Note, however, that that the unemployment rate decreases as a result of the minimum wage: Prob$[un|p_1^C]$ <Prob$[un|p_1^*]$ < Prob$[un|p_1^P]$.

Now consider an unemployment benefits scheme that gives b to every worker who was unable to find a job and is financed by lump-sum taxation. This scheme is simply a redistribution of resources and therefore it does not affect the efficient allocation. For the equilibrium analysis, we normalize all values to take into account the unemployment benefits: let $\hat{x}_j = x_j - b$ and $\hat{w}_j = w_j - b$ be the productivity and wage, respectively, in excess of the workers' unemployment benefits (or, outside option). Treating $\hat{x}_j$ as the firms productivity and $\hat{w}$ as the wage, the equilibrium can be characterized in the same way as in section 3.1. Equation (8) becomes

$$\frac{x_1 - b}{x_2 - b} = \frac{1 - p_2(b)}{1 - p_1(b)} \frac{1 + 2\ p_1(b)}{1 + 2\ p_2(b)} \frac{2 - p_1(b)}{2 - p_2(b)} \tag{14}$$

Equation (14) defines the equilibrium worker strategies for given b, $p_1(b)$. The ratio $(x_1 - b)/(x_2 - b)$ is strictly increasing in b and $p_1(b)$ can achieve any value between $[0, 1)$ by varying b within $[0, x_2)$. $p_1(x_2) = 1$ is too high because it is inefficient to price the low productivity

[13] A minimum wage below $w_2^U(x_1, x_2)$ has no effect because it does not bind, while if $\underline{w} \geq x_2$ the low productivity firm is priced out of the market. It is clear from equation (11) that it is never efficient to leave one of the firms without applications. Depending on parameter values, closing firm 2 down could improve efficiency but this is not a policy that we will consider.

firm out of the market and $p_1(0)$ is too low, as was shown in section 3.2. Therefore, there is a unique $b^* \in (0, x_2)$ such that $p_1(b^*) = p_1^P$ and efficiency is restored.

The main lesson of section 3.2 is that the market power of firms results in inefficiencies. This, of course, is not a new result. What is novel in our model is how the inefficiencies manifest themselves and the resulting implications with respect to two policies that can reduce the firms' market power. In contrast to frictionless models of monopsony where the inefficiencies are due to the underutilization of labor and where a (carefully chosen) minimum wage helps move towards efficiency, our model shows that the allocative inefficiencies are important and they are actually made worse by a minimum wage. While the minimum wage results in a redistribution of the surplus from the firms to the workers, this is not sufficient to be efficiency-improving. Intuitively, a minimum wage mostly affects the low productivity firms, while it is the high productivity firms that are principally responsible for the inefficiency. And, in fact, the minimum wage exacerbates the distortions caused by (productive, or large) firms' market power. Introducing an appropriately measured unemployment benefit can help overcome the inefficiencies because it affects the workers' decisions with respect to both types of firm. It is worth reiterating that in other frictional models that exhibit inefficiencies, but where prices do not have an allocative role, the welfare effects of introducing a minimum wage are qualitatively similar to those of introducing unemployment benefits, unlike the results of our model. For instance, see Acemoglu and Shimer (1999), Acemoglu (2001) or Manning (2001).

4 The General Case

We now extend our results to the general case of arbitrary but finite numbers of workers and firms. We replicate the analysis of section 3. The existence and characterization of equilibrium is analyzed in Galenianos and Kircher (2007) so section 4.1 simply defines the problem. The following sections extend our inefficiency result and examine the effects of policy.

4.1 Equilibrium Characterization

In this section we describe the agents' maximization problem for the general case of n workers and m firms.

We begin by examining the subgame that follows an arbitrary wage announcement $\mathbf{w}$. In the symmetric subgame equilibrium the workers' strategies, $\mathbf{p}(\mathbf{w})$, are such that the expected utility received by a worker is the same at all firms where he applies, i.e. $G(p_j)\ w_j = G(p_k)\ w_k = U(\mathbf{w})$, whenever $p_j, p_k > 0$.

