Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk*

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Abstract

We study the functioning and possible breakdown of the interbank market due to asymmetric information about counterparty risk. We allow for privately observed shocks to the distribution of asset risk across banks after the initial portfolio of liquid and illiquid investments is chosen. Our model generates several interbank market regimes: 1) low interest rate spread and full participation; 2) elevated spread and adverse selection; and 3) liquidity hoarding leading to a market breakdown. The regimes are in line with observed developments in the interbank market before and during the 2007-09 financial crisis. We use the model to discuss various policy responses.

JEL classification: G01, G21, D82

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Neither the recent massive money injections, the coordinated lowering of interest rates nor the use of public funds to recapitalize banks have done much to restart interbank lending. This action did not solve the underlying problem preventing interbank lending: extreme information asymmetry.

Financial Times, November 9, 2008

1 Introduction

Interbank markets play a key role in banks' liquidity management and the transmission of monetary policy. The interest rate in the unsecured three-month interbank market acts as a benchmark for pricing fixed-income securities throughout the economy. In normal times, interbank markets are among the most liquid in the financial sector. Since August 2007, however, the functioning of interbank markets has become severely impaired around the world. As the financial crisis deepened in September 2008, liquidity in the interbank market has further dried up as banks preferred hoarding cash instead of lending it out even at short maturities. Central banks' massive injections of liquidity did little to restart interbank lending. The failure of the interbank market to redistribute liquidity has become a key feature of the 2007-09 crisis (see, for example, Allen and Carletti, 2008, and Brunnermeier, 2009).

Why has the interbank market been dysfunctional for so long? What frictions can explain these developments? How do they relate to the roots of the financial crisis? In particular, how can the illiquidity of banks' assets depress activity in what used to be one of the most liquid markets? And how do the policy responses that are discussed or implemented around the world hold up against these frictions?

This paper provides a model of the unsecured interbank market with asymmetric information about counterparty risk. Various regimes in the interbank market arise depending on the level and distribution of counterparty risk. In the first regime, there is full participation of borrowers and lenders in the interbank market. The market functions smoothly and interest rates are low despite the presence of asymmetric information. Riskier banks exert an externality on safer banks as the latter subsidize the liquidity of the former. But the cost is small compared to the cost of alternatives to the unsecured market. In the second regime, the interbank market is characterized by adverse selection. The externality on safer banks is too costly and they leave the unsecured market. Liquidity is still traded but the interest rate rises since only riskier banks are active in the market. In the third regime, the interbank market breaks down. This happens either because lenders prefer to hoard liquidity instead of lending it out to an adverse selection of borrowers, or because even riskier borrowers find the unsecured interest rate too high and choose to obtain liquidity elsewhere.

The three regimes derived in our model line up well with the observed developments. Figure 1 plots the spread between the three-month unsecured rate and the overnight index swap in three months' time,[1] a standard measure of interbank market tensions (red line), and the amounts of liquidity parked by banks at the ECB (light and dark blue bars). Until August 9, 2007 (the start of the financial crisis), the unsecured euro interbank market is characterized by a very low spread, around five basis points, and infinitesimal amounts of liquidity parked at the ECB. In normal times, banks prefer to lend out excess cash since the rate offered by the ECB's overnight deposit facility is punitive relative to rates available in interbank markets. The phase between August 9, 2007 and the last weekend of September, 2008 is characterized by a significantly higher spread, yet there is still no parking of funds (except the 2007 year-end effect). As of September 28, 2008, the spread increases even further to a maximum of 186 basis points. But the distinguishing feature of this phase is a dramatic increase in the amounts banks bring to the ECB. The amounts increase more than 1800-fold between the week of September 1, 2008 and the week of September 29, 2008.[2]

[1] The overnight index swap is a measure of what the market expects the overnight unsecured rate to be over a three-month period and thus controls for interest rate expectations.

[2] The amounts deposited with the ECB rise from a daily average of €0.09 billion in the week starting September 1, 2008 to €169.41 billion in the week of September 29, 2008. The ECB only announced a more extensive provision of liquidity on October 8, 2008. It was partially implemented a day later and came into full force on October 15. We examine the events in September and October 2008 in more detail in Section 4.

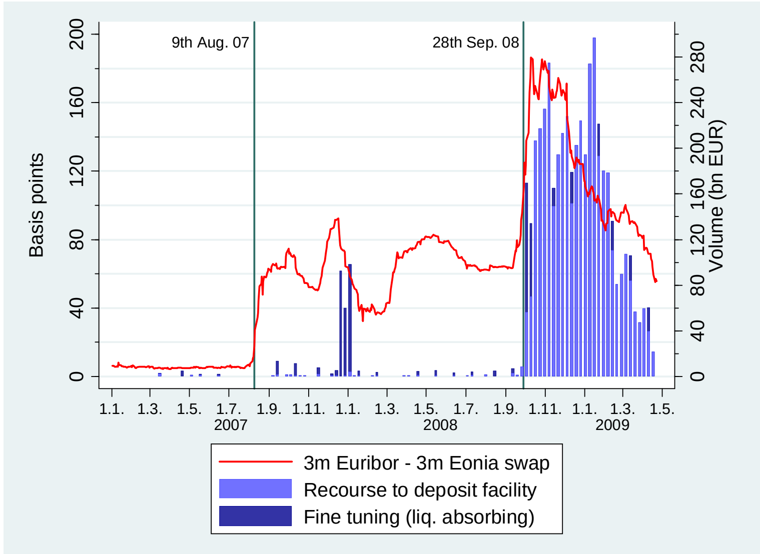

Figure 1: Interbank spread, recourses to the ECB deposit facility (daily average per week), and liquidity-absorbing open market operations (daily average per week), 01/2007 - 04/2009.

Banks are hoarding liquidity rather than lending it out. A similar pattern of three distinct phases can be observed in the spread for the United States (Figure 2).[3]

The transition across various regimes in our model implies a change in the underlying level and distribution of counterparty risk that is consistent with the development of actual events: a sharp market-wide reassessment of risk in the summer of 2007, after subprime-mortgage backed securities were discovered in portfolios of banks and bank-sponsored conduits, and a further increase in the level and the dispersion of counterparty risk following the events in September 2008. Asymmetric information as an underlying friction can also rationalize the prolonged nature of interbank market tensions, despite an unprecedented increase in the liquidity provision by central banks.

We model the interbank market in the spirit of Diamond and Dybvig (1983). Banks may need to realize cash quickly due to demands of customers who draw on committed lines of credit or on their demandable deposits. Banks in need of liquidity can borrow from banks with a surplus of liquidity as in Bhattacharya and Gale (1987) and Bhattacharya

[3]The spread in the interbank market secured by mortgage-backed securities (MBS) in the US followed a similar pattern, albeit at lower levels.

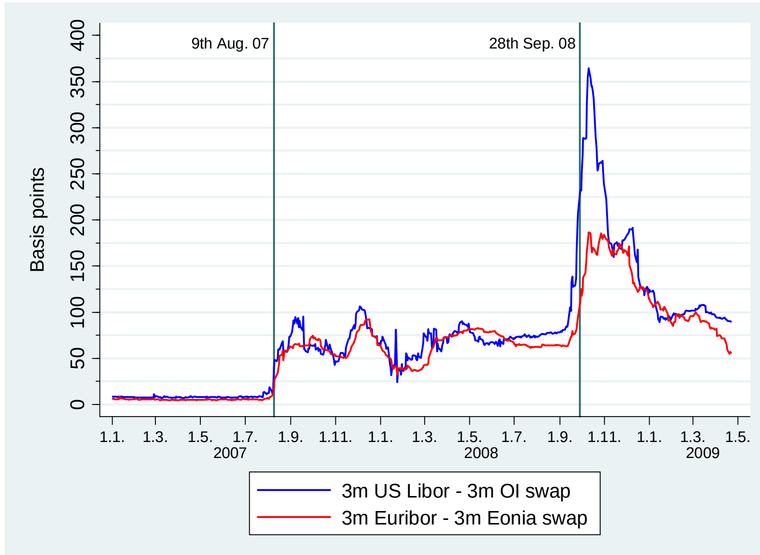

Figure 2: Interbank spreads US and euro area, 01/2007 - 04/2009

and Fulghieri (1994). Banks' profitable but illiquid assets are risky. Hence, banks may not be able to repay their interbank loan. The novel feature we add to this framework is asymmetric information about counterparty risk.[4] Banks become privately informed about the risk of their illiquid assets after they chose the portfolio of liquid and illiquid investments. Asymmetric information about counterparty risk creates frictions in the interbank market as suppliers of liquidity protect themselves against lending to "lemons".

