Bailouts and Bank Runs

Todd Keister
Research and Statistics Group
Federal Reserve Bank of New York
Todd.Keister@ny.frb.org

September 24, 2009

PRELIMINARY AND INCOMPLETE

Abstract

How does the belief that policy makers will bail out the financial sector in the event of a crisis affect individual behavior and economic outcomes? I study this question in a model of financial intermediation with limited commitment. In the event of a crisis, policy makers may choose to transfer real resources into the financial sector. When this allocation decision is made *ex post*, a moral hazard effect arises and banks choose *ex ante* contracts with excessive leverage. If policy makers could credibly commit to not provide any bailout, banks would choose less leverage. However, the anticipation of a bailout also lessens the incentive for individuals to withdraw their funds and can thereby have a stabilizing effect on the economy. I show that committing to a no-bailout policy *increases* the scope for financial fragility in this model. Such a commitment can either increase or decrease *ex ante* welfare, depending on parameter values.

Prepared for the Cornell-Penn State Conference on Financial Fragility. I thank Vijay Narasiman for excellent research assistance. The views expressed herein are my own and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

It is often claimed that government bailouts of the financial sector during a crisis serve to plant the seed of future crises. By insulating banks, other financial institutions, and individuals from the consequences of bad outcomes, bailouts are said to encourage risky behavior and thereby increase the potential likelihood of a future crisis. In other words, the expectation that the government will respond to a crisis by transferring resources into the financial sector generates a moral hazard effect. This effect has been discussed at length during the current financial crisis and has been the basis for sharp criticism of the actions of the public sector in the U.S. and elsewhere. This reasoning has led some observers to argue that the economy would be better off if the government could credibly commit not to undertake such bailouts, since the private sector would then have greater incentive to prepare for adverse outcomes, which would make such outcomes less frequent and/or less severe.

The anticipation of a government bailout can also have a stabilizing effect on the financial sector, however. Financial crises are often believed to have an important self-fulfilling component, with individual investors or depositors each withdrawing funds because they fear the withdrawals of others will deepen the crisis and create further losses. The anticipation of a bailout provides these individuals with a form of insurance, since it lessens the potential loss they will face if they do not withdraw their funds. As such, bailouts tend to decrease the incentive for individuals to withdraw, which, in turn, tends to make the financial sector less susceptible to self-fulfilling crises. One can think of government-sponsored deposit insurance programs, for example, as a type of bailout policy that is explicitly designed to play this stabilizing role.

Given these two competing effects, how does the belief that the government will bail out the financial sector in the event of a crisis affect individual behavior and economic outcomes? Would it be desirable for policy makers to commit to never bail out financial institutions and their creditors? This paper addresses these questions in a model of financial intermediation and fragility based on the classic paper of Diamond and Dybvig [4]. In particular, it studies an environment with idiosyncratic liquidity risk and with limited commitment by policy makers, as in Ennis and Keister [8]. Individuals can deposit resources in banks, and these resources are invested in a nonstochastic production technology. Banks perform maturity transformation by offering contracts that insure depositors against idiosyncratic liquidity risk. This maturity transformation may make a bank susceptible to a self-fulfilling run by depositors. Policy makers can react to a run by rescheduling banks' liabilities and, potentially, by "bailing out" the banking sector with a transfer of real resources.

Fiscal policy is introduced into this framework by adding a public good that is financed by taxing households' endowments. In the event of a crisis, some of this revenue may instead be given to the remaining depositors in the banking system. The policy maker chooses the size of this bailout payment in order to achieve an *ex post* efficient allocation of the remaining resources in the economy. Of course, banks and depositors anticipate this reaction when making ex ante decisions. The model thus captures both the moral hazard effect and the insurance effect of bailouts in a natural way.

Diamond and Dybvig [4] studied a form of deposit insurance in their model that was financed by taxing depositors after they have withdrawn from the bank. Wallace [16] pointed out that such a policy violates the sequential service constraint used to motivate the assumption that banks offer demand-deposit contracts. In other words, Wallace [16] emphasized that a coherent explanation of financial fragility requires that policy makers be constrained by the same frictions as private agents. The model presented here is entirely consistent with this view. The policy maker taxes endowments before agents deposit in banks. During normal times, all tax revenue is used to produce the public good. If, however, the policy maker discovers that a run is underway, this fact changes the marginal opportunity cost of providing the public good. The efficient response will be to transfer resources to the banking sector, which decreases the level of the public good but increases private consumption for the remaining depositors.

I compare the equilibrium outcomes in this model under three different policy scenarios. First, I study a benchmark economy in which a benevolent policy maker is able to set the banking contract, the tax rate, and the bailout policy. The policy maker cannot commit to future actions and chooses each policy as an optimal response to the strategies of depositors. For some parameter values, bank runs can occur with positive probability in this setting. Nevertheless, because the banking contract is determined by a benevolent policy maker, no moral hazard arises and the equilibrium allocation satisfies an appropriate notion of constrained efficiency. This outcome provides a natural benchmark against which the outcomes under different policies can be compared.

I then study the model when individual banks compete for deposits by announcing deposit contracts. This standard approach leads all banks to offer the contract that, in equilibrium, maximizes the expected utility of all depositors. The tax rate and any bailout payment are still determined by the benevolent policy maker. Importantly, banks and depositors recognize that in the event of a crisis, the policy maker will intervene to bring about the efficient allocation of the remaining resources, which removes the private incentive to prepare for such scenarios. As a result, the equilibrium banking contract has a higher degree of maturity transformation than in the first-best allocation. This is the moral hazard effect in the model: the anticipation of a bailout in the event of a crisis leads banks to take on more leverage.

For the third policy scenario, I suppose that the government is able to commit to not providing any bailout payment. This policy can be thought of as a simple rule that could be written into the economy's legal code or constitution. Banks still compete for deposits, and the tax rate is again determined by the benevolent policy maker. The only difference in this scenario is that all tax revenue must be used to provide the public good. The commitment not to bail out the banking sector removes the moral hazard effect described above. If fact, it leads banks to perform *less* maturity transformation than in the first-best allocation.

I compare the equilibrium outcomes under the different policy regimes along two dimensions. First, I examine the scope for financial fragility, that is, the set of parameter values of the model for which a bank run can occur with positive probability in equilibrium. Somewhat surprisingly, committing not to provide any bailout always increases the scope for financial fragility. While the decrease in leverage that accompanies the no-bailout policy tends to make the financial system more stable, this effect is more than offset by the increased incentive for depositors to withdraw if they expect others to do so. Contrary to the claims of many commentators, therefore, there is a well-defined sense in which a commitment to a no-bailouts policy *increases* the fragility of the banking sector. Finally, I examine the welfare benefits of committing to a no-bailout policy. I show through examples that whether or not a no-bailout policy is desirable depends on parameter values. In particular, for some parameter values committing to provide no bailout does raise ex ante welfare.

There is a large literature in which versions of the Diamond-Dybvig model are used to address issues related to banking policy and financial fragility. This paper continues that tradition, but, unlike many papers in that literature, no ad hoc restrictions are placed on banking contracts to simplify the analysis. The paper follows Green and Lin [11], Peck and Shell [15] and other recent work in specifying an explicit sequential service constraint and allowing any banking contract that is consistent with the information flow generated by that constraint. The paper also focuses on the implications of a lack of commitment power on the part of policy makers, an issue that has received relatively little attention in the context of banking policy. Notable examples of such work included Mailath and Mester [14] and Acharya and Yorulmazer [1].