We now determine the probability of getting a job. When the strategy of all other workers is to apply to firm j with probability $p_j > 0$, the probability of being hired by firm j is

$$G(p_j) = \frac{1-(1-p_j)^n}{n\ p_j},$$

where $G(0) \equiv \lim_{p_j \to 0} G(p_j) = 1$. Intuitively, the probability of getting the job is equal to the probability that at least one worker applies there divided by the expected number of applicants.[14] Note that $G(p_j)$ is decreasing in its argument.

Turning to the firms' problem, firm j will hire a worker unless it receives no applicants which occurs with probability $(1-p_j)^n$. Therefore, firm j hires with probability

$$H(p_j) = 1-(1-p_j)^n.$$

Firm j takes as given the wage announcements of other firms, $\mathbf{w}_{-j}$ and the equilibrium response of workers and solves the following problem:

$$\begin{aligned} \pi_j(w_j, \mathbf{w}_{-j}) &= \max_{w_j}\ H(p_j)\ (x_j - w_j) \\ \text{s.t.}\quad G(p_j)\ w_j &= U(w_j, \mathbf{w}_{-j}). \end{aligned} \tag{15}$$

In a companion paper, Galenianos and Kircher (2007), we prove the existence of a directed search equilibrium and show that, under an additional condition, wage dispersion is driven by productivity: more productive firms post higher wages and firms with the same productivity post the same wage. This condition will be important for the rest of our results, as it guarantees that the firms' problem is characterized by the first order conditions. It states that every firm can profitably attract workers' applications even when all of its competitors post a wage equal to their productivity level.

C1: If $w_j = x_j$, then $p_j(w_j, \overline{\mathbf{w}}_{-j}) > 0$ where $\overline{\mathbf{w}}_{-j} = (x_1, ..., x_{j-1}, x_{j+1}, ..., x_m)$, for all $j \in M$.

[14] One can derive this equation by summing over the binomial coefficients or by looking at the equivalent derivation in BSW.

4.2 Efficiency Properties of Equilibrium

We now generalize the results of section 3.2. We show that constrained efficiency does not obtain except for the special case of homogeneous firms and that in equilibrium the more productive firms hire less frequently than is efficient.

The planner's optimization problem is given by

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{j=1}^{m} H(p_j)\ x_j \\ \text{s.t.} \quad & \sum_{j=1}^{m} p_j = 1 \text{ and } p_j \geq 0\ \forall j \in M \end{aligned} \tag{16}$$

Let $\{p_1^P, ..., p_m^P\}$ denote the planner's constrained efficient allocation and $\{p_1^*, ..., p_m^*\}$ denote the equilibrium allocation.

Proposition 4.1 *Assume (C1) holds.*
(1) If $x_j \neq x_k$ for some $j, k \in M$, then constrained efficiency does not obtain in equilibrium. Furthermore, there exists an $r \in \{1, 2, ..., m\}$ such that $p_j^* < p_j^P$ for $j \in \{1, ..., r\}$ and $p_j^* > p_j^P$ for $j \in \{r+1, ..., m\}$.
(2) If $x_j = x_k$ for all $j, k \in M$, then constrained efficiency obtains in equilibrium.

Proof. See the appendix. ■

4.3 Policy Implications

We now generalize the policy implications of section 3.3 to an environment with many firms ($m \geq 2$). We first show analytically that the results of section 3.3 hold when firms have two productivity levels. We then provide some computational evidence that they extend to more general productivity distributions, although we have not been able to provide a proof.

Let $x_1 > x_2$ and suppose that m_1 and m_2 is the number of high and low productivity firms, respectively. Assume that condition (C1) holds. The characterization results in Galenianos and Kircher (2007) guarantee that in the unconstrained equilibrium all high productivity firms post w_1^U and all low productivity firms post w_2^U.

We first consider the welfare effect of imposing a binding minimum wage.

Proposition 4.2 *Any unconstrained equilibrium $\{w_1^U, w_2^U\}$ is strictly more efficient than an equilibrium with a minimum wage $\underline{w} \in (w_2^U, x_2)$.*

Proof. See the appendix. ■

In contrast, unemployment benefits can implement the efficient outcome.