Our modelling assumptions are designed to reflect the insights from broad analyses of the 2007-09 financial crisis. First, asymmetric information about the size and location of risk, and the accompanying fear of counterparty default, which was created by the complexity of securitization, are at the heart of the financial crisis (see Gorton, 2008, 2009). Second, maturity mismatch is a key factor contributing to the fragility of modern financial systems that can become clogged by illiquid securities (see, for example, Diamond and Rajan, 2008a, and Brunnermeier, 2009). Hence, we employ the standard model of banking introduced by Diamond and Dybvig (1983) that allows us to consider the tradeoff between liquidity and

[4] Our model therefore applies to money market segments in which credit risk concerns play a role, namely unsecured (term) markets and markets secured by risky collateral.

return in bank's portfolio decisions. A further advantage of this model is that it naturally creates a scope for interbank markets (see Bhattacharya and Gale, 1987, and Bhattacharya and Fulghieri, 1994).[5]

Our paper analyzes the effects of asymmetric information about credit risk on the functioning and possible breakdown of the interbank market. In that respect, our work builds on the contributions by Stiglitz and Weiss (1981), Broecker (1990) and Flannery (1996). Freixas and Holthausen (2004) study interbank market integration across countries when there is better information about the solvency of domestic banks than of foreign banks.

Bolton, Santos, and Scheinkman (2009) examine asymmetric information between short-term and long-term investors. Longer-term investors, as potential buyers of assets, do not know whether short-term investors sell because the asset failed to produce a return or because they need liquidity and the asset has not yet matured. Delaying the sale deepens the information problem and adverse selection may inefficiently accelerate asset liquidation. They distinguish between outside and inside liquidity (asset sales versus cash), which connects to our analysis where banks hold liquid and illiquid securities and the former can be traded in exchange for risky claims on the latter. Brunnermeier and Pedersen (2009) similarly distinguish between market liquidity and funding liquidity. In our model, banks can obtain funding liquidity in the interbank market by issuing claims on assets with limited market liquidity.

In Diamond and Rajan (2009), illiquidity can depress lending and low prices for illiquid assets go hand in hand with high returns on holding liquidity. They do not consider asymmetric information. Instead, potential buyers may want to wait for asset prices to decline further. At the same time, the managers of selling banks may want to gamble for resurrection. These two effects feed on each other and may lead to a market freeze.

Allen, Carletti, and Gale (2009) present a model of a market freeze without asymmetric information or counterparty risk. Banks can stop trading due to aggregate liquidity risk, i.e.

[5] An important complement to liquidity within the financial sector is the demand and supply of liquidity within the real sector (see Holmström and Tirole, 1998).

banks hold similar rather than offsetting positions. Aggregate shortages are also examined in Diamond and Rajan (2005) where bank failures can be contagious due to a shrinking of the pool of available liquidity. Freixas, Parigi, and Rochet (2000) analyze systemic risk and contagion in a financial network and its ability to withstand the insolvency of one bank. In Allen and Gale (2000), the financial connections leading to contagion arise endogenously as a means of insurance against liquidity shocks.

Acharya, Gromb, and Yorulmazer (2008) and Freixas, Martin, and Skeie (2008) both study rationales for central bank intervention in the interbank market. In Acharya et al., market power makes it possible for liquidity-rich banks to extract surplus from banks that need liquidity. A central bank provides an outside option for the banks suffering from such liquidity squeezes. In Freixas et al., multiple equilibria exist in interbank markets, some of which are more efficient than others. By steering interest rates, a central bank can act as a coordination device for market participants and ensure that a more efficient equilibrium is reached. Freixas and Jorge (2008) examine how financial imperfections in the interbank market affect the monetary policy transmission mechanism beyond the classical money channel.

The remainder of the paper is organized as follows. In Section 2, we describe the setup of the model. In Section 3, we derive and characterize the various interbank market regimes. In Section 4, we discuss the empirical implications of the model and relate it to the developments during the financial crisis. In Section 5, we employ our model to discuss policy responses. In Section 6, we offer concluding remarks. All proofs are in the Appendix.

2 The model

There are three dates, $t = 0, 1$, and 2, and a single homogeneous good that can be used for consumption and investment. There is no discounting and no aggregate uncertainty.

Banks. There is a $[0, 1]$ continuum of identical, risk neutral banks. Banks manage the funds on behalf of risk neutral customers with future liquidity needs. To meet the liquidity needs of customers, banks offer them claims worth d_1 and d_2 that can be withdrawn at $t = 1$ and $t = 2$, respectively, e.g. demand deposits or lines of credit. We assume that the liquidity needs are strictly positive at each date so that $d_1 > 0$ and $d_2 > 0$.

The aggregate demand for liquidity is certain: a fraction λ of customers withdraws their claims at $t = 1$. The remaining fraction $1 - \lambda$ withdraws at $t = 2$. At the individual bank level, however, the demand for liquidity is uncertain. A fraction π_h of banks face a high liquidity demand $\lambda_h > \lambda$ at $t = 1$ and the remaining fraction $\pi_l = 1 - \pi_h$ of banks faces a low liquidity demand $\lambda_l < \lambda$. Hence, we have $\lambda = \pi_h\lambda_h + \pi_l\lambda_l$. Let the subscript $k = l, h$ denote whether a bank faces a low or a high need for liquidity at $t = 1$. We assume that banks' idiosyncratic liquidity shocks are not contractible: A bank's liabilities cannot be contingent on whether it faces a high or a low liquidity shock at $t = 1$ and $t = 2$. This is the key friction that will give rise to an interbank market.

Assets and banks' portfolio decision. At $t = 0$, banks can invest in two types of real assets, a long-term illiquid asset and a short-term liquid asset. We assume that each bank has one unit of the good under management at $t = 0$. Each unit invested in the liquid asset offers a return equal to 1 after one period (costless storage). Each unit invested in the illiquid asset yields an uncertain payoff at $t = 2$. The illiquid asset can either succeed and return R or fail and return zero. In the latter case, a bank is insolvent and it is taken over by the deposit insurance fund. Let α^I denote the fraction invested in the illiquid asset at $t = 0$. The remaining fraction $1 - \alpha^I$ is invested in the liquid asset.

Importantly, banks are uncertain about the riskiness of their illiquid investment when they make their portfolio allocation at $t = 0$. With probability q, the illiquid investment succeeds with probability p_s and with probability $1 - q$, it succeeds with probability $p_r < p_s$. Let p denote the expected success probability: $p = qp_s + (1 - q)p_r$. Each bank becomes privately informed about the risk of its illiquid investment at $t = 1$. While the overall level of risk, p, is known, banks have private information whether their illiquid investment is safer

($p_s > p$) or riskier ($p_r < p$) than expected. The uncertainty about liquidity demand is assumed to be independent of the uncertainty about the risk of the illiquid asset. Let the subscript $\theta = s, r$ denote whether a bank's illiquid asset is safer or riskier than expected.

The investment in the illiquid asset is ex ante efficient: $pR > 1$. This does not, however, preclude an illiquid investment that turns out to be riskier than expected to be unprofitable ex post: $p_r R < 1$. Any fraction α^L of the illiquid investment can be converted into liquidity using a private liquidation technology at $t = 1$, for a constant unit return of less than one (costly liquidation). We interpret this broadly as a cost of accessing sources of funding other than unsecured borrowing. We assume that safer investments are easier to convert into liquidity, $1 > l_s > l_r$.[6] This structure makes riskier assets also more illiquid, a feature particularly pronounced in the current crisis. In case $p_r R < 1$, we assume that $p_r R > l_r$ so that even if the illiquid investment turns out to be riskier than expected, banks prefer to keep it to maturity. In sum, banks face a trade-off between liquidity and return when making their portfolio decision. The illiquid asset is ex ante more productive but it is costly to convert it into liquidity at $t = 1$.

Banks are protected by limited liability in case they make losses. To prevent any risk-shifting behavior due to limited liability, the regulator dictates the structure of banks' liabilities, i.e. he imposes d_1 and d_2. Banks then choose their portfolio at $t = 0$ to maximize profits.

Interbank market and liquidity management. Given that banks face differing liquidity demands at $t = 1$, an interbank market can develop. Banks with low withdrawals at $t = 1$ can lend any excess liquidity to banks with high $t = 1$ withdrawals. Let L_l and L_h denote the amount lent and borrowed, respectively, and let r denote the interest rate on interbank loans.[7]

[6] For example, such a technology would allow to realize a constant fraction γ of the illiquid asset's expected value: $l_\theta = \gamma p_\theta R$. Our results would be qualitatively unchanged if we instead assumed that a riskier illiquid asset returns more, $R_s < R < R_r$, and that illiquid assets can be converted into liquidity at the same rate, $l_s = l_r = l$. We show this in Appendix B. What is needed is that the opportunity cost of liquidation, $\frac{R}{l}$, is higher for a riskier bank.

[7] Note that screening of borrowers is not possible in this set-up as all banks demand the same loan size

Due to the risk of the illiquid asset, a borrower as well as a lender in the interbank market may be insolvent at $t = 2$ when the loan repayment is due. A solvent borrower must always repay his interbank loans. If his lender is insolvent, the repayment goes to the deposit insurance fund. In contrast, a solvent lender is only repaid if his borrowers are solvent, too. Hence, lenders in the interbank market are exposed to the possibility that the interbank loans they made are not repaid, or *counterparty risk*. We denote the probability that an interbank loan is repaid by $\tilde{p}$.

We assume that the interbank market is competitive, i.e. banks act as price takers, and that banks are completely diversified across interbank loans. Hence, $\tilde{p}$ is also a proportion of interbank loans that will be repaid at $t = 2$.