The remainder of the paper is organized as follows. The next section presents the model together with the first-best allocation of resources in this environment. It also discusses the type of bank-run equilibrium that will be considered here. Section 3 then analyzes equilibrium under the efficient banking policy, where the benevolent policy maker chooses the banking contract as well as the fiscal policy. Section 4 studies the model with competitive banks and the ex post efficient bailout policy, while Section 5 studies the case where the policy maker has committed to no bailout. A numerical example is presented in Section 6.

2 The Model

The model follows that in Ennis and Keister [8], augmented to include a public good. I begin by describing the physical environment and deriving the first-best allocation in this environment. I then discuss how the public good is provided in equilibrium and the type of strategy profiles that are considered in this paper.

2.1 The environment

There are three time periods: $t = 0, 1, 2$. There is a continuum of depositors, indexed by $i \in [0, 1]$. Each depositor has preferences given by

$$u\left(c_1, c_2, d; \theta_i\right) = \frac{\left(c_1 + \theta_i c_2\right)^{1-\gamma}}{1-\gamma} + \delta \frac{d^{1-\gamma}}{1-\gamma},$$

where c_t is consumption of the private good in period t and d is the level of public good. The parameter δ measures the relative importance of the public good and is assumed to be common to all depositors. The public good is consumed in period 1. The parameter θ_i is a binomial random variable with support $\Theta = \{0, 1\}$. If the realized value of θ_i is zero, depositor i is *impatient* and only cares about consumption in period 1. A depositor's type θ_i is revealed to her in period 1 and remains private information. Let π denote the probability with which each individual depositor will be impatient. By a law of large numbers, π is also the fraction of depositors in the population who will be impatient. As in Diamond and Dybvig [4], the coefficient of relative risk aversion γ is assumed to be greater than 1.

The economy is endowed with one unit of the good per capita in period 0. As in Diamond and Dybvig [4], there is a single, constant-returns-to-scale technology for transforming this endowment into private consumption in the later periods. A unit of the good invested in period 0 yields $R > 1$ units in period 2, but only one unit in period 1.

A banking technology allows depositors to pool resources and insure against individual liquidity risk. The banking technology is operated in a central location. As in Wallace [16], [17], depositors are isolated from each other in periods 1 and 2 and no trade can occur among them. However, each depositor has the ability to visit the central location once, either in period 1 or in period 2, and withdraw from the pooled resources after her type has been realized. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the productive technology described above in period 0.

Depositors' types are revealed in a fixed order determined by the index i; depositor i discovers her type before depositor i' if and only if $i < i'$. A depositor knows her own index i and, therefore, knows her position in this ordering. This construction follows Green and Lin [10] and is a simplified version of that in Green and Lin [11]. Upon discovering her type, each depositor must decide whether or not to visit the central location in period 1. If she does, she must consume immediately; the consumption opportunity in period 1 is short-lived. This timing assumption implies that the payment a depositor receives from the banking technology cannot depend on any information other than the number of depositors who have withdrawn prior to her arrival. This sequential-service constraint follows Wallace [16], [17] and others.

Under sequential service, the payments made from the banking technology in period 1 can be summarized by a (measurable) function $x : [0, 1] \to \mathbb{R}_+$, where the number $x(\mu)$ has the interpretation of the payment given to the μ^{th} depositor to withdraw in period 1. Note that the arrival point μ of a depositor depends not only on her index i but also on the actions of depositors with lower indexes. In particular, μ will be strictly less than i if some of these depositors choose not to withdraw in period 1. In period 2, the bank will pay an equal share of the matured assets in the banking technology to each remaining depositor. Therefore, the operation of the banking technology is completely described by the function x, which I call the *banking policy*. Feasibility of the banking policy requires that total payments in period 1 not exceed the short-run value of assets, even if all depositors choose to withdraw in that period, that is,

$$\int_0^1 x(\mu)\, d\mu \leq 1 - d. \tag{1}$$

There is also an extrinsic, "sunspot" random variable that is observed by all depositors in period 1. As is well known, there cannot be an equilibrium of this model in which a bank run occurs with certainty – if everyone expects a run to occur, the optimal banking policy will be "run proof" (see, for example, Cooper and Ross [3]). A run can only occur in equilibrium if, at the time the policy x is set, agents are unsure whether or not a run will occur. Introducing a sunspot variable is the standard way to allow for this possibility.[1] I assume, without any loss of generality, that the sunspot variable is uniformly distributed on $S = [0, 1]$.

The behavior of depositor i can be summarized by a (measurable) function $y_i : \Theta \times S \to \{0, 1\}$ that assigns a particular action to each possible realization of her type and of the sunspot variable. Here $y_i = 0$ represents withdrawing in period 1 and $y_i = 1$ represents waiting until period 2. I refer to the function y_i as the *withdrawal strategy* of depositor i, and we use y to denote the profile of withdrawal strategies for all depositors.

An *allocation* in this environment consists of an assignment of consumption levels to each depositor in each period along with a level of public good provision. An individual depositor's private consumption is completely determined by the banking policy $x,$ the level of the public good $d,$ the profile of withdrawal strategies y, and the realization of her own type θ_i. We can, therefore, define the (indirect) expected utility of depositor i as

$$v_i(x, y, d) = E\left[u\left(c_{1,i}, c_{2,i}, d; \theta_i\right)\right],$$

where E represents the expectation over θ_i. Define U to be the average of all depositors' expected utilities, *i.e.*,

$$U(x, y, d) = \int_0^1 v_i(x, y, d)\, di. \tag{2}$$

This expression can be given the following interpretation. Suppose that, at the beginning of period 0, depositors are assigned their index i randomly, with each depositor having an equal chance of occupying each space in the unit interval. Then U measures the expected utility of each depositor

[1] See Diamond and Dybvig [4], Cooper and Ross [3], and Peck and Shell [15].

before places are assigned. I use U as the measure of aggregate welfare throughout the paper (as in Green and Lin [10], [11]) and others.

2.2 The first-best allocation

The first-best allocation of resources in this environment is the allocation that would be chosen by a fictitious planner who could observe each depositor's type and allocate resources accordingly. The planner would give the same amount of consumption to all impatient depositors in period 1; let c_1 denote this amount. It would also give the same amount of consumption to all patient depositors in period 2; denote this amount by c_2. Then the first-best allocation is the solution to the following maximization problem

$$\max_{\{c_1,c_2,\tau\}} \pi\frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma} + \delta\frac{d^{1-\gamma}}{1-\gamma}$$

subject to

$$(1-\pi)c_2 = R\left(1-d-\pi c_1\right).$$

Note that this problem combines two very standard elements: the division of resources between private consumption and a public good, plus the allocation of private consumption between patient and impatient agents. The solution is given by

$$c_1^{FB} = \frac{1}{\delta^{\frac{1}{\gamma}} + \pi + (1-\pi)\,A_0}, \qquad c_2^{FB} = \frac{RA_0}{\delta^{\frac{1}{\gamma}} + \pi + (1-\pi)\,A_0}, \qquad \text{and}$$

$$d^{FB} = \frac{\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + \pi + (1-\pi)\,A_0},$$

where

$$A_0 \equiv R^{\frac{1-\gamma}{\gamma}} < 1.$$

Define

$$r_1 \equiv \frac{c_1}{1-d},$$

so that r_1 represents the private consumption of an impatient depositor relative to the per-capita period-1 value of the resources designated for private consumption. In the decentralized economies discussed below, r_1 will have the interpretation of the return offered by banks on deposits withdrawn early, or as the leverage of the banking system. In the first-best allocation presented above, we have

$$r_1^{FB} = \frac{1}{\pi + (1-\pi) A_0}.$$

Note that r_1^{FB} is independent of δ; in fact, this is the same value that would characterize the first-best allocation in a standard Diamond-Dybvig model with no public good.