Proposition 4.3 *There exist unemployment benefits $b^* > 0$, such that a directed equilibrium with such unemployment benefits is constrained efficient.*

Proof. See the appendix. ■

The following figures provide numerical evidence that the logic behind the above results holds in the general environment with a larger number of different productivity levels. In both figures there are five equidistant productivity levels ($x_1 = 3, x_2 = 2.5, ..., x_5 = 1$) with s firms at each level ($m_j = s$ for all j). In each case the number of worker is equal to the number of firms ($n = 5s$). The vertical axis denotes the percentage change in welfare due to a policy change as a proportion of the original inefficiency.

In figure 1, the policy in question is the minimum wage which is denoted as a ratio over the lowest unconstrained equilibrium wage on the horizontal axis. Consistent with our previous results, it is clear that the efficiency loss increases when the minimum wage is introduced. Different specifications of productivity and s yield qualitatively similar graphs which leads us to believe that this is a more general result.

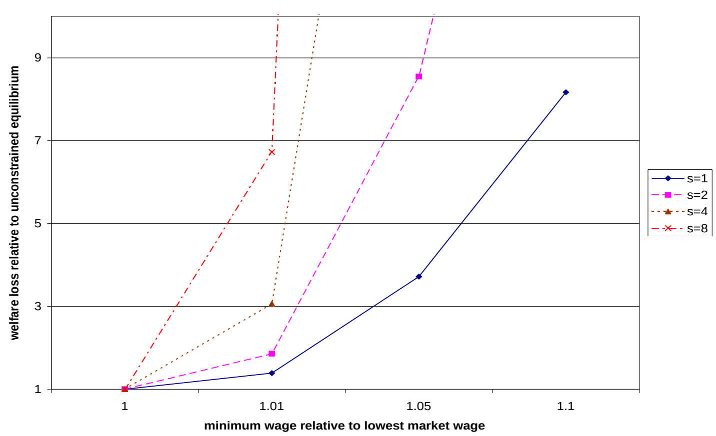

Figure 1: Illustration of the efficiency loss relative to the unconstrained equilibrium when a minimum wage is introduced.

In figure 2, the level of the unemployment benefit is on the horizontal axis. The productivity levels and number of agents are determined in the same way as above. This figure shows two things: first, moderate unemployment benefits improve welfare; second, the optimal level of unemployment benefits decreases in the market size (s) which reflects the fact that the decentralized allocation approaches efficiency as the market becomes larger. While it is not always possible to fully achieve efficiency due to the interaction between various productivity levels that cannot be completely fine-tuned with a single policy instrument, the efficiency losses in our illustrative example are reduced substantially even under multiple productivity levels when unemployment benefits are chosen optimally. Again, the features we present are representative of the numerical examples with different number of firms and productivity levels.

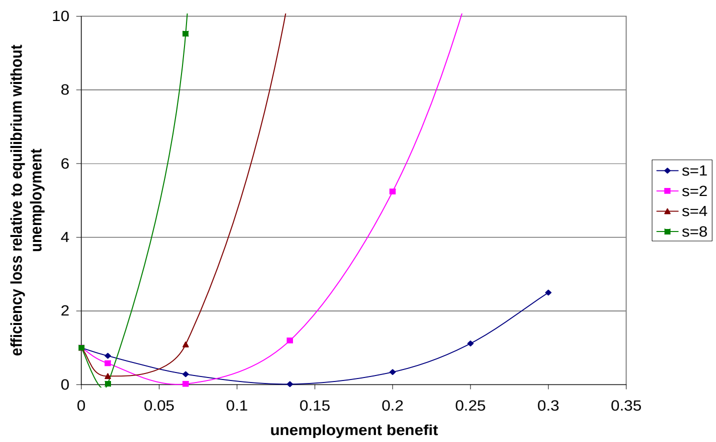

Figure 2: Illustration of the change in welfare from the introduction of unemployment benefits.