In sum, a bank can manage its liquidity at $t = 1$ in three ways: 1) by borrowing/lending in the interbank market, 2) by converting the illiquid asset into liquidity, and 3) by investing in the liquid asset for another period.

The sequence of events is summarized in Figure 3 below.

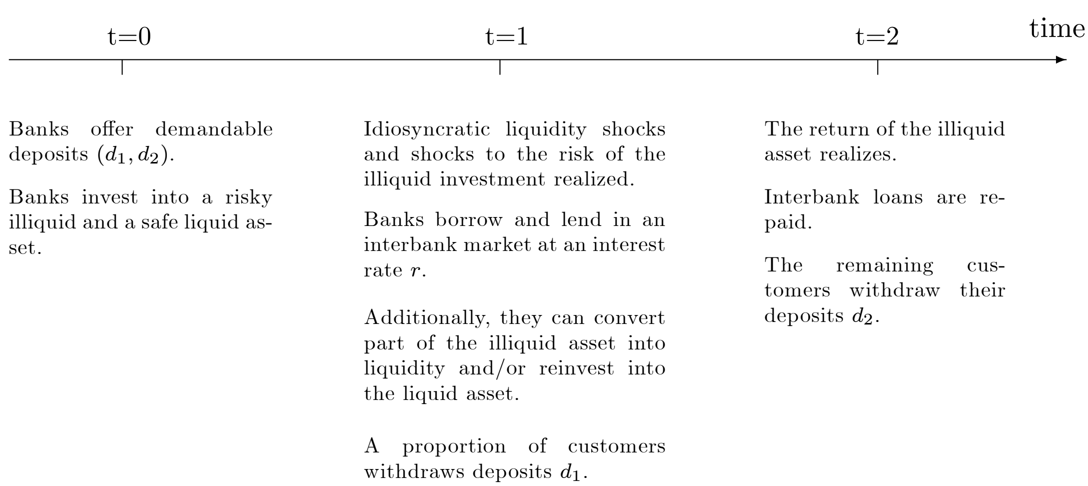

Figure 3: The timing of events

and there is no readily available collateral they can pledge.

3 Analysis

In this section we solve the model backwards by first examining banks' liquidity management at $t = 1$ and then deriving the price of liquidity from banks' portfolio allocation at $t = 0$. We derive different regimes in the unsecured interbank market. First, there can be full participation of borrowers and lenders in the market. Second, there can be adverse selection in the unsecured market when borrowers with safer illiquid investments prefer to obtain liquidity outside this money market segment. Third, the interbank market can break down. This happens either because all lenders prefer to hoard liquidity instead of lending it out or because all borrowers drop out of the market. Which of the regimes occurs depends on the underlying parameters of the model.

3.1 Regime I: Full participation of borrowers and lenders

In order to characterize the regime with full participation in the interbank market, we start by assuming that there is indeed full participation and then verify for which parameters the assumption is met.

Liquidity management. Having received liquidity shocks, $k = l, h$, and being privately informed about the risk of their illiquid investment, $\theta = s, r$, banks need to manage their liquidity at $t = 1$ in order to maximize profits at $t = 2$.

A bank that faces a low level of withdrawals at $t = 1$, type-(l, θ), solves the following problem:

$$\max_{\alpha^L_{l,\theta}, \alpha^R_{l,\theta}, L_{l,\theta}} p_\theta [R(1 - \alpha^L_{l,\theta})\alpha^I + \alpha^R_{l,\theta}((1 - \alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta) + \tilde{p}(1 + r)L_{l,\theta} - (1 - \lambda_l)d_2] \tag{1}$$

subject to

$$\lambda_l d_1 + \alpha^R_{l,\theta}((1 - \alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta) + L_{l,\theta} \leq (1 - \alpha^I) + \alpha^L_{l,\theta}\alpha^I l_\theta.$$

A type-(l, θ) bank has spare liquidity since the level of $t = 1$ withdrawals is low. The bank can thus lend $L_{l,\theta}$ at a rate r in the interbank market. The bank can also reinvest a fraction $\alpha^R_{l,\theta}$ in the liquid asset. Finally, it can convert a fraction $\alpha^L_{l,\theta}$ of its illiquid investment into liquidity.

The budget constraint requires that the outflow of liquidity at $t = 1$ (deposit withdrawals, reinvestment into the liquid asset and interbank lending) be matched by the inflow (return on the liquid asset and liquidation proceeds).

A bank that has received a high liquidity shock, type-(h, θ), will be a borrower in the interbank market, solving:

$$\max_{\alpha^L_{h,\theta},\alpha^R_{h,\theta},L_{h,\theta}} p_\theta[R(1-\alpha^L_{h,\theta})\alpha^I + \alpha^R_{h,\theta}((1-\alpha^I)+\alpha^L_{h,\theta}\alpha^I l_\theta) - (1+r)L_{h,\theta} - (1-\lambda_h)d_2] \quad (2)$$

subject to

$$\lambda_h d_1 + \alpha^R_{h,\theta}((1-\alpha^I)+\alpha^L_{h,\theta}\alpha^I l_\theta) \leq (1-\alpha^I)+\alpha^L_{h,\theta}\alpha^I l_\theta + L_{h,\theta}.$$

A type-(h, θ) bank has a liquidity shortage. It can borrow an amount $L_{h,\theta}$ in the interbank market. It could also convert some of its illiquid asset into liquidity and reinvest into the liquid asset.

There are two key differences between the optimization problems of a lender and a borrower. The first difference is in the budget constraint. The interbank loan is an outflow for a lender and an inflow for a borrower. The second difference is in the objective function. A borrower expects having to repay $p_\theta(1 + r)L_{h,\theta}$ while a lenders expects a repayment $p_\theta\tilde{p}(1 + r)L_{l,\theta}$. A lender will not be repaid if the illiquid investment of his counterparty fails. With full participation in the interbank market, a lender expects his counterparty to be solvent and repay the interbank loan with probability $p = qp_s + (1 - q)p_r$ since he cannot distinguish safer and riskier borrowers. Hence, we have that

$$\tilde{p} = p$$

when participation is full.

We characterize banks' liquidity management at $t = 1$ in a number steps. First, we obtain the marginal value of liquidity from the Lagrange multiplier on the budget constraint, denoted by $\mu^{k,\theta}$.

Lemma 1 (Marginal value of liquidity I) *With full participation in the interbank market, the marginal value of liquidity is* $\mu^{l,\theta} = pp_\theta(1+r)$ *for a lender and* $\mu^{h,\theta} = p_\theta(1+r)$ *for a borrower.*

A lender values liquidity at $t = 1$ since he can lend it out at an expected return $p_\theta p(1+r)$. A borrower values liquidity since it saves the cost of borrowing in the interbank market, $p_\theta(1+r)$. The marginal value of liquidity is lower for a lender because of counterparty risk.

The following result describes banks' decision to reinvest into the liquid asset.

Lemma 2 (Liquid reinvestment I) *With full participation in the interbank market, a borrower does not reinvest in the liquid asset at* $t = 1$*:* $\alpha^R_{h,\theta} = 0$*. A lender does not reinvest in the liquid asset if and only if* $p(1+r) \geq 1$*.*

It cannot be optimal for a type-(h, θ) bank to borrow in the interbank market at rate $1+r$ and to reinvest the obtained liquidity in the liquid asset since it would yield a negative net return. The same is not true for a lender since his rate of return on the lending in the interbank market is only $p(1+r)$ due to counterparty risk. But if a lender stores his liquidity instead of lending it out, then the interbank market cannot be active.

To have full participation in the interbank market, borrowers must not convert long-term investments into liquidity at $t = 1$. Otherwise, a borrower could never repay the interbank loan. If he liquidates, he has no inflows at $t = 2$ since he does not reinvest into the liquid asset at $t = 1$ (Lemma 2). Knowing that, no bank would lend in the interbank market.

The next result characterizes banks' decision to convert illiquid investments into liquidity.

Lemma 3 (No liquidation I) *With full participation in the interbank market, a borrower does not convert his illiquid investment into liquidity if and only if* $1+r \leq \frac{R}{l_\theta}$*. A lender does not convert his illiquid investment into liquidity if and only if* $p(1+r) \leq \frac{R}{l_\theta}$*.*

The decision depends on the benefit of liquidation relative to its opportunity cost. The benefit is given by the expected return on an interbank loan. It is lower for a lender due to counterparty risk. The opportunity cost of liquidation, $\frac{R}{l_\theta}$, is the rate at which the return on the illiquid asset can be converted into liquidity at $t = 1$. The opportunity cost is higher for a safer bank since its investment is easier to convert. It follows that i) safer banks convert their illiquid investments into liquidity earlier, i.e. at lower interbank rates, than riskier ones, and ii) borrowers convert earlier than lenders.

Banks' liquidity management at $t = 1$ determines an interval of feasible interbank interest rates.