2.3 Public good provision

Note that the amount of public good in the first-best allocation is strictly positive. Since there is a continuum of depositors, individuals will not voluntarily provide any of the public good from their own funds. In the decentralized economy, I assume that the public good is provided by a benevolent policy maker who has the ability to tax endowments in period 0. The revenue from this tax is placed into the investment technology and transformed into period 1 goods. In period 1, the policy maker can use these goods to produce units of the public good *or* can transfer these goods into the banking technology. I refer to this latter option as a "bailout" payment to the banking system. Letting τ denote the tax revenue collected in period 1, the policy maker's budget constraint can be written as

$$d = \tau - (1-\pi) b, \tag{3a}$$

where $(1-\pi) b \geq 0$ is the bailout payment made to the banking system in period 1.[2]

Notice that the type of bailout policy I consider here is entirely consistent with the sequential service constraint in the model. The only opportunity to tax agents comes in the first period, before bank deposits are made.[3] To keep the notation simple, I assume that the policy maker does not transfer any funds to the banking system in the event that there is no run. This assumption is without any loss of generality.

2.4 Partial run strategies

In this paper, I focus on outcomes in which a bank run occurs with positive probability. As shown in Ennis and Keister [8], there cannot be an equilibrium in this type of model in which all depositors run with some probability, because the bank and the policy maker would eventually discover that a

[2] The reason for the $(1-\pi)$ term will become clear later.

[3] If the environment allowed the government to tax depositors after it has observed the total level of withdrawal demand, then it must also be possible for the bank to require advance notice of a depositor's intention to withdraw. By making its payments to depositors contingent on total withdrawal demand, the bank could then rule out the possibility of runs.

run is underway and would react in a way the removes the incentive for depositors to run. Any run in this setting must be *partial*, with only some agents participating. I consider outcomes in which all depositors choose the following *partial run* strategy:

$$\begin{array}{lll} \text{For } s > q: & y_i(\theta_i, s) = \theta_i & \text{for all } i \\ \text{For } s \leq q: & y_i(\theta_i, s) = \left\{ \begin{array}{c} 0 \\ \theta_i \end{array} \right\} & \text{for} \left\{ \begin{array}{c} i \leq \pi \\ i > \pi \end{array} \right\}. \end{array} \tag{4}$$

for some $q \in [0,1]$. Under this strategy profile, with probability $1-q$ the sunspot state will satisfy $s > q$ and each depositor will withdraw early if and only if she is impatient. This case corresponds to "normal" times in which there is no run on the banking system. With probability q, however, the sunspots state will satisfy $s \leq q$. In this case, depositors with an index $i \leq \pi$ will withdraw early regardless of their type. In other words, depositors who have an opportunity to withdraw from the bank relatively early in the course the crisis will run. Depositors with $i > \pi$, however, choose to wait until period 2 if they are patient under this profile. Following Peck and Shell [15], I call q the "propensity to run" and treat it as a parameter of the economy.

Ennis and Keister [8] study the strategy profile in (4) along with a class of more complex profiles in which some depositors with $i > \pi$ choose to run in some sunspot states. They show that the conditions on parameter values under which equilibria based on these more complex strategy profiles exist are exactly the same as for the basic partial-run profile in (4). In this paper, I will focus on the basic case and examine how the addition of a public good and the possibility of bailout payments affects the scope for financial fragility.

3 The Efficient Banking Policy

In this section, I derive the efficient allocation in this model *conditional* on depositors following the strategy profile in (4). The first-best allocation described above requires that depositors withdraw from the banking technology in period 1 if and only if they are impatient. The exercise here is different: depositors are assumed to follow strategy profile (4) if this is consistent with equilibrium, so that a bank run occurs with positive probability. The benevolent policy maker chooses the banking contract, tax rate, and bailout policy as a best response to this behavior. Recall that there is limited commitment. In particular, while the tax rate is chosen in period 0, the payments to depositors and the size of the bailout are not determined until period 1, when information about withdrawal decisions begins to be revealed.

3.1 The optimal continuation allocation

The equilibrium of the model is constructed by working backward. First, suppose the realized state s is less than q, so that a partial run occurs. A fraction π of depositors will withdraw before the policy maker is able to infer the state. Let ϕ denote the remaining resources in the banking system in per-capita terms after these π withdrawals have taken place, that is,

$$\phi = \frac{1 - \tau - \int_0^{\pi} x(\mu)\, d\mu}{1 - \pi}.$$

Suppose that the bailout policy b has already been decided, so the the level of the public good is already determined. Then, given that the run has stopped, the efficient allocation of the remaining private consumption requires that the payments to the remaining depositors solve

$$\max_{\{\widehat{c}_1, \widehat{c}_2\}} \pi \frac{(\widehat{c}_1)^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(\widehat{c}_2)^{1-\gamma}}{1-\gamma}$$

subject to

$$(1-\pi)\widehat{c}_2 = R\left(\phi + b - \pi \widehat{c}_1\right)$$

and non-negativity constraints.[4] The solution to this problem is

$$\widehat{c}_1^* = (\phi + b) \frac{1}{\pi + (1-\pi) A_0} \quad \text{and} \quad \widehat{c}_2^* = (\phi + b) \frac{R A_0}{\pi + (1-\pi) A_0}. \tag{5}$$

This solution will apply in all of the policy scenarios considered in this paper. In the event that a run occurs, it will halt after a fraction π of depositors have withdrawn, in accordance with the strategy profile (4). The resources available in the banking system at that point will be allocated among the remaining depositors according to (5). Define the value function V_0 to measure the utility from private consumption that the remaining $1 - \pi$ depositors will receive in the event of a run, that is,

$$V_0(\phi, b) \equiv (1-\pi) \left\{ \pi \frac{(\widehat{c}_1^*)^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(\widehat{c}_2^*)^{1-\gamma}}{1-\gamma} \right\}.$$

[4] Note that ϕ and b are both expressed in terms of resources per remaining depositor. The measure of remaining depositors is $1 - \pi$, so that total resources devoted to the bailout equals $(1-\pi)\, b$, as indicated in the policy maker's budget constraint (3a).

Substituting in the solution (5) and simplifying shows that the value function can be rewritten as

$$V_0(\phi, b) = (1-\pi) A_1^{\gamma} \frac{(\phi + b)^{1-\gamma}}{1-\gamma},$$

with

$$A_1 \equiv (\pi + (1-\pi) A_0).$$

Given this distribution rule for private consumption, the policy maker will choose the bailout policy b to solve

$$\max_{0 \le b \le \frac{\tau}{1-\pi}} V_0(\phi, b) + \delta \frac{d^{1-\gamma}}{1-\gamma}$$

subject to

$$d = \tau - (1-\pi) b.$$

Note that b is the size of the bailout payment per remaining depositor, so $(1-\pi) b$ is the total size of the payment. Using the definitions above and substituting in the constraint, this problem can be written as

$$\max_{0 \le b \le \frac{\tau}{1-\pi}} (1-\pi) A_1^{\gamma} \frac{(\phi + b)^{1-\gamma}}{1-\gamma} + \delta \frac{(\tau - (1-\pi) b)^{1-\gamma}}{1-\gamma}.$$