5 Conclusions

We develop a frictional model of the labor market with two main characteristics: wages play an important role in allocating labor across heterogeneous firms and there is room for welfare-improving policy because the decentralized equilibrium is inefficient. The inefficiencies are driven by the market power that firms enjoy. However, we show that not all policies favoring workers are good for welfare. In particular, a minimum wage actually reduces welfare by further distorting the allocation. Unemployment benefits, on the other hand, can increase welfare because they make workers more willing to wait and look for better employment options, thus, in effect, limiting firms' market power. These results are interesting because they put the recent debate on minimum wages into perspective: while most papers have focused on the employment effects of a minimum wage, we show that the welfare implications of such policy can be more complicated. Even when it is desirable to transfer surplus towards workers (in our case because of the firms' market power), a minimum wage may have additional undesirable distortionary effects.

We highlight a particular channel through which inefficiencies may arise, namely the market power that firms enjoy in the context of a small market. Since many labor markets are fragmented across occupational and geographical lines, we think that this is a case worth studying. For instance, following Shimer (2007) and restricting a labor market to an occupation and geographical area combination leads to as few as 10 unemployed workers per labor market on average with a correspondingly small number of hiring firms.[15] While workers arguably search across some geographical borders and across some occupational boundaries, this calculation suggests that a strategic view of the hiring process might be relevant for the labor market experience of a significant proportion of workers.

The question of whether the message of this paper is more broadly applicable to different channels that lead to inefficiencies is left for future research. We conjecture that at least in the case of risk aversion, which was examined by Acemoglu and Shimer (1999), our results carry over once we introduce heterogeneity in firm total factor productivity: workers' risk aversion would lead them to apply too frequently to the safer, low-productivity firm, leading to a misallocation of labor similar to the one described in this paper. Therefore, we conjecture that the two policies that we examine would have the effects that we describe in this paper.

[15]Shimer (2007) proposes the combination of occupation and geographical unit as a labor market. With a total of 362 metropolitan and 560 micropolitan statistical areas (regions with at least one urbanized area of more than 50,000 inhabitants and 10,000 to 50,000 urban inhabitants, respectively) and about 800 occupations listed in the Occupational Employment Statistics (OES) and he obtains a total of about 740,000 combinations of occupations and geographic areas. For an unemployment level of 7.6 million in the Current Employment Statistics (CES) of December 2007 this yields on average 10.4 unemployed people per combination of occupation and geographical area.

6 Appendix

We introduce the following definitions to facilitate exposition. Let $g(p_j) \equiv G'(p_j) = -[G(p_j) - (1-p_j)^{n-1}]/p_j$ for $p_j > 0$ and $g(0) \equiv \lim_{p_j \to 0} g(p_j) = -(n-1)/2$; $u_j(\mathbf{w}) \equiv g(p_j(\mathbf{w}))\ w_j$; and $h(p_j) \equiv H'(p_j) = n\ (1-p_j)^{n-1}$.

Proposition 4.1:
Proof. Equation (16) yields the following first order conditions:

$$\begin{aligned} h(p_j)\ x_j &\leq \lambda \\ &= \lambda \text{ if } p_j > 0, \quad \forall j \in M, \end{aligned}$$

where λ is the Lagrange multiplier. This condition requires that the shadow value in terms of expected output has to be equal across all firms that receive applications. Therefore, for any two firms j and k with $p_j, p_k > 0$ the following has to hold:

$$\frac{x_j}{x_k} = \frac{h(p_k^P)}{h(p_j^P)} = (\frac{1-p_k^P}{1-p_j^P})^{n-1}. \tag{17}$$

One implication of equation (17) is that with homogeneous firms ($x_1 = ... = x_m$), efficiency obtains when workers apply to each firm with with identical probability which is established to occur in equilibrium in Galenianos and Kircher (2007).

We turn to the case with productivity heterogeneity. For simplicity, the rest of the proof assumes that no two firms have the same productivity level, though the complimentary case can be handled with minor modifications. Assume, without loss of generality, that $x_1 > x_2 > ... > x_m$. Consider the firms' problem. Under (C1), all firms attract applications and the solutions to the firms' problem are characterized by their first order conditions. Furthermore, our characterization result from section 4.1 implies that $p_1 > p_2 > ... > p_m > 0$. The first order conditions of the firm's problem are

$$\frac{\partial \pi_j}{\partial w_j} = -H(p_j) + h(p_j)\ [x_j - w_j]\ \frac{\partial p_j}{\partial w_j}. \tag{18}$$