Proposition 1 (Feasible interbank loan rates I) *Full participation regime is an equilibrium in the interbank market if and only if the interbank interest rate satisfies:*

$$\frac{1}{p} \leq 1 + r \leq \frac{R}{l_s}.$$

The lower bound on the interest rate is given by the participation constraint of lenders. Their outside opportunity is to reinvest in the liquid asset. The upper bound is given by the participation constraint of safer borrowers. Their outside opportunity is to convert their illiquid investments into liquidity. Safer borrowers drop out of the unsecured interbank market earlier than riskier ones since their illiquid investment is easier to convert. The upper bound, unlike the lower one, depends on banks' risk type.

Pricing liquidity. The price of unsecured interbank loans, $1 + r$, which banks take as given when making their portfolio choice, must be consistent with an interior portfolio allocation, $0 < \alpha^I < 1$. The profitability of the illiquid asset implies that a bank would never want to invest everything into the liquid asset and thus $\alpha^I > 0$. The need for a positive payout to customers at $t = 1$, $d_1 > 0$, implies that banks will not be able to invest everything into the illiquid asset, $\alpha^I < 1$.

An interior portfolio allocation α^I maximizes expected profits. Under full participation, a bank solves:

$$\max_{\alpha^I} \pi_l p[R\alpha^I + p(1+r)L_l - (1-\lambda_l)d_2] \tag{3}$$
$$+ \pi_h p[R\alpha^I - (1+r)L_h - (1-\lambda_h)d_2]$$

subject to

$$L_l = (1-\alpha^I) - \lambda_l d_1 \tag{4}$$
$$L_h = \lambda_h d_1 - (1-\alpha^I). \tag{5}$$

where we have used the fact that $\alpha^R_{k,\theta} = \alpha^L_{k,\theta} = 0$ (Lemma 2, Lemma 3 and Proposition 1). Since all banks are assumed to borrow or lend in the interbank market, L_k is given by the budget constraint at $t = 1$. The amounts lent and borrowed are independent of the risk-type of the illiquid investment, θ.

The first-order condition for a bank's optimal portfolio allocation across the liquid and illiquid assets requires that:

$$\pi_h p_\theta (1+r) + \pi_l p_\theta p(1+r) = \pi_h p_\theta R + \pi_l p_\theta R$$

or, equivalently,

$$(\pi_h + \pi_l p)(1+r) = R. \tag{6}$$

The interbank interest rate r, the price of liquidity traded in the interbank market, is given by a no-arbitrage condition. The right-hand side is the expected return from investing an additional unit into the illiquid asset, R. The left-hand side is the expected return from investing an additional unit into the liquid asset. With probability π_h, a bank will have a shortage of liquidity at $t = 1$ and one more unit of the liquid asset saves on borrowing in the interbank market at an expected cost of $p_\theta(1+r)$. With probability π_l, a bank will have excess liquidity and one more unit of the liquid asset can be lent out at an expected return $p_\theta p(1+r)$. Lenders' expected counterparty risk is the average probability of repayment at $t=2$ given that all borrowers participate in the interbank market, $p = qp_s + (1-q)p_r$. Note that banks' own probability of being solvent at $t=2$, p_θ, cancels out in (6) since it affects the expected return on the liquid and the illiquid investment symmetrically.

We rewrite (6) as:

$$\delta(1+r) = R \tag{7}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{\pi_h + \pi_l p} > 1 \tag{8}$$

is the premium of lending in the interbank market due to counterparty risk. Liquidity becomes more costly when i) there are fewer suppliers of liquidity ($\pi_l = 1 - \pi_h$ decreases), and ii) counterparty risk increases. Counterparty risk increases when is it less likely that the illiquid investment turns out to be safer than expected (lower q) or when the probability of success decreases (lower p_θ).

The next result summarizes the discussion on the pricing of liquidity at $t=0$, taking into account the conditions obtained from the management of liquidity at $t=1$ (Proposition 1).

Proposition 2 (Pricing I) *In the full participation regime, the risk premium is smaller than the illiquidity premium of the safer illiquid asset:* $\frac{1}{\delta} \leq \frac{1}{l_s}$. *The interbank interest rate is given by* $1+r = \frac{R}{\delta}$.

Under full participation in the interbank, there is no impairment to market functioning due to asymmetric information about counterparty risk. The price of liquidity reflects the opportunity cost of not investing into illiquid asset, R, and the premium due to *average* counterparty risk, $\frac{1}{\delta}$.

Portfolio allocation. The amounts invested in the liquid and illiquid asset are determined by market clearing in the interbank market. Using (4) and (5), market clearing in the interbank market, $\pi_l L_l = \pi_h L_h$, yields:

$$1 - \alpha^I = \lambda d_1. \tag{9}$$

The amount invested in the liquid asset exactly covers the expected amount of withdrawals at $t = 1$.

It is useful to consider the benchmark case when there is no counterparty risk, $\tilde{p} = 1$.

Corollary 1 (No counterparty risk) *Without counterparty risk, i) there is always full participation in the interbank market, and ii) the interest rate is equal to R.*

Without counterparty risk there is no friction in the economy. All banks participate in the interbank market since lending is riskless and obtaining liquidity by converting illiquid asset into liquidity is more costly.

3.2 Regime II: Adverse selection in the interbank market

The previous section analyzed the regime with full participation in the unsecured market. In that regime, borrowers whose illiquid investment is safer than expected subsidize borrowers whose illiquid investment turns out to be riskier. The subsidy becomes too costly when the risk premium is larger than the liquidation premium, $\frac{1}{\delta} > \frac{1}{l_s}$ (Proposition 2). In this case, the interest rate in the interbank market is so high that safer banks prefer to obtain their liquidity outside the unsecured market. Lenders therefore face an adverse selection of risky borrowers.

We follow the same steps as in the previous section. We start by assuming that there is adverse selection in the interbank market and then verify for which parameters there is indeed adverse selection. As before, we first examine banks' liquidity management at $t = 1$ and then consider banks' portfolio choice at $t = 0$.

Let r_r denote the interest rate and α_r^I the fraction invested in the illiquid asset when there is an adverse selection of risky borrowers in the interbank market.[8] Lenders' objective function $t = 1$ is the same as under full participation (equation (1)), except that the probability of a repayment of the interbank loan $\tilde{p}$ is now given by p_r instead of p. Similarly, borrowers' expected interest repayment is $1 + r_r$ instead of $1 + r$ (as in equation (2)). The budget constraint of banks active in the interbank market is unchanged. The analogue of Lemma 1 under adverse selection is:

Lemma 4 (Marginal value of liquidity II) *With adverse selection in the interbank market, the marginal value of liquidity is $\mu^{l,\theta} = p_r p_\theta (1 + r_r)$ for a lender and $\mu^{h,r} = p_r(1 + r_r)$ for a risky borrower.*

Adverse selection affects the marginal value of liquidity. It increases counterparty risk, $p_r < p$, and it changes the interest rate. As before, the marginal value of liquidity is higher for borrowers than for lenders.

The changes in the marginal value of liquidity modify banks' decisions to reinvest in the liquid asset and to convert the illiquid asset into liquidity.

Lemma 5 (Liquid reinvestment II) *With adverse selection in the interbank market, a risky borrower does not reinvest in the liquid asset at $t = 1$: $\alpha^R_{h,r} = 0$. A lender does not reinvest in the liquid asset if and only if $p_r(1 + r_r) \geq 1$.*

Lemma 6 (No liquidation II) *With adverse selection in the interbank market, a risky borrower does not convert his illiquid investment into liquidity if and only if $(1 + r_r) \leq \frac{R}{l_r}$. A lender does not convert his illiquid investment into liquidity if and only if $p_r(1 + r_r) \leq \frac{R}{l_\theta}$.*

As in the case with full participation in the interbank market, banks' liquidity management at $t = 1$ determines an interval of feasible interest rates under adverse selection.

[8] For notational simplicity, we do not index by r the other choice variables.

Proposition 3 (Feasible interbank loan rates II) *Adverse selection regime is an equilibrium in the interbank market if and only if the interbank interest rate satisfies:*

$$\frac{1}{p_r} \leq 1 + r_r \leq \frac{R}{l_r}.$$

Under adverse selection, the lower bound on the interest rate is higher than with full participation (Proposition 1). Facing only risky borrowers, lenders' outside opportunity of reinvesting in the liquid asset is more attractive. Since only riskier banks borrow, the upper bound is also higher.