The first-order condition for an interior solution is

$$(1-\pi) A_1^{\gamma} (\phi + b)^{-\gamma} = (1-\pi) \delta (\tau - (1-\pi) b)^{-\gamma},$$

which is solved by

$$b^* = \frac{A_1 \tau - \delta^{\frac{1}{\gamma}} \phi}{\delta^{\frac{1}{\gamma}} + (1-\pi) A_1}. \tag{6}$$

Next, define V_1 to be the utility from private consumption for the remaining $1-\pi$ depositors plus the total utility that *all* depositors receive from the public good. Then V_1 satisfies

$$V_1(\phi, \tau) \equiv V_0(\phi, b^*(\phi, \tau)) + \delta \frac{(\tau - (1-\pi) b^*)^{1-\gamma}}{1-\gamma},$$

which can be rewritten as

$$V_1(\phi, \tau) = A_3 \frac{(\tau + (1-\pi) \phi)^{1-\gamma}}{1-\gamma}, \tag{7}$$

where the constant A_3 is given by

$$A_3 = (1-\pi)^{\gamma} A_1^{\gamma} \lambda^{1-\gamma} + \delta (1-\lambda)^{1-\gamma}$$

and λ is the fraction of the total resources available to the policy maker that are used to provide private consumption to the remaining depositors. This value is given by

$$\lambda \equiv \frac{(1-\pi)\,A_1}{\delta^{\frac{1}{\gamma}} + (1-\pi)\,A_1}.$$

The remaining fraction $(1-\lambda)$ of these resources is used to provide the public good.

We have now determined how the remaining resources in the economy will be allocated in the event of a run. After π depositors have withdrawn and consumed, the available resources will be divided between private consumption by the remaining depositors and provision of the public good according to (5) and (6). The next step is to determine how the policy maker will set the payments to the first π depositors to withdraw and the tax rate, given that everyone anticipates this particular allocation of resources in the event of a run.

3.2 Banking and fiscal policy

First note that the policy maker will give the same payment to each depositor who is among the first fraction π to withdraw; let c_1 denote this payment. Unlike in Wallace [17], Green and Lin [11], and Peck and Shell [15], the policy maker gains no information about the aggregate state as these withdrawals take place. Only after the fraction of depositors who are withdrawing goes beyond π is the policy maker able to infer anything about the state s, and in this case it knows that $s \leq q$ must hold. In period 0, then, the policy maker will choose the tax rate τ and the initial payment c_1 to maximize the welfare of depositors. The policy maker takes as given that depositors will follow the partial-run strategy profile (4) if that profile is consistent with equilibrium given the chosen policy. However, the policy maker also has the option of choosing a run-proof policy, in which case it is assured that only impatient depositors will withdraw in period 1. To determine which of these represents a better course of action, I examine each in turn.

3.2.1 The best runs-permitting policy

If the policy maker does not choose a run-proof policy, then she will choose τ and c_1 to solve

$$\max_{\{c_1,c_2,\tau\}} (1-q)\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma} + \delta\frac{\tau^{1-\gamma}}{1-\gamma}\right) + q\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma} + V_1\left(\frac{1-\tau-\pi c_1}{1-\pi}, \tau\right)\right)$$

subject to

$$
\begin{aligned}
(1-\pi)c_2 &= R\,(1-\tau-\pi c_1) \\
&\text{and} \\
c_2 &\geq c_1.
\end{aligned}
$$

The last constraint is the incentive-compatibility condition that ensures there is an equilibrium in which patient depositors wait until period 2 to withdraw. In this section, the interior solution will always satisfy this constraint for the usual reasons. Note that I have ignored the constraint $b \geq 0$, but it is easy to show that this constraint will never be binding, assuming τ was chosen optimally, because the marginal utility of private consumption is higher in the run state than in the no-run state.

Using (7), the first-order conditions for an interior solution can be written as

$$
c_1^{-\gamma} - (1-q)\,R^{1-\gamma}\,(1-\pi)^{\gamma}\,(1-\tau-\pi c_1)^{-\gamma} - qA_3\,(1-\pi c_1)^{-\gamma} = 0
$$

and

$$
\delta\tau^{-\gamma} = R^{1-\gamma}\,(1-\pi)^{\gamma}\,(1-\tau-\pi c_1)^{-\gamma}\,.
$$

Note that q does not appear in the latter equation. In the run state, what matters for welfare is the total amount of resources available to be divided between private consumption for remaining depositors and provision of the public good; the optimal decision rule for this division is already included in the objective function. The value of τ is, therefore, chosen so that the desired level of public good is provided in the no-run states.

Solving the first-order conditions yields

$$
c_1^{EP} = \frac{1}{\pi + A_5} \tag{8}
$$

and

$$
\tau^{EP} = A_4 \frac{A_5}{\pi + A_5}, \tag{9}
$$

where

$$
\begin{aligned}
A_4 &\equiv \frac{\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (1-\pi)\,A_0} \qquad \text{and} \\
A_5 &= \left((1-q)\,R^{1-\gamma}\,(1-\pi)^{\gamma}\,(1-A_4)^{-\gamma} + qA_3\right)^{\frac{1}{\gamma}}.
\end{aligned}
$$

Note that the tax rate τ^{EP} is always strictly positive. This policy represents the best response of the policy maker, taking as given the fact that depositors are following the strategy profile (4).

The rate of return on early withdrawals is then given by

$$r_1^{EP} = \frac{c_1^{EP}}{1-\tau^{EP}}.$$

This value can also be thought of as measuring the amount of maturity transformation done by the banking system or as the *leverage* of the banking system, since it represents the total short-term liabilities of the system relative to the resources contributed by depositors (or the current value of assets).

3.2.2 The best run-proof policy

Instead of accepting that a run will occur with probability q, the policy maker has the option of ensuring that (4) is not an equilibrium strategy profile by choosing a run-proof contract. In the single-technology model, finding the best run-proof contract is straightforward since there is no portfolio choice: a contract is run-proof in this setting if and only if c_1 is no greater than the amount each agent deposits in the banking system. In other words, for a given value of τ, the best run-proof contract sets $c_1 = (1-\tau)$ and $c_2 = R(1-\tau)$. The best run-proof policy therefore solves

$$\max_{\{\tau\}} \pi\frac{(1-\tau)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(R(1-\tau))^{1-\gamma}}{1-\gamma} + \delta\frac{\tau^{1-\gamma}}{1-\gamma}.$$

The solution is given by

$$\tau^{RP} = \frac{\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (\pi + (1-\pi)R^{1-\gamma})^{\frac{1}{\gamma}}},$$

$$c_1^{RP} = \frac{(\pi + (1-\pi)R^{1-\gamma})^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (\pi + (1-\pi)R^{1-\gamma})^{\frac{1}{\gamma}}}, \quad \text{and} \quad c_2^{RP} = \frac{R(\pi + (1-\pi)R^{1-\gamma})^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (\pi + (1-\pi)R^{1-\gamma})^{\frac{1}{\gamma}}}.$$

Substituting this solution into the objective function and simplifying, we can write the level of welfare achieved by the best run-proof policy as

$$W^{RP} = \frac{1}{1-\gamma}\left(\delta^{\frac{1}{\gamma}} + \left(\pi + (1-\pi)R^{1-\gamma}\right)^{\frac{1}{\gamma}}\right)^{\gamma}. \tag{10}$$

Note that this value is independent of q.