Equating (18) to zero for all j and rearranging leads to:

$$\frac{x_j}{x_k} = \frac{h(p_k)}{h(p_j)}\ \frac{h(p_j)\ w_j + \frac{H(p_j)}{\partial p_j / \partial w_j}}{h(p_k)\ w_k + \frac{H(p_k)}{\partial p_k / \partial w_k}}, \tag{19}$$

We proceed to compare the probabilities implied by the solution to the planner's problem to the ones from the decentralized firms' problem. First, note that $p_j^P = 0$ is a possibility for some $j \in M$. In that case, it is straightforward to show that there exists some t such that $p_1^P > p_2^P > ..., p_t^P > 0$ and $p_{t+1}^P = ... = p_m^P = 0$, which means that the low productivity firms (below the tth) hire too often in the decentralized allocation, as the statement of the proposition suggests.

In what follows, we restrict attention on those firms that is efficient to attract applications. The efficient probabilities are given by equation (17) which we compare to the probabilities from the decentralized allocation (19).

We want to show that for all $j < k$

$$\frac{1-p_k^P}{1-p_j^P} > \frac{1-p_k}{1-p_j}, \tag{20}$$

which implies the claim of the proposition. We establish equation (20) for $j = 1$ and $k = 2$ but the proof is identical for other values of j and k.

Recalling that $U(\mathbf{w}) = G(p_i(\mathbf{w}))\ w_i$, for all $p_i > 0$, the problem of firm 1 is

$$\begin{aligned} &\max_{w_1}\ H(p_1(\mathbf{w}))\ (x_1 - w_1) \\ \text{s.t.}\ \ &G(p_1(\mathbf{w}))\ w_1 = G(p_2(\mathbf{w}))\ w_2 \end{aligned}$$

which is equivalent to: $\max_{w_1} [(1-(1-p_1)^n)\ x_1 - n\ p_1\ G(p_2)\ w_2]$ where the argument $\mathbf{w}$ has been omitted for brevity. Setting the first order condition of this problem to zero yields

$$(1-p_1)^{n-1} x_1 \frac{\partial p_1}{\partial w_1} = w_2[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}].$$

Performing the same calculation for firm 2 and combining the results yields

$$\frac{w_1}{w_2} = (\frac{1-p_2}{1-p_1})^{n-1} \frac{x_2}{x_1} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2} p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]}.$$

Using the indifference condition of the buyers, $G(p_1)\ w_1 = G(p_2)\ w_2$, leads to

$$\frac{x_1}{x_2} = (\frac{1-p_2}{1-p_1})^{n-1} \frac{G(p_1)}{G(p_2)} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2} p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]}.$$

If

$$\frac{G(p_1)}{G(p_2)} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2} p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]} > 1 \tag{21}$$

then equation (20) holds and we have our result. The rest of the proof establishes (21).

Equation (21) holds if and only if

$$\frac{p_1\ g(p_2)\ \frac{\partial p_2}{\partial w_1}/\frac{\partial p_1}{\partial w_1}}{p_2\ g(p_1)\ \frac{\partial p_1}{\partial w_2}/\frac{\partial p_2}{\partial w_2}} > \frac{G(p_2)}{G(p_1)}. \tag{22}$$

We want to characterize $\partial p_i/\partial w_l$. Note that $p_1+...+p_m = 1 \Rightarrow \partial p_1/\partial w_i+...+\partial p_m/\partial w_i = 0$. Let $\rho_i \equiv g(p_i)/G(p_i)$. We can differentiate the equality $G(p_1)\ w_1 - G(p_i)\ w_i = 0$ for $i > 2$ with respect to w_2 to get (where the equality was used again to substitute out the wages)

$$\frac{\partial p_i}{\partial w_2} = \frac{\partial p_1}{\partial w_2}\ \frac{\rho_1}{\rho_i}$$

Therefore,

$$\begin{aligned} \frac{\partial p_2}{\partial w_2} = -[\frac{\partial p_1}{\partial w_2} + \frac{\partial p_2}{\partial w_2} + ... + \frac{\partial p_m}{\partial w_2}] &= -\frac{\partial p_1}{\partial w_2}\ \rho_1\ [\frac{1}{\rho_1} + \frac{1}{\rho_3} + ... + \frac{1}{\rho_m}] \\ \Rightarrow \frac{\partial p_1}{\partial w_2}/\frac{\partial p_2}{\partial w_2} &= -\frac{\frac{1}{\rho_1}}{\frac{1}{\rho_1} + \frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m}}. \end{aligned}$$