The portfolio allocation between the liquid and the illiquid asset at $t = 0$ determines again the interest rate in the interbank market. Anticipating adverse selection in the interbank market, a bank solves:[9]

$$\max_{0<\alpha_r^I<1} \quad \pi_l p[R\alpha_r^I + p_r(1 + r_r)L_l - (1 - \lambda_l)d_2]$$
$$+\pi_h(1 - q)p_r[R\alpha_r^I - (1 + r_r)L_h - (1 - \lambda_h)d_2]$$

subject to

$$L_l = (1 - \alpha_r^I) - \lambda_l d_1$$
$$L_h = \lambda_h d_1 - (1 - \alpha_r^I)$$

where we used the results in Lemma 5, Lemma 6 and Proposition 3. Compared to full participation (equation (3)), banks' objective function at $t = 0$ under adverse selection differs in two respects. First, the interest rate is given by r_r instead of r. Second, a bank expects not to participate in the unsecured interbank market if it receives a high liquidity

[9] Lemma 6 implies that there can be two cases under adverse selection regime: 1) a case in which none of the lenders convert illiquid investments into liquidity, $p_r(1 + r_r) \leq \frac{R}{l_s}$; and 2) a case in which safer lenders choose to convert their illiquid investments and to lend their excess liquidity in the interbank market, $\frac{R}{l_s} < p_r(1 + r_r) < 1 + r_r \leq \frac{R}{l_r}$. We will focus on the former case as the other case does not add any new features to the results. Moreover, it did not seem to play a central role in the interbank market developments in the 2007-09 crisis. This is because liquidity hoarding, which we document above, cannot occur in this case: $p_r(1 + r_r) > \frac{R}{l_s} > 1$. We therefore proceed under the assumption that $p_r(1 + r_r) \leq \frac{R}{l_s}$, which is equivalent to $p_r \frac{1}{\delta_r} \leq \frac{1}{l_s}$, i.e. the lender's risk premium under adverse selection is smaller than the illiquidity premium of the safer illiquid asset.

shock and if its illiquid investment is safer than expected. With probability $\pi_h q$, a bank therefore gets liquidity by converting its illiquid asset. As before, the amounts lent and borrowed per bank are denoted by L_l and L_h, respectively.

The first-order condition for an optimal portfolio allocation under adverse selection is given by:

$$(\pi_l p p_r + \pi_h (1-q) p_r)(1+r_r) = (\pi_l p + \pi_h (1-q) p_r) R. \tag{10}$$

Comparing (10) to the condition with full participation (6) shows that adverse selection has two effects on the price of liquidity in the interbank market. First, lenders get repaid less often, $p_r < p$. Second, composition of banks in the interbank market changes since only riskier banks borrow, which is reflected by the term $\pi_h (1-q) p_r$.

We can rewrite the no-arbitrage condition (10) as:

$$\delta_r (1+r_r) = R \tag{11}$$

where

$$\frac{1}{\delta_r} \equiv \frac{\pi_l + \pi_h \zeta}{\pi_l p_r + \pi_h \zeta} \tag{12}$$

and

$$\zeta \equiv \frac{1}{1 + \frac{q}{1-q} \frac{p_s}{p_r}}. \tag{13}$$

Adverse selection affects the risk premium in the interbank market $\frac{1}{\delta_r}$ first via higher counterparty risk and second via the composition effect ζ. Higher counterparty risk (lower p_r) and a worse composition (lower ζ) both increase the risk premium. Adverse selection in the interbank market therefore unambiguously increases the price of liquidity: $1 + r_r > 1 + r$. This is because the risk premium under adverse selection is higher, $\frac{1}{\delta_r} > \frac{1}{\delta}$.

The next Proposition summarizes the pricing of liquidity under adverse selection in the interbank market.

Proposition 4 (Pricing II) *In the adverse selection regime: i) the risk premium must be* *smaller than the illiquidity premium of the riskier illiquid asset:* $\frac{1}{\delta_r} \leq \frac{1}{l_r}$*; and ii) the risk discount must be smaller than the expected return of the riskier illiquid asset,* $\delta_r \leq p_r R$*. The interbank interest rate is given by* $1 + r_r = \frac{R}{\delta_r}$*.*

Using the market clearing in the interbank market, it is easy to show that the portfolio choice in Regime II is given by:

$$\alpha_r^I = \frac{1 - \lambda d_1 - \pi_h q \left(1 - \lambda_h d_1\right)}{\pi_l + \pi_h (1 - q)}.$$

Banks hold a more illiquid portfolio in Regime II compared to Regime I: $\alpha_r^I > \alpha^I$. To see this, note that

$$\frac{1 - \lambda d_1 - \pi_h q \left(1 - \lambda_h d_1\right)}{\pi_l + \pi_h (1 - q)} - 1 + \lambda d_1 = \frac{\pi_h q \left(\lambda_h - \lambda\right)}{1 - \pi_h q} > 0.$$

3.3 Regime III: Breakdown of the interbank market

When the interest rate under adverse selection is outside the bounds imposed by Proposition 3, then the unsecured interbank market may break down. Liquidity will no longer flow from banks with small liquidity shocks to banks with large liquidity shocks. The market can break down either because lenders prefer to keep their liquidity instead of lending it out to an adverse selection of borrowers (liquidity hoarding) or because even riskier banks find it too expensive to borrow and drop out of the market.

Lenders drop out. Adverse selection in the interbank market leads to a higher interest rate. But is the increase in the interest rate high enough to compensate lenders for the larger counterparty risk when facing an adverse selection of borrowers? Lenders prefer to hoard liquidity by reinvesting it in the liquid asset when the lower bound in Proposition 3 is violated:

$$p_r (1 + r_r) < 1. \tag{14}$$

The condition can also be written as in Proposition 4:

$$p_r R < \delta_r. \tag{15}$$

Since $\delta_r < 1$, lenders only hoard liquidity if the illiquid investment not only turns out to be riskier than expected, but it is also unprofitable. Note that this is compatible with the assumption about the ex ante efficiency of the illiquid investment, $pR > 1$.

Borrowers drop out. Even riskier borrowers may choose to leave the unsecured market segment if adverse selection drives up the interest rate too much. The upper bound on the interest rate in Proposition 3 is violated when:

$$\frac{1}{\delta_r} > \frac{1}{l_r},$$

i.e. when the risk premium under adverse selection is higher than the illiquidity premium for riskier borrowers (see Proposition 4).

Our assumption that banks prefer to keep their illiquid investments to maturity even if it turns out to be riskier than expected, $p_r R > l_r$, implies that the interbank market breaks down either because all borrowers drop out or because all lenders drop out, i.e. the two situations are mutually exclusive. This is because the condition for all borrowers to leave the market, $l_r > \delta_r$, together with $p_r R > l_r$ imply that $p_r R > \delta_r$ and thus liquidity hoarding cannot occur.

3.4 Multiple equilibria

In this subsection, we summarize conditions under which a particular regime constitutes the unique equilibrium in the interbank market.

Regime I is the unique equilibrium if and only if

$$1 + r < 1 + r_r \leq \frac{R}{l_s}$$

or, equivalently,

$$\delta_r \geq l_s.$$

The result follows from Proposition 1. When the interest rate that would arise under adverse selection is relatively low, safer borrowers prefer to stay in the market and hence adverse selection regime cannot be an equilibrium.

Similarly, Regime II is the unique equilibrium when

$$\frac{R}{l_s} < 1 + r < 1 + r_r$$

and conditions of Proposition 3 hold or, equivalently, when

$$l_s > \delta \text{ and } p_r R \geq \delta_r \geq l_r.$$

Liquidity hoarding is the unique equilibrium if and only if $l_s > \delta$ and $\delta_r > p_r R$. Market breakdown due to the drop out of borrowers is the unique equilibrium if and only if $l_s > \delta$ and $l_r > \delta_r$.

It follows that there is an open set of parameters such that we have multiple equilibria in the model. This occurs when

$$1 + r \leq \frac{R}{l_s} < 1 + r_r$$

holds implying that $\delta \geq l_s > \delta_r$.

Both Regimes I and II are equilibria if $1 + r \leq \frac{R}{l_s} < 1 + r_r$ and conditions of Proposition 3 are satisfied so that

$$\delta \geq l_s > \delta_r \text{ and } p_r R \geq \delta_r \geq l_r.$$

If banks expect Regime I to be an equilibrium, all banks participate in the interbank market and the resulting interest rate $1 + r$ is smaller than $\frac{R}{l_s}$, thus justifying banks' expectations.

However, if banks expect Regime II to be an equilibrium, safer banks in need of liquidity drop out of the interbank market and the interest rate is given by $1 + r_r$.

Similarly, both Regimes I and III (liquidity hoarding) are equilibria when $\delta \geq l_s > \delta_r > p_r R$. Both Regimes I and III (borrowers drop out) are equilibria when $\delta \geq l_s > l_r > \delta_r$.

Possibility of multiple equilibria creates a scope for a policy intervention which can coordinate banks' expectations. We discuss this further in Section 5.2.

4 Discussion and empirical implications

Depending on parameters, three different interbank market regimes can occur as an equilibrium in our model: i) full participation and no impairment to the functioning of the interbank market, ii) adverse selection and higher interest rates, and iii) market breakdown. Figure 4 shows which regime occurs under different parameters for the average success probability, p, and the dispersion of risk, $\Delta p \equiv p_s - p_r$.[10] Since banks have private information about the risk of the illiquid asset, Δp is a measure of the severity of the asymmetric information problem.

When the average level of counterparty risk is low (high p), there is full participation in the interbank market (Regime I) regardless of the dispersion of counterparty risk. Asymmetric information about the risk of illiquid assets does not impair the functioning of the interbank market in this case. When average counterparty risk rises (p decreases), driving up the interest rate in the interbank market beyond a certain threshold, safer banks with a liquidity shortage prefer to obtain liquidity outside the unsecured interbank market. Only an adverse selection of riskier banks keeps borrowing unsecured, causing the interest rate to increase even further. Once there is adverse selection in the interbank market (Regime II), the dispersion of counterparty risk matters. An increase in the dispersion of risk alone (higher Δp), without an increase in the level of risk, can lead to a breakdown of the interbank market

[10] For ease of exposition, we consider parameter space such that each regime occurs as the unique equilibrium.