3.3 Conditions for equilibrium

The last step in the construction of the partial-run equilibrium is to verify that the strategy profile (4) is indeed an equilibrium of the depositors' game generated by the policy response derived above. It is easy to see that $\widehat{c}_2^* > \widehat{c}_1^*$ always holds, so the strategies specified for depositors $i > \pi$ in states where a run occurs do indeed comprise an equilibrium of the continuation game. The remaining conditions for (4) to be an equilibrium of the depositors' game are

$$c_1^{EP} \geq \widehat{c}_2^{EP} \quad \text{and} \quad W^{EP} \geq W^{RP}. \tag{11}$$

The first condition guarantees that patient depositors with $i \leq n$ will choose to participate in the run and withdraw early. The second condition guarantees that it is not optimal for the policy maker to switch to a run-proof contract. When parameter values are such that both of these conditions hold, we have constructed a partial run equilibrium of the model.

One way to measure the scope for financial fragility is to examine the set of parameter values for which the run equilibrium exists for *some* values of q. This can be done by looking at the case where $q = 0$ and using a continuity argument to show that the results also hold for small enough values of q. I will say that the banking system is *fragile* if there is a strict equilibrium in which all depositors follow the strategy profile (4) when q is equal to zero.

When $q = 0$ holds, it is straightforward to show that c_1, τ, and (by extension) ϕ all equal the first-best values derived in the previous section. The condition $c_1^{EP} > \widehat{c}_2^{EP}$ can then be written as[5]

$$R^{\frac{1}{\gamma}} \frac{\delta^{\frac{1}{\gamma}} + (1-\pi) A_0}{\delta^{\frac{1}{\gamma}} + (1-\pi)(\pi + (1-\pi) A_0)} < 1. \tag{12}$$

When $\delta = 0$, this condition reduces to condition (18) in Ennis and Keister [8].

Proposition 1 *The set of parameter values such that the banking system is fragile under the efficient policy response is given by* (12).

It is straightforward to show that the expression on the left-hand side is increasing in δ and, hence, the presence of the public good works against the existence of the run equilibrium in this model. The fact that the policy maker has a stock of resources available in period 1 that can be transferred to the banking system in the event of a run raises the payments received by depositors who do

[5] Note that the strict inequality is needed for the continuity argument.

not participate in the run. This lower incentive to participate in the run translates into a decreased scope for financial fragility. This fact is recorded in the following corollary.

Corollary 1 *The set of values of (R, γ, π) such that the banking system is fragile under the efficient policy response is strictly decreasing in δ.*

4 Competitive Equilibrium with Bailouts

I now examine the outcome of the decentralized model where banks compete for deposits by announcing banking contracts. The contract offered by a bank is characterized by promised payments (c_1, c_2) in period 1 and 2, respectively. If no run occurs, the policy maker does not intervene and a depositor's consumption is as specified by her bank's contract.

In the event of a run, however, the policy maker intervenes in two ways. First, it makes a payment $(1-\pi)\,b$ to the banking system. This payment is divided among banks so that each one has the same level of resources per capita, *regardless of the amount of funds remaining in the bank*. In other words, banks that have fewer resources (because they had a higher value of c_1) will get a larger bailout. Note that this is the ex post efficient policy. In equilibrium, of course, all banks will choose the same c_1 and receive the same bailout payment. In addition, the policy maker also reschedules payments to depositors. Since I consider single-wave run equilibria, the efficient ex post policy is to offer a contract that implements the first-best continuation allocation, as derived in the previous section. In other words, the policy maker makes sure that the remaining funds in the banking system (including the bailout payment) are divided among the remaining depositors in an efficient way.[6]

As a result of these assumptions, banks and depositors take as given the consumption that the remaining depositors will receive in the event of a run. They also take as given the level of provision of the public good in each state. From a private point of view, then, the choice of banking contract only affects the consumption of the first π depositors to withdraw from the bank in the event of a run. The consumption of the remaining depositors will be determined by the total amount of resources left in the economy at that point. In other words, there is an external effect: an individual

[6] It is worth noting that the approach here implicitly assumes a form of commitment in normal times. If banks were to choose different levels of c_1, patient depositors would be receiving different levels of c_2. When the late period arrives, the policy maker would like to intervene and equalize consumption across the remaing depositors. I do not allow this type of intervention.

bank's choice of contract affects the consumption available to other banks' depositors in the event of a run. This external effect is the source of the moral hazard problem that arises in this model.

Banks choose contracts and complete for deposits after the policy maker has chosen the tax rate τ. In choosing τ, the policy maker recognizes that banks will react to this choice.

4.1 A competitive bank's decision problem

4.1.1 The runs-permitting contracts

Competition for deposits will lead banks to offer, in equilibrium, the deposit contract that maximizes an individual depositor's expected utility. If the contract is run-permitting, it must solve the following problem

$$\max_{\{c_1,c_2\}} (1-q)\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma}+\delta\frac{\tau^{1-\gamma}}{1-\gamma}\right)+q\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+V_1\right)$$

subject to

$$\begin{aligned}(1-\pi)c_2 &= R\left(1-\tau-\pi c_1\right)\\ &\text{and}\\ c_2 &\geq c_1.\end{aligned}$$

Note that the tax rate τ and the continuation value V_1 are taken as given in this problem. This resembles a standard Diamond-Dybvig problem, but now the incentive compatibility constraint $c_2 \geq c_1$ may bind at the solution. The first-order condition for an interior solution is

$$\pi c_1^{-\gamma}=(1-q)\left(1-\pi\right)c_2^{-\gamma}R\frac{\pi}{1-\pi}$$

or

$$u'\left(c_1\right)=(1-q)\,Ru'\left(c_2\right).$$

Note that an increase in the probability of a run q will tend to raise the chosen value of c_1, since when a run occurs the bank's depositors do not care how many resources their own bank has left. This is the heart of the moral hazard problem. We can see from the last expression that the incentive compatibility constraint will be satisfied at the interior solution as long as

$$(1-q)\,R \geq 1 \tag{13}$$

but will otherwise be violated. For this reason, we need to split the analysis into two cases. First, suppose that (13) holds. In this case, solving the usual way yields

$$\begin{aligned}
c_1 &= A_6\,(1-\tau) \qquad \text{where} \qquad A_6 = \frac{1}{\pi + (1-q)^{\frac{1}{\gamma}}\,(1-\pi)\,A_0} \\
&\text{and} \\
c_2 &= A_7\,(1-\tau) \qquad \text{where} \qquad A_7 = \frac{RA_0\,(1-q)^{\frac{1}{\gamma}}}{\pi + (1-q)^{\frac{1}{\gamma}}\,(1-\pi)\,A_0}.
\end{aligned}$$

When (13) is violated, however, the incentive compatibility constraint will be binding at the solution. In this case, combining the two constraints in the optimization problem yields the solution

$$c_1 = c_2 = A_8\,(1-\tau) \qquad \text{where} \qquad A_8 = \frac{1}{\pi + (1-\pi)\,R^{-1}}.$$

The equations above show that the leverage of the banking system, $r_1^B = c_1^B / \left(1-\tau^B\right)$, is given by A_6 when (13) is satisfied and A_8 otherwise. It is straightforward to show that when $q=0$, A_6 is equal to the amount of leverage in the first-best allocation. For $q>0$, A_6 is strictly increasing in q, while A_8 is independent of q. Combining these facts yields the following result.