We can characterize $(\partial p_2/\partial w_1)/(\partial p_1/\partial w_1)$ in a similar way. Using these results, we can rewrite (22) as

$$\begin{aligned} &\frac{p_1\rho_1(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}{p_2\rho_2(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})}\frac{\frac{1}{\rho_1}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}}{\frac{1}{\rho_2}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}} \\ &\qquad > \frac{1+(1-p_2)+...+(1-p_2)^{n-1}}{1+(1-p_1)+...+(1-p_1)^{n-1}}. \end{aligned} \tag{23}$$

The definition of ρ implies that

$$\frac{\rho_1}{\rho_2} = \frac{(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})}{(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}\frac{1+(1-p_2)+...+(1-p_2)^{n-1}}{1+(1-p_1)+...+(1-p_1)^{n-1}}. \tag{24}$$

Case 1: $0 > \rho_1 > \rho_2$

In this case

$$\frac{\frac{1}{\rho_1}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}}{\frac{1}{\rho_2}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}} > 1$$

and thus (23) follows from

$$\frac{p_1\rho_1(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}{p_2\rho_2(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})} > \frac{1+...+(1-p_2)^{n-1}}{1+...+(1-p_1)^{n-1}}.$$

However, using (24) this last inequality is equivalent to $p_1 > p_2$ which holds since $x_1 > x_2$.

Case 2: $0 > \rho_2 \geq \rho_1$

In this case it holds that

$$\frac{\rho_1}{\rho_2} \frac{\frac{1}{\rho_1} + \frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m}}{\frac{1}{\rho_2} + \frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m}} = \frac{1 + \rho_1(\frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m})}{1 + \rho_2(\frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m})} \geq 1,$$

because $\rho_i < 0$ for all i. Then (23) follows from

$$\frac{p_1(1 + 2(1-p_2) + ... + (n-1)(1-p_2)^{n-2})}{p_2(1 + 2(1-p_1) + ... + (n-1)(1-p_1)^{n-2})} > \frac{1 + (1-p_2) + ... + (1-p_2)^{n-1}}{1 + (1-p_1) + ... + (1-p_1)^{n-1}},$$

which can be rewritten as

$$\frac{1 - (1-p_1)^n}{1 + ... + (n-1)(1-p_1)^{n-2}} > \frac{1 - (1-p_2)^n}{1 + ... + (n-1)(1-p_2)^{n-2}}.$$

This inequality holds holds because $p_1 > p_2$. ■

Proposition 4.2:

Proof. Let p_i^U be the equilibrium application probability of workers to firm i when no minimum wage is introduced, and let $p_i(\underline{w})$ be the application probability when the minimum wage is introduced. Let $(w_1(\underline{w}), w_2(\underline{w}))$ refer to an equilibrium wage offer profile if a minimum wage requirement is introduced.

First, it is easy to show that

$$w_1(\underline{w}) \geq w_2(\underline{w}).$$

Moreover, $w_1(\underline{w}) = w_2(\underline{w})$ implies that $w_1(\underline{w}) = w_2(\underline{w}) = \underline{w}$, i.e. the minimum wage is so high that it is binding even for the high productivity firm. In this case obviously $p_1(\underline{w}) = p_2(\underline{w})$ holds and thus the equilibrium application levels are further from the constrained efficient allocation than without a minimum wage, since without the minimum wage at least $p_1 > p_2$ could be ensured.