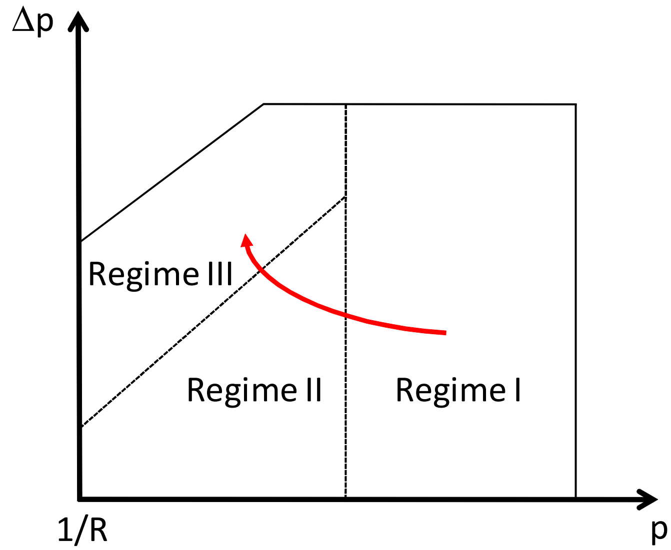

Figure 4: Comparative statics: Transition between regimes

and liquidity hoarding. Lenders prefer to keep liquidity instead of lending it out despite the high rates borrowers would be willing to pay.

The arrow in Figure 4 depicts a change in the level and the dispersion of counterparty risk and a corresponding transition between regimes that echoes the experience in interbank markets before and during the financial crisis of 2007-09. Three different phases described in Figure 1, i.e. i) normal times, ii) elevated spreads but no recourse to the ECB's deposit facility, and iii) further increase in spreads and substantial amounts deposited overnight with the ECB, resemble the different regimes of our model: i) no impairment, ii) adverse selection, and iii) liquidity hoarding. Moreover, the transition across regimes implies a change in the underlying level and dispersion of counterparty risk that is consistent with the development of actual events. First, the transition from Regime I to II occurs at the start of the crisis in August 2007. At that time, subprime-mortgage backed securities were discovered in portfolios of banks and bank-sponsored conduits (SIVs) leading to a reassessment of the level of risk. The extent of exposures was unknown and counterparties could not distinguish safe from risky banks. The transition from Regime II to III occurs at the moment of the dramatic events surrounding the last weekend of September 2008 when the financial crisis spread outside the realm of investment banking and into the global financial system.[11] These events can be interpreted as a further increase in the level, and importantly, in the dispersion of counterparty risk making the adverse selection problem more severe.[12] In the context of our model, one can similarly view the effect of the rescue of Bear Stearns as placing a lower bound on the perceived probability of default of counterparties. But letting Lehman fail then led to a drastic revision of expected default probabilities.

Since the possibility of a market breakdown due to liquidity hoarding by lenders is an important feature of our model, we examine the empirical evidence on the hoarding of liquidity more closely. The major developments at the time of the transition from Regime II to III are depicted in relation to flows and stocks of liquidity using daily data in Figures 5 and 6. The amounts deposited with the ECB start rising after the collapse of Washington Mutual, ten days after the Lehman failure (September 15, 2008). Importantly, the rise precedes the ECB announcement of a change in its tender procedure and standing facilities corridor on October 8, 2008.[13] In the week of September 29, 2008, the daily amounts of liquidity absorbed by the ECB averaged more than €169 billion (Figure 5).

At exactly the same time as banks started to bring funds to the ECB, the average daily volume in the overnight unsecured interbank market (Eonia) halved and the net amount of central bank liquidity outstanding dropped significantly (Figure 6).[14] The net amount of

[11] Before the weekend of September 27-28, 2008 Washington Mutual, the largest S&L institution in the US was seized by the FDIC and sold to JPMorgan Chase. At the same time, negotiations on the TARP rescue package stalled in US Congress. Over the weekend, it was reported that British mortgage lender Bradford & Bingley had to be rescued and Benelux announced the injection of €11.2 billion into Fortis Bank. On the following Monday, Germany announced the rescue of Hypo Real Estate, and Iceland nationalized Glitnir.

[12] The fact that banks no longer trust each other amid perceptions that other banks are at risk of default was also pointed out by market commentators at the time, see, for example, "Central Banks Add Funds to Money Markets," The Wall Street Journal, September 29, 2008 and "Why the ECB Can't Fix Europe," Business Week, October 8, 2008.

[13] As of October 9, the deposit facility rate was increased from 100 to 50 basis points below the policy rate, thus making deposits relatively more attractive. The marginal lending facility rate was reduced from 100 to 50 basis points above the policy rate. Moreover, as from the operation settled on October 15, 2008, the weekly main refinancing operation is carried out through a fixed rate tender procedure with full allotment at the policy rate.

[14] At the onset of the crisis in August 2007, the Eonia saw an *increase* in volume. The average daily volume was €40.91 billion in the year prior to August 9, 2007. It increased by 27%, to an average of €52.12 billion, between August 9, 2007 and September 26, 2008. This increase could reflect a substitution towards more short-term financing in the interbank market in Regime II as liquidity in longer-term segments of the market

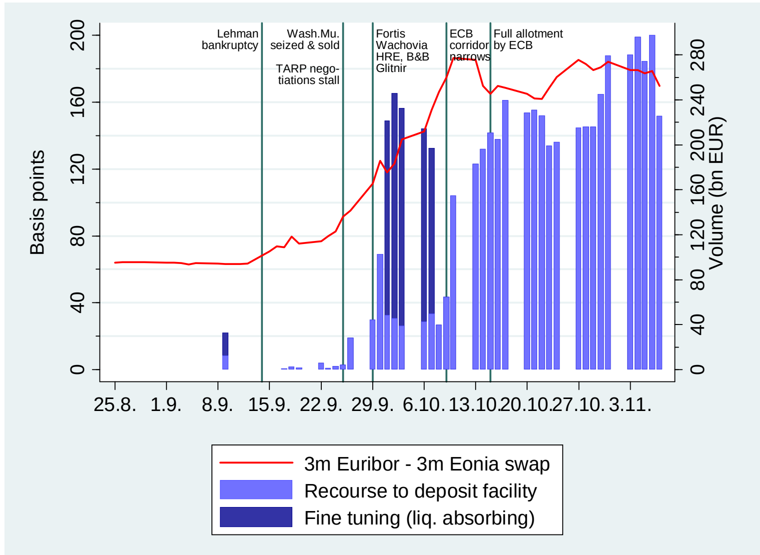

Figure 5: Interbank spread, recourses to the ECB deposit facility, and liquidity-absorbing fine tuning operations, 08/2008 - 11/2008

central bank liquidity outstanding is the total stock of liquidity provided minus the amount absorbed in all open market operations and recourses to its standing facilities. The Figures also show that although the ECB provided large amounts of liquidity (see the spikes in the net stock of liquidity) throughout September 2008, banks were not depositing funds until the end of the month. Moreover, there is evidence that the set of banks participating in the liquidity-absorbing operations of the ECB is not the same as the set of banks participating in its liquidity-providing operations. It follows that, as of the last weekend of September 2008, banks were hoarding their own liquidity and parking it at the ECB rather than lending it out.

If the interbank market suffers from liquidity hoarding, two further implications follow from our model. First, a necessary condition for liquidity hoarding is that some banks are insolvent, i.e. $p_r R < 1$ (see condition 15). Tackling the roots of the problem therefore requires finding out who these banks are and recapitalizing (or closing) them. Indeed, the US government and banking regulators are assessing banks' risk and viability through a

dried up. The drop in overnight volumes of more than €29 billion observed at the end of September 2008 is thus even more dramatic.

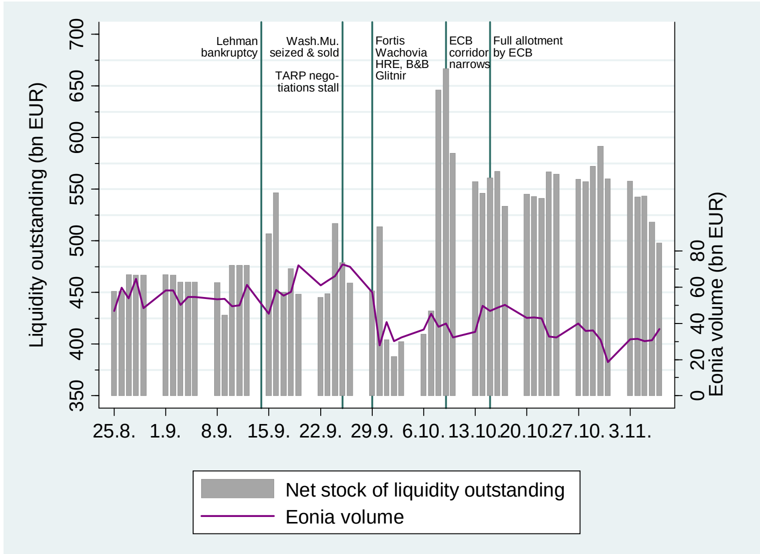

Figure 6: Net stock of central bank liquidity outstanding (left scale) and overnight unsecured market volumes (right scale), 08/2008 - 11/2008

comprehensive "stress testing" exercise since February 25, 2009. Second, increasing the rate at which banks can park funds with the central bank reinforces liquidity hoarding. To see this, consider an increase in the right-hand side of (14) from 1 to $1+\tau$, where τ is the interest earned by depositing with the central bank. After increasing the deposit facility rate from 100 to 50 basis points below the policy rate on October 9, 2008, the ECB lowered it back to 100 basis point on January 21, 2009.