Proposition 2 *For any $q>0$, we have $r_1^B > r_1^{FB}$.*

4.1.2 Run-proof contracts

An individual bank could instead choose a run-proof contract. Note that while such a contract will assure that the bank choosing it will not be subject to a run, other banks will still experience a run with probability q. This fact matters for the depositors because the amount of public good that is provided will depend on the bailout provided to other banks. The expected utility of a depositor in a run-proof bank would be

$$W_{RP}^B(q) = \left(\pi + (1-\pi)\,R^{1-\gamma}\right)\frac{\left(1-\tau^B\right)^{1-\gamma}}{1-\gamma} + \delta\left((1-q)\,\frac{\left(\tau^B\right)^{1-\gamma}}{1-\gamma} + q\frac{\left((1-\lambda)\left(1-\pi c_1^B\right)\right)^{1-\gamma}}{1-\gamma}\right).$$

4.2 The tax rate

Next we solve for the policy maker's optimal choice of τ. When (13) holds, the maximization problem is

$$\max_{\{\tau\}} \ (1-q)\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma}+\delta\frac{\tau^{1-\gamma}}{1-\gamma}\right)+q\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+A_3\frac{(1-\pi c_1)^{1-\gamma}}{1-\gamma}\right) \tag{14}$$

subject to

$$c_1 = A_6\,(1-\tau) \qquad \text{and} \qquad c_2 = A_7\,(1-\tau)\,.$$

Notice the last term of the objective function, which is the same as in the previous section. In the event of a run, the remaining resources in the economy $(1-\pi c_1)$ will be divided between the remaining depositors and the public good in the ex post efficient manner. The first-order condition for this problem can be written as

$$\left(A_6^{1-\gamma}\pi+(1-q)\,A_7^{1-\gamma}\,(1-\pi)\right)(1-\tau)^{-\gamma}=(1-q)\,\delta\tau^{-\gamma}+q\pi A_3A_6\,(1-\pi A_6\,(1-\tau))^{-\gamma}\,. \tag{15}$$

This equation implicitly defines the policy maker's optimal choice τ^B in the case where the ex-post efficient bailout policy will be followed and (13) holds.

When (13) is violated, the policy maker's again aims to maximize (14), but the constraints in the problem become

$$c_1 = c_2 = A_8\,(1-\pi)\,.$$

In this case, the objective function can be rewritten as

$$\max_{\{\tau\}} \ (1-q)\left(A_8^{1-\gamma}\frac{(1-\tau)^{1-\gamma}}{1-\gamma}+\delta\frac{\tau^{1-\gamma}}{1-\gamma}\right)+q\left(\pi A_8^{1-\gamma}\frac{(1-\tau)^{1-\gamma}}{1-\gamma}+A_3\frac{(1-\pi A_8\,(1-\tau))^{1-\gamma}}{1-\gamma}\right)$$

and the first-order condition is

$$((1-q)+\pi q)\,A_8^{1-\gamma}\,(1-\tau)^{-\gamma}=(1-q)\,\delta\tau^{-\gamma}+q\pi A_3A_8\,(1-\pi A_8\,(1-\tau))^{-\gamma}\,. \tag{16}$$

The solution to this equation is the policy maker's optimal choice for τ^B when (13) does not hold.

4.3 Equilibrium conditions

In order for this construction to be an equilibrium of the model, we need the following two conditions to hold

$$c_1^B \geq \widehat{c}_2^B \quad \text{and} \quad W^B \geq W_{RP}^B. \tag{17}$$

Starting with the latter condition, suppose a bank deviates from this candidate equilibrium and chooses a run-proof contract. Note that individual banks take as given the tax rate τ and the level of provision of the public good, which still depends on whether or not a run occurs. For the single-wave run equilibrium based on the strategy profile (4) to exist, this value must satisfy $W^B(q) \geq W_{RP}^B(q)$. Note that this is necessarily the case for values of q that are close enough to zero (given the assumption that $\gamma > 1$).

We now look at the scope for financial fragility by asking for what parameter values the condition $c_1^B \geq \widehat{c}_2^B$ holds for some values of q. As before, this is done by evaluating the two expressions at $q = 0$ and appealing to a continuity argument. When q is zero, the first-order condition for τ in (15) can be solved in closed form and, after some algebra, can be shown to be equal to the first-best value τ^{FB}. Similarly, the values of c_1, c_2, and, by extension, ϕ can all be shown to be equal to the first-best values. The intuition is clear: when $q = 0$ there is no moral hazard problem. As a result, the condition for financial fragility in the competitive model with the ex-post efficient bailout policy is exactly the same as in the previous section and is given by equation (12).

Proposition 3 *The set of parameter values such that the banking system is fragile under the ex post efficient bailout policy is given by* (12).

5 No Bailouts

Now suppose that the policy maker can commit in period 0 to setting $b = 0$ in all states of nature. This idea is perhaps best interpreted as the ability to write an enforceable law prohibiting bailout payments to the banking sector. The question is whether such a law improves welfare relative to the competitive equilibrium under the ex-post optimal bailout policy calculated in the previous section.

In the event of a run, each bank reschedules payments to implement the efficient continuation allocation among its own depositors. This allocation is that same as that derived in Section 3, but with b set to zero:

$$\widehat{c}_1 = \phi \frac{1}{\pi + (1-\pi) A_0} \quad \text{and} \quad \widehat{c}_2 = \phi \frac{RA_0}{\pi + (1-\pi) A_0}.$$

5.1 The banking contract

Competitive banks take the tax rate τ as given and recognize that the level of the public good will be equal to τ in all states. The optimization problem characterizing the equilibrium banking contract is then:

$$\max_{\{c_1,c_2\}} (1-q) \left(\pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(c_2)^{1-\gamma}}{1-\gamma} \right) + q \left(\pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + V_0(\phi, 0) \right) + \delta \frac{\tau^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{array}{rcl} (1-\pi)c_2 & = & R(1-\tau-\pi c_1) \\ \phi & = & \dfrac{1-\tau-\pi c_1}{1-\pi} \\ & & \text{and} \\ c_2 & \geq & c_1. \end{array}$$

Define the constant

$$A_9 \equiv \left((1-q)(R)^{1-\gamma} + q(\pi + (1-\pi) A_0)^\gamma \right)^{\frac{1}{\gamma}}.$$

Then the solution to the above problem is given by

$$\begin{array}{rclcl} c_1 & = & A_{10}(1-\tau) & \text{where} & A_{10} \equiv \dfrac{1}{\pi + (1-\pi) A_9} \end{array} \tag{18}$$

and

$$\begin{array}{rclcl} c_2 & = & A_{11}(1-\tau) & \text{where} & A_{11} = \dfrac{RA_9}{\pi + (1-\pi) A_9}. \end{array} \tag{19}$$

It is reasonably straightforward to show that $RA_9 > 1$ holds and, hence, that the incentive compatibility constraint will never bind in the solution to this problem.

The leverage of the banking system under this policy is given by $r_1^{NB} = A_{10}$. Straightforward algebra shows that the constant A_9 is strictly increasing in q, which implies that A_{10} is strictly decreasing in q. Furthermore, one can show that A_{10} reduces to the leverage associated with the first-best allocation when q is zero. Together, these facts establish the following result.

Proposition 4 *For any $q > 0$, we have $r_1^{NB} < r_1^{FB}$.*

Combining the results in Propositions 2 and 4 shows that committing to a no-bailout policy reduces the leverage of the banking system whenever a run occurs with positive probability.