Consider now the case where $w_1 > w_2 \geq \underline{w}$. In this case a high productivity firm chooses his wage offer such that his marginal profit is zero, since such a firm does not face a binding minimum wage. Let $w_1(p_1, w_2, \widehat{w}_1)$ denote the (unique) wage level that a high productivity firm needs to offer to obtain an application probability of p_1, if low productivity firms offer a wage of w_2 and the other high productivity firms offer a wage of $\widehat{w}_1$. It is easy to show that

$$w_1(p_1, \alpha w_2, \alpha \widehat{w}_1) = \alpha w_1(p_1, w_2, \widehat{w}_1).$$

Now, we show that if w_2 increases, then the high productivity firms obtain lower application probabilities in equilibrium, which would prove the result, since this means that increasing the minimum wage (and thus w_2) moves the allocation even further from the constrained efficient allocation. To prove this claim take any given p_1 and let $\widehat{w}_1$ be such that if all high productivity firms offer this wage, then each of them is visited with probability p_1. Let us denote this value as $w_1^*(p_1, w_2)$ and note that

$$w_1(p_1, w_2, w_1^*(p_1, w_2)) = w_1^*(p_1, w_2).$$

Suppose that a high productivity firm, firm i considers a deviation in his wage to change the application probability he receives. If he achieves an application probability of $\widetilde{p}_1$, then his profit can be written as

$$\pi_1(\widetilde{p}_1, w_2, p_1) = (x_1 - w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2)))H_1(\widetilde{p}_1).$$

Linearity of w_1^* in w_2 implies that

$$w_1^*(p_1, \alpha w_2) = \alpha w_1^*(p_1, w_2),$$

and thus linearity of function w_1 in w_2 and $\widehat{w}_1$ implies that

$$w_1(\widetilde{p}_1, \alpha w_2, w_1^*(p_1, \alpha w_2))) = \alpha w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2))).$$

Now, let us study the marginal profit of firm i from increasing $\widetilde{p}_1$ when $\widetilde{p}_1 = p_1$ and the low productivity firms offer w_2. This marginal profit can be written as

$$\beta(w_2, p_1) = \frac{\partial(x_1 - w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2)))H_1(\widetilde{p}_1)}{\partial \widetilde{p}_1} |_{\widetilde{p}_1 = p_1} =$$

$$= \frac{\partial(x_1 - w_2 w_1(\widetilde{p}_1, 1, w_1^*(p_1, 1)))H_1(\widetilde{p}_1)}{\partial \widetilde{p}_1} |_{\widetilde{p}_1 = p_1} .$$

It is immediate that

$$\frac{\partial \beta(w_2, p_1)}{\partial w_2} < 0$$

holds. Therefore, if $w_2 > w_2^U$ then $\beta(w_2, p_1^U) < 0$. Moreover, it holds that $\beta(w_2, 0) > 0$, since attracting an extra customer when no one is planning to visit is always profitable. By continuity of function β it follows that there exists a value $\overline{p}_1 \in (0, p_1^U)$, such that $\beta(w_2, \overline{p}_1) = 0$. By construction, if all other high productivity firms offer a wage of $w_1^*(\overline{p}_1, w_2)$ and low productivity firms offer w_2, then any given high productivity firm cannot gain by changing his wage offer by deviating from $w_1^*(\overline{p}_1, w_2)$ slightly. However, Galenianos and Kircher (2007) show that under assumption C1 the profit function is concave in the own wage variable and thus offering $w_1^*(\overline{p}_1, w_2)$ is a best reply for all high productivity firms.

Therefore, for every wage level of the low productivity firms, such that $w_2 > w_2^U$, there is an equilibrium in the game between only the high productivity firms (i.e. taking w_2 as given) such that the workers' application probability to high productivity firms goes down if low productivity firms all offered wage w_2, because $\overline{p}_1 < p_1^U$.

The proof is completed by showing that for every w_2 the best response in the game of high productivity firms is unique. The tedious algebra for this result can be obtained from the authors upon request (or see the technical appendix on http://www.econ.psu.edu/∼ eug10/). ■

Proposition 4.3:

Proof. Let $p_j(b)$ denote the probability with which a worker applies to some firm of productivity x_j when the level of unemployment benefits is given by b. Assume without loss of generality that $x_1 > x_2$. Under (C1) both firms attract applications when $b = 0$. If $p_2^* = 0$, then setting $b = x_2$ implements the constrained efficient allocation. The rest of the proof considers the case where $p_2^P > 0$.