When the unsecured interbank market is still functioning, albeit at elevated interest rates (Regime II), our model implies that only an adverse selection of riskier banks is willing to borrow at those rates. Safer banks have left the unsecured market since they have better alternatives to obtain liquidity than riskier banks. This is consistent with anecdotal evidence about the reluctance of banks to borrow at high rates since the onset of the crisis in order to avoid "signaling" that they are bad banks. Moreover, there is evidence of "tiering" in interbank markets consistent with our model where differences in the risk of banks' illiquid assets translates into differences in their alternatives to unsecured borrowing. With the onset of the financial crisis in August 2007, the spread between the rate in interbank market secured by government bonds (Eurepo) and the rate of secured borrowing in ECB auctions rose significantly. Banks with high quality collateral could borrow more cheaply than banks bidding in the ECB auctions where a larger set of collateral is accepted (Tapking and Weller, 2008).

Prior to September 2008 and in light of committed credit lines to SIVs, aggregate liquidity risk, in addition to credit risk, was suggested as a reason for the high level of interbank rates.[15] This is consistent with our model since the interbank interest rate r unambiguously increases following an increase in the proportion of banks with the liquidity shortage (higher π_h). However, aggregate liquidity risk by itself cannot explain why banks with sufficient liquidity refused to lend funds in the market even at short maturities. Since the ECB moved to fully satisfy banks' demand for liquidity against a wide set of collateral and committed itself to uphold the full allotment for a considerable amount of time, concerns about aggregate liquidity shortages are greatly reduced (see also Taylor, 2009).

5 Policy responses

The aim of this section is to shed light on some of the policy responses that were discussed or implemented in order to relieve the tensions observed in interbank markets since August 2007. We examine five responses through the lens of our model: market transparency, liquidity requirements, central bank liquidity provision, loan guarantees, and asset purchases.

5.1 Transparency

Asymmetric information about the riskiness of the illiquid asset is at the heart of the adverse selection problem in our model. Safer borrowing banks subsidize riskier banks since lenders cannot tell them apart. If this subsidy becomes too large, safer banks prefer to obtain

[15] A number of studies assess the relative importance of credit and liquidity risk in interbank interest rate speads (see, for example, Taylor and Williams, 2009, and Schwarz, 2009). Acharya and Merrouche (2008) establish a causal link between aggregate liquidity held by large settlement banks in the UK and interest rates in secured and unsecured interbank markets.

liquidity outside the unsecured market. Lenders are then facing an adverse selection of borrowers that could result in the hoarding of liquidity.

A natural regulatory response is therefore to improve transparency in the banking sector.[16] If, for example, bank supervisors could assess banks' risk and communicate it to the market, then lenders would be able to distinguish safer and riskier borrowers. Two markets would emerge, one for riskier banks with an interest rate, r^r_{tr}, and one for safer banks with an interest rate, r^s_{tr}.[17] The following proposition states the interest rates under market transparency:

Proposition 5 (Market transparency) *Market transparency in the form of a regulator assessing and publicly announcing the risk type θ of banks' illiquid investment, leads to the interest rates $\delta(1+r^s_{tr}) = \frac{p}{p_s}R$ and $\delta(1+r^r_{tr}) = \frac{p}{p_r}R$.*

The two interest rates are determined by two "no-arbitrage" conditions. One condition follows again from banks' portfolio allocation at $t = 0$. The second condition requires that in equilibrium, lenders are indifferent between lending to safer or riskier borrowers.

For market transparency to avoid adverse selection, the interest rate, r^s_{tr}, must be low enough to bring safer borrowers back to the interbank market:

$$1 + r^s_{tr} \leq \frac{R}{l_s}, \tag{16}$$

The condition is equivalent to $\frac{p}{p_s} \leq \frac{\delta}{l_s}$, which is possible since $\frac{p}{p_s} < 1$ and $\frac{\delta}{l_s} < 1$ when there is adverse selection (Regime II).

[16] Increased transparency is a key recommendation of the de Larosière report, which examines the organization of supervision of financial institutions and markets in the EU. Similar recommendations are made by the UK's Turner Review and the Group of 30 Report by Paul Volcker.

[17] Assessing banks' risk is indeed the aim of the "stress testing" exercise undertaken by the US government and banking regulators (Board of Governors of the Federal Reserve System, 2009). Such an exercise can also help to restore normal trading conditions in the interbank market by reducing the degree of asymmetric information, Δp. Moreover, it can help to bring back the supply of funds that withdrew in fear of lending to unprofitable "lemons" (see equation (15)). Either the regulator is able to find out which bank is unprofitable and close it down, or it can convince market participants that there are no such banks around.

Market transparency lowers the interest rate for safer banks in need of liquidity since they are no longer pooled with riskier banks. But riskier banks will be charged a higher interest rate. As long as:

$$1 + r_{tr}^{r} \leq \frac{R}{l_r}, \tag{17}$$

or, equivalently, $\frac{p}{p_r} \leq \frac{\delta}{l_r}$, riskier banks participate in the interbank market. Again, this is possible. Although $\frac{p}{p_r} > 1$, we also have that $\frac{\delta}{l_r} > 1$ since riskier banks would still borrow in Regime II, $l_r < \delta_r < \delta$ (see Proposition 4). Market transparency thus enlarges the set of parameters for which all types of borrowers participate in the interbank market.

Market transparency does not distort the price of liquidity. However, the drawback is that it does not achieve full participation in the market when the conditions (16) and (17) fail to hold. By contrast, liquidity requirements, while distorting, can always avert adverse selection.

5.2 Liquidity requirements

In the wake of the financial crisis, bank regulators are investigating a strengthening of liquidity requirements. These requirements are supposed to ensure that financial institutions are able to withstand liquidity stresses of varying magnitude and duration.

In our model, requiring banks to hold a certain amount of liquidity can also act as a coordination device when multiple equilibria in the interbank market are possible. Recall that Regime I is not a unique equilibrium when $\delta \geq l_s > \delta_r$. In this parameter space, the regulator can achieve Regime I in the interbank market if he requires banks to hold liquidity in the amount of

$$1 - \alpha^I = \lambda d_1$$

at $t = 0$. Then, the interbank market will clear at the interest rate r and full participation equilibrium will ensue.

5.3 Liquidity provision by the central bank

A central bank can offer to provide liquidity directly to banks in need. Indeed, increased liquidity provision was a common reaction by central banks around the world to the 2007-2009 financial crisis.[18]

Suppose that an unanticipated adverse shock to counterparty risk, p, moves the economy from full participation (Regime I) to adverse selection (Regime II).[19] Assuming that a central bank that has no informational advantage over the market, it has to offer liquidity to all banks at the same rate, r_{CB}. The highest rate at which safer banks are willing to borrow from the central bank is:

$$1 + r_{CB} = \frac{R}{l_s}.$$

The central bank's net return from lending (an amount $\pi_h L_h$) to *all* banks is:

$$\pi_h L_h \left(p\frac{R}{l_s} - 1 \right),$$

which is positive since $pR > 1 > l_s$. Even though the central bank lends at a subsidized rate, it makes a profit. The reason is that a central bank can raise liquidity at unit cost. That is, it can "print money". In contrast, the private supply of liquidity is costly since banks have to forgo investing in the illiquid asset if they want to be able to provide liquidity at $t = 1$. Moreover, banks have to bear liquidity and counterparty risk. Condition (6) shows that the cost of private liquidity is $\frac{R}{\delta} > R > 1$.

[18] At the onset of the crisis on 9 August 2007, when overnight rates temporarily spiked up by 60 basis points, the Eurosystem provided €94 billion of liquidity via collateralized, overnight lending. From August 2007 until September 2008, the Eurosystem was able to stabilize the overnight interbank rates without increasing the aggregate supply of liquidity by adjusting the time path of its liquidity provision ("frontloading" liquidity within each maintenance period). From October 2008, it introduced a full allotment procedure in its market operations which led to a significant increase in the liquidity provision. As a result, the size of the ECB's balance sheet temporarily increased by roughly €600 billion. The Federal Reserve introduced the Term Auction Facility (TAF), which allowed the auctioning of term funds to all depository institutions. In early 2009, the outstanding volume in the TAF was almost \$500 billion, and the total short-term liquidity provided by the Federal Reserve to financial institutions totalled around \$850 billion (Bernanke 2009).

[19] Since we assume that the shock to counterparty risk is unanticipated, the regulatory response to the crisis is also unexpected. Thus, we abstract from moral hazard issues that can be an important consideration when examining policy responses to crises (for a recent analysis see, e.g., Diamond and Rajan, 2008b).