Corollary 2 *For any $q > 0$, we have $r_1^{B} > r_1^{NB}$.*

5.2 The tax rate

Next, I derive the policy maker's optimal choice of the tax rate τ. The maximization problem is

$$\max_{\{\tau\}} \; (1-q)\left(\pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma}\right)$$
$$+q\left(\pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)^{\gamma} A_1^{\gamma}\frac{(1-\tau-\pi c_1)^{1-\gamma}}{1-\gamma}\right) + \delta \frac{\tau^{1-\gamma}}{1-\gamma}$$

subject to the relationships in (18) and (19). Substituting in the constraints and rearranging terms, we can rewrite the objective function as

$$\max_{\{\tau\}} \; A_{12}^{\gamma}\frac{(1-\tau)^{1-\gamma}}{1-\gamma} + \delta \frac{\tau^{1-\gamma}}{1-\gamma}$$

where

$$A_{12} \equiv \left(\pi A_{10}^{1-\gamma} + (1-q)\,(1-\pi) A_{11}^{1-\gamma} + q\,(1-\pi)^{\gamma} A_1^{\gamma}\,(1-\pi A_{10})^{1-\gamma}\right)^{\frac{1}{\gamma}}.$$

The first-order condition is

$$\tau^{\gamma} A_{12}^{\gamma} = \delta\,(1-\tau)^{\gamma}$$

which is solved by

$$\tau^{NB} = \frac{\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + A_{12}}.$$

5.3 Equilibrium conditions

In order for this construction to be an equilibrium of the model, we need the following two conditions to hold

$$c_1^{NB} \geq \widehat{c}_2^{NB} \quad \text{and} \quad W^{NB} \geq W_{RP}^{NB}.$$

Starting with the latter condition, suppose a bank deviates from this candidate equilibrium and chooses a run-proof contract. The expected utility of a depositor in a run-proof bank would be

$$W_{RP}^{NB}(q) = \left(\pi + (1-\pi) R^{1-\gamma}\right) \frac{\left(1-\tau^{NB}\right)^{1-\gamma}}{1-\gamma} + \delta \frac{\left(\tau^{NB}\right)^{1-\gamma}}{1-\gamma}.$$

For the partial run equilibrium based on the strategy profile (4) to exist, this value must satisfy $W^{NB}(q) \geq W_{RP}^{NB}(q)$ must be non-negative. We again know that this is necessarily the case for values of q that are close enough to zero.

Turning to the former condition, we again examine the scope for financial fragility by asking for what parameter values the condition $c_1^B > \widehat{c}_2^B$ holds when $q = 0$. It is straightforward to show that when q is zero, the values of τ, c_1, c_2, and ϕ are all equal to the first-best values. The difference in this section comes with $\widehat{c}_2$, because there is now no bailout payment. When $q = 0$, the condition $c_1^{NB} > \widehat{c}_2^{NB}$ can then be written as

$$R \frac{R^{1-\gamma}}{\left(\pi + (1-\pi) A_0\right)^{\gamma}} < 1, \tag{20}$$

which is exactly condition (18) in Ennis and Keister [8].

Proposition 5 *The set of parameter values such that the banking system is fragile under a no-bailout policy is given by* (20)*.*

Combining the results in Propositions 1 and 5 (and, by extension, that in Corollary 1) yields the following.

Corollary 3 *The set of parameter values such that the banking system is fragile is strictly larger under the no-bailout policy than under the ex-post efficient bailout policy.*

In other words, for some parameter values, committing to a no-bailout policy *introduces* the possibility of a run equilibrium. This results shows that, contrary to the claims of many commentators, there is a well-defined sense in which a commitment to a no-bailouts policy increases the fragility of the banking sector.

6 An Example

To further illustrate the properties of the model and of equilibrium under the two different policy regimes, I now present a numerical example. The parameter values are given by $R = 1.1$, $\gamma = 6$, $\pi = 0.5$, and $\delta = 0.01$. These values satisfy (12), so that the banking system is fragile under both policy regimes.

The first three figures depict the equilibrium of the model under the ex-post efficient bailout policy as the probability of a run q varies. Figure 1 shows the returns offered on early and late withdrawals in states where no run occurs. Consistent with Proposition 2, the early return is initially increasing in q. Eventually, condition (13) is violated and the incentive compatibility constraint $c_2 \geq c_1$ binds; in such cases $r_1^B = r_2^B$ holds. Figure 2 presents the tax rate and the level of provision of the public good in the event of a run; the difference between these lines is the size of the bailout payment. The figure shows that the tax rate increases as a run becomes more likely. Figure 3 plots the equilibrium conditions (17). The fact that both lines are positive confirms that the partial run equilibrium exists for all values of q shown in these figures.

Figures 4 through 6 present the same information for the equilibrium of the model under the no-bailouts policy. Consistent with Proposition 4, the return on early withdrawals in this case is strictly decreasing in q. An increased probability of a run now leads banks to choose less leverage in order to give more resources to their remaining depositors in the event that a run occurs. As a consequence, the return on late withdrawals r_2 is an increasing function of q. When there is no bailout, level of public good provision is equal to the tax rate regardless of whether or not a run occurs. For this reason, the two lines in Figure 5 lie on top of each other. This figure shows that the tax rate (and the level of provision of the public good) falls as the probability of a run increases. Figure 6 presents the equilibrium conditions for the no-bailout case. and shows that the partial-run equilibrium again exists for all values of q presented in these graphs. Interestingly, the maximum probability of a run in this case is equal to approximately 0.18; beyond this point, the policy maker would prefer to choose a run-proof contract. Looking back at Figure 3 shows that the maximum probability of a run is higher under the bailout policy, since neither of the two lines has yet reached zero. In this sense, one could argue that in this example the no-bailout policy has a type of stabilizing effect: that it lowers the maximum probability of a run consistent with equilibrium.

Figure 7 presents the increase in *ex ante* welfare that comes from committing to a no-bailout