The strategy of the proof is to show that there is a b^* such that the firms' first order conditions coincide with the ones of the planner. Constrained efficiency is given by

$$\frac{x_1}{x_2} = (\frac{1 - p_2^P}{1 - p_1^P})^{n-1} > (\frac{1 - p_2^*}{1 - p_1^*})^{n-1},$$

where the inequality follows from the argument used in the proof of Proposition 4.1 (noting that in our notation $p_j^* = p_j(0)$).

As shown in Section 3, an unemployment benefit is mathematically equivalent to lowering the productivity of every firm by b leading to the following equilibrium condition for the decentralized economy:

$$\frac{x_1 - b}{x_2 - b} = (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1} p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2} p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}$$

We use the intermediate value theorem to conclude the proof. Note that for b close enough to x_2

$$\frac{x_1 - b}{x_2 - b} > (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1} p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2} p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}.$$

If $b = 0$, then

$$\frac{x_1 - b}{x_2 - b} = (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} < (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1}p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2}p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}.$$

Therefore, an appropriate value of $b \in [0, x_2)$ works to replicate the constrained efficient allocation. ■

References

[1] Acemoğlu, Daron (2001): "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19, pp. 1-22.

[2] Acemoğlu, Daron and Robert Shimer (1999): "Efficient Unemployment Insurance," *Journal of Political Economy*, 107, 893-928.

[3] Albrecht, James, Pieter A. Gautier, and Susan Vroman (2006): "Equilibrium Directed Search with Multiple Applications." *Review of Economic Studies*, 73, 869-891.

[4] Albrecht, James, Pieter A. Gautier, Serene Tan, and Susan Vroman (2004): "Matching with Multiple Applications Revisited." *Economics Letters*, 84, 311-314.

[5] Bhaskar, V., Alan Manning and Ted To (2002): "Oligopsony and Monopsonistic Competition in Labor Markets." *Journal of Economic Perspectives*, 16 (2), 155-174.

[6] Burdett, Kenneth, Shouyong Shi and Randall Wright (2001): "Pricing and Matching with Frictions," *Journal of Political Economy*, 109, 1060-1085.

[7] Card, David and Alan B. Krueger (1994): "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review*, 84 (4), 772-793.

[8] Coles, Melvyn and Jan Eeckhout (2003a): "Indeterminacy and Directed Search," *Journal of Economic Theory*, 111, 265-276.

[9] Coles, Melvyn and Jan Eeckhout (2003b): "Heterogeneity as a Coordination Device,"mimeo.

[10] Galenianos, Manolis and Philipp Kircher (2005): "Directed Search with Multiple Job Applications" *Journal of Economic Theory*, forthcoming.

[11] Galenianos, Manolis and Philipp Kircher (2007). "Heterogeneous Firms in a Finite Directed Search Economy." PIER working paper.

[12] Halko, Marja-Liisa, Klaus Kultti and Juha Virrankoski (2008): "Search Direction and Wage Dispersion," *International Economic Review*, 49, 111-134.

[13] Kircher, Philipp (2007): "Efficiency of Simultaneous Search." mimeo.

[14] Moen, Espen R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385-411.

[15] Montgomery, James D. (1991): "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 106, 163-179.

[16] Neumark, David and William Wascher (2000): "A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Comment," *American Economic Review*, 90 (5), 1362-1396.

[17] Neumark, David and William Wascher (2000): "Minimum Wages and Employment," *Foundations and Trends in Microeconomics*, 3, 1-182.

[18] Peters, Michael (1991): "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica*, 59 1425-1454.

[19] Peters, Michael (2000): "Limits of Exact Equilibria for Capacity Constrained Sellers with Costly Search," *Journal of Economic Theory*, 95, 139-168.

[20] Shi, Shouyong (2001): "Frictional Assignment I: Efficiency," *Journal of Economic Theory*, 98, 232-260.

[21] Shimer, Robert (2005): "The Assignment of Workers to jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113/5, 996-1025.

[22] Shimer, Robert (2007): "Mismatch," *American Economic Review*, 97/4, 1074-1101.

[23] Shimer, Robert and Ivan Werning (2008): "Liquidity and Insurance for the Unemployed," *American Economic Review*, forthcoming.

[24] Virag, Gabor (2008): "Collusive equilibria in directed search models," mimeo.