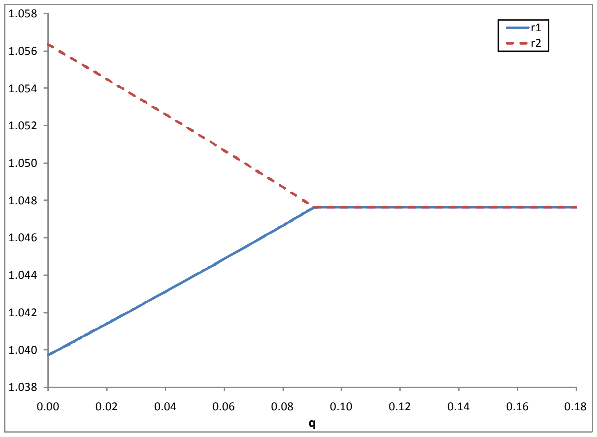

Figure 1: Return on Early and Late Withdrawals: With Bailout

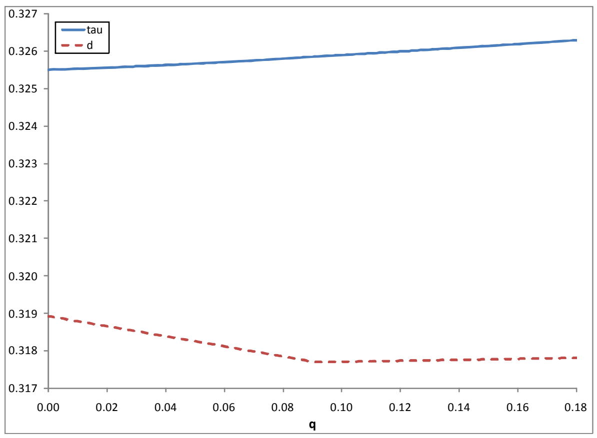

Figure 2: Tax Rate and Public Good Provision: With Bailout

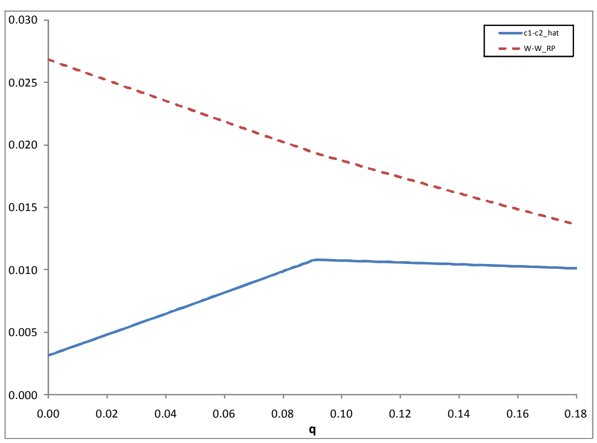

Figure 3: Equilibrium Conditions: With Bailout

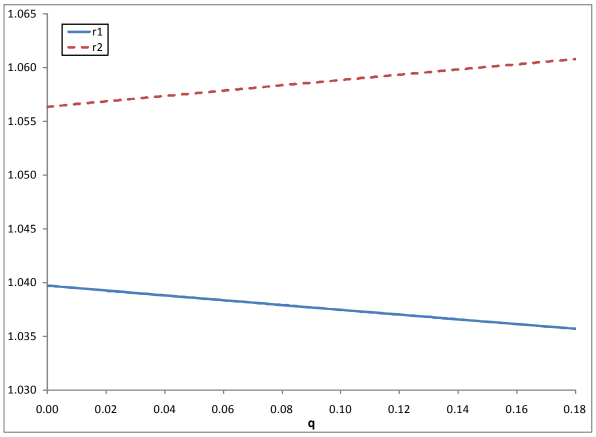

Figure 4: Return on Early and Late Withdrawals: No Bailout

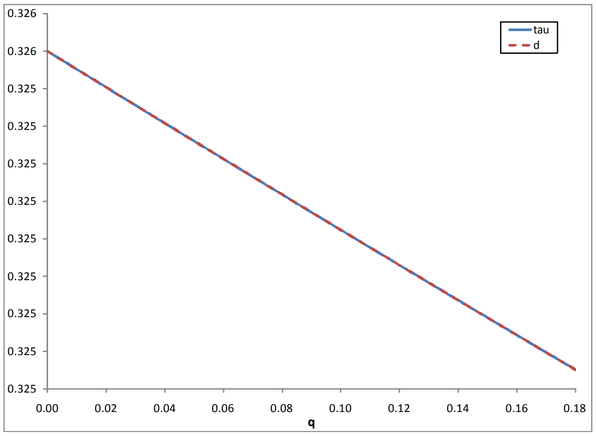

Figure 5: Tax Rate and Public Good Provision: No Bailout

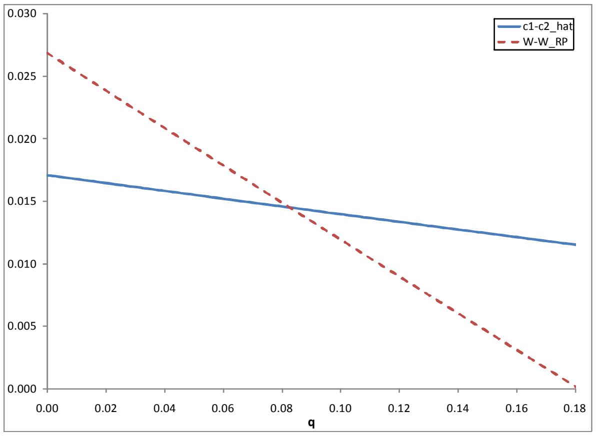

Figure 6: Equilibrium Conditions: No Bailout

policy. For this example, the gain is negative for low values of q, indicating that committing to no bailouts lowers welfare. For large enough values of q, however, the gain becomes positive and a no-bailout commitment is desirable.

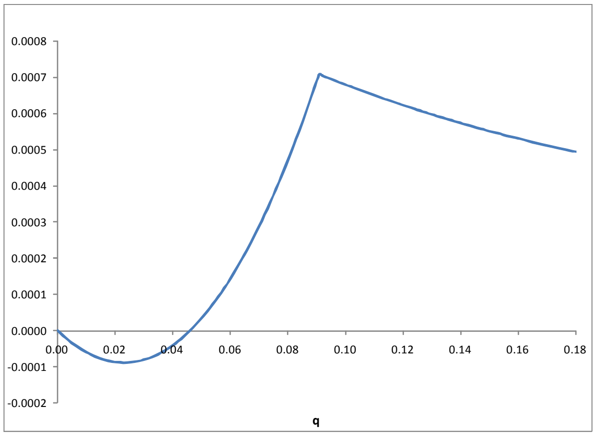

Figure 7: The Welfare Gain from No Bailout: $W^{NB} - W^{B}$

7 Concluding Remarks

[To be written.]

References

[1] Acharya, Viral V. and Tanju Yorulmazer (2007) "Too many to fail: an analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation* 16: 1-31.

[2] Allen, Franklin and Douglas Gale (2000) "Financial contagion," *Journal of Political Economy* 108: 1-33.

[3] Cooper, Russell and Thomas W. Ross (1998) "Bank runs: liquidity costs and investment distortions," *Journal of Monetary Economics* 41: 27-38.

[4] Diamond, Douglas W. and Phillip H. Dybvig (1983) "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy* 91: 401-419.

[5] Ennis, Huberto M. and Todd Keister (2006) "Bank runs and investment decisions revisited," *Journal of Monetary Economics* 53: 217-232.

[6] Ennis, Huberto M. and Todd Keister (2009) "Bank runs and institutions: The perils of intervention," *American Economic Review* 99: 1588-1607.

[7] Ennis, Huberto M. and Todd Keister (2009) "Run equilibria in the Green-Lin model of financial intermediation," *Journal of Economic Theory* 144: 1996-2020.

[8] Ennis, Huberto M. and Todd Keister (2008) "Commitment and Equilibrium Bank Runs," Federal Reserve Bank of New York Staff Report no. 274.

[9] Goldstein, Itay and Ady Pauzner (2005) "Demand deposit contracts and the probability of bank runs," *Journal of Finance* 60: 1293-1327.

[10] Green, Edward J. and Ping Lin (2000) "Diamond and Dybvig's classic theory of financial intermediation: What's missing?" Federal Reserve Bank of Minneapolis *Quarterly Review* 24: 3-13.

[11] Green, Edward J. and Ping Lin (2003) "Implementing efficient allocations in a model of financial intermediation," *Journal of Economic Theory* 109: 1-23.

[12] Gu, Chao (2008) "Asymmetric information and bank runs," mimeo., University of Missouri.

[13] Kydland, Finn E. and Edward C. Prescott (1977) "Rules rather than discretion: the inconsistency of optimal plans," *Journal of Political Economy* 85: 473-492.

[14] Mailath, George J. and Loretta J. Mester (1994) "A positive analysis of bank closure," *Journal of Financial Intermediation* 3: 272-299.

[15] Peck, James and Karl Shell (2003) "Equilibrium bank runs," *Journal of Political Economy* 111: 103-123.

[16] Wallace, Neil (1988) "Another attempt to explain an illiquid banking system: the Diamond and Dybvig model with sequential service taken seriously," *Federal Reserve Bank of Minneapolis Quarterly Review* 12 (Fall): 3-16.

[17] Wallace, Neil (1990) "A banking model in which partial suspension is best," *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Fall): 11-23.