If a central bank provides liquidity to banks with a liquidity shortage, then it crowds out the private supply of liquidity. Banks with excess liquidity are no longer able to find a counterparty. In order to have a more balanced intervention, the central bank can offer to take on the excess liquidity and, possibly, offer a return on it. The central bank would effectively become an intermediary. It would be the counterparty for all liquidity transactions and replace the interbank market.[20]

A central bank can always provide liquidity at a lower cost than the interbank market. This is true even without a crisis. While such an intervention may seem desirable ex post (thus disregarding any moral hazard issues), it can have substantial costs. One important consideration is the role of interbank markets in information aggregation, price discovery, and peer monitoring (see, for example, Rochet and Tirole, 1996).

5.4 Interbank loan guarantees

Several countries have introduced loan guarantees in order to revive the interbank market.[21] Depending on their scope, loan guarantees reduce or even eliminate counterparty risk, thus lowering the interbank interest rate and inducing safer banks to borrow again.

Consider first the case of full interbank loan guarantees. Counterparty risk is eliminated and all banks participate in the interbank market. The interest rate in the interbank market is $1 + r_{FG} = R$, where r_{FG} denotes the interest rate under full guarantees. The cost of this intervention to the guarantor is

$$p\,(1 + r_{FG})\,\pi_h L_h - (1 + r_{FG})\,\pi_h L_h$$

[20] See also Buiter (2008). As of October 15, 2008 the ECB is de facto intermediating: it fully satisfies demand for liquidity in its weekly Main Refinancing Operations and, at the same time, banks deposit significant amounts with the ECB (see also the discussion in section 4).

[21] One example is Italy, where the Banca d'Italia and the owners of the e-Mid trading platform have established the Mercato interbancario collateralizzato (MIC). Even though its trading activity is in principle collateralized, the Banca d'Italia guarantees timely repayment of all loans in MIC. The reason is that the crisis also affected secured interbank lending as there were credit risk concerns due to uncertain collateral values.

or, equivalently,

$$-R\pi_h L_h (1-p). \tag{18}$$

The guarantor has to pay for all losses due to the risk of the illiquid investment.

Consider next partial guarantees that increase the probability of repayment from p to $\hat{p}$, where $\hat{p}$ is high enough to guarantee full participation in the interbank market:

$$1 + r_{PG} = \frac{R}{l_s},$$

and where r_{PG} is the interest rate under partial loan guarantees.[22] The cost to the guarantor is:

$$p(1+r_G)\pi_h L_h - \hat{p}(1+r_G)\pi_h L_h$$

or, equivalently,

$$-\frac{R}{l_s}\pi_h L_h (\hat{p}-p). \tag{19}$$

The following proposition shows that interbank loan guarantees should be sufficiently comprehensive to be cost-efficient for the public sector.

Proposition 6 (Partial guarantees) *The cost of partial guarantees that yield an interest rate just ensuring full participation,* $1 + r_{PG} = \frac{R}{l_s}$*, always exceeds the cost of full guarantees.*

A guarantee covers both, principal and interest. While a partial guarantee reduces the cost on the principal it increases the cost on the interest as the interest rate rises to compensate lenders for the remaining counterparty risk.

5.5 Asset purchases

An alternative to borrowing in the interbank market is to convert the risky illiquid asset into liquidity. One way to do this is to acquire high quality collateral that can be used in

[22] To ensure that lenders are willing to lend, the guarantee must be sufficiently high: $\hat{p}(1+r_{PG}) > 1$.

the repo market or with central banks. Selling illiquid assets can be costly, $l_\theta < 1$ in the context of our model. In a financial crisis, this cost is particularly acute due to "fire-sale" prices. If banks bring more illiquid assets to the market than there are funds available to buy them, the market will be characterized by "cash-in-the-market pricing". In other words, illiquid assets are particularly subject to market liquidity risk (as in, for example, Shleifer and Vishny, 1992, Allen and Gale, 2004, or Gorton and Huang, 2004).

A central bank or a government authority does not face liquidity risk. Since liquidity risk does not need to be priced in, they can offer to buy illiquid assets from banks at a higher price, $P > l_\theta$.[23] The price only needs to reflect the credit risk of assets. Moreover, by setting the price appropriately, the central bank or government can attract both safer and riskier borrowers and take advantage of pooling assets.

In particular, the price P could be set equal to the expected return on the illiquid asset, pR. This ensures that the central bank or government do not suffer losses on average. Such pricing effectively lowers the opportunity cost of liquidity to 1. This is beneficial for borrowers, who would otherwise have to pay a premium for obtaining liquidity in the interbank market since they have to compensate lenders for counterparty risk.

6 Conclusion

Interbank markets underwent dramatic changes during the ongoing financial crisis, with interest rates rising to previously unseen levels and trading activity declining significantly in some market segments. Unsecured, longer-term interbank lending and lending secured with risky collateral were particularly affected. Motivated by these facts, we present a model of the interbank market with asymmetric information about counterparty risk. We show that depending on parameters, reflecting in particular the level and distribution of counterparty risk among banks, an equilibrium with full participation of borrowers and

[23] The Eurosystem has for example widened the set of eligible collateral for its refinancing operations. While the Federal Reserve cannot purchase assets other than Treasuries, agencies and agency MBS, the US government is purchasing assets via the Troubled Asset Relief Program (TARP).

lenders in the interbank market and the smooth reallocation of liquidity may not be reached. The functioning of the interbank market can be impaired by adverse selection, possibly leading to a market breakdown. The interbank market regimes obtained in the model echo the developments prior to and during the financial crisis of 2007-09.

Although a number of factors affect banks' decisions to trade in interbank markets, our model highlights the role of counterparty risk as a necessary ingredient to explain the qualitatively different phases observed in the interbank markets over the past two years. At the same time, the asymmetry of information can rationalize the prolonged nature of interbank market tensions despite an unprecedented liquidity provision by central banks.

The model can shed light on the effects of various policy responses that were put in place to relieve the tensions. It can help interpret a number of empirical issues. For example, the model predicts that interbank market spreads can be higher than what would be implied solely by credit risk. The reason is that adverse selection has an amplifying effect on spreads.

The model can be extended along a number of dimensions. In particular, potential spillovers between the secured and unsecured money market segments can be analyzed more closely. What led to the significant degree of interbank market segmentation observed since the onset of the crisis? What determines the willingness of banks to pay at central bank refinancing operations? How broad should the list of eligible collateral be? These questions are left for future research.

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Appendix

Proof of Lemma 1

Let $\mu_2^{k,\theta}$ be the Lagrange multiplier on the feasibility constraint $L_{k,\theta} \geq 0$. The first-order condition for a type-(l,θ) w.r.t. $L_{l,\theta}$ is:

$$p_\theta p(1+r) - \mu^{l,\theta} + \mu_2^{l,\theta} = 0 \tag{20}$$

while the first-order condition for a type-(h,θ) bank w.r.t. to $L_{h,\theta}$ is:

$$-p_\theta(1+r) + \mu^{h,\theta} + \mu_2^{h,\theta} = 0. \tag{21}$$

Since we assume that all banks borrow and lend in the interbank market, we have $L_{k,\theta} > 0$ so that $\mu_2^{k,\theta} = 0$. Then (20) and (21) become:

$$\begin{aligned} p_\theta p(1+r) &= \mu^{l,\theta} \\ p_\theta(1+r) &= \mu^{h,\theta}. \end{aligned}$$

Proof of Lemma 2

Let $\mu_3^{k,\theta}$ and $\mu_4^{k,\theta}$ be the Lagrange multipliers on $0 \leq \alpha_{k,\theta}^R \leq 1$. The first-order condition for a type-(k,θ) bank w.r.t. to $\alpha_{k,\theta}^R$ is:

$$((1-\alpha^I) + \alpha_{k,\theta}^L \alpha^I l_\theta)(p_\theta - \mu^{k,\theta}) + \mu_3^{k,\theta} - \mu_4^{k,\theta} = 0 \tag{22}$$

Substituting $\mu^{h,\theta} = p_\theta(1+r)$ (Lemma 1) into (22) yields:

$$((1-\alpha^I) + \alpha_{h,\theta}^L \alpha^I l_\theta)(-r) = -\mu_3^{h,\theta} + \mu_4^{h,\theta} < 0 \tag{23}$$

since left hand side is negative. It cannot be zero since $\alpha^I = 1$ and $\alpha_{h,\theta}^L = 0$ cannot be optimal. A type-(h,θ) bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $r > 0$ whereas it could just store some liquidity without cost using the short-term asset. Since $-\mu_3^{h,\theta} + \mu_4^{h,\theta} < 0$ we have $\alpha_{h,\theta}^R = 0$.
Consider now the case of a lender. Substituting $\mu^{l,\theta} = p_\theta p(1+r)$ (Lemma 1) into (22) yields:

$$((1-\alpha^I) + \alpha_{l,\theta}^L \alpha^I l_\theta) p_\theta (1 - p(1+r)) = -\mu_3^{l,\theta} + \mu_4^{l,\theta}.$$

The first term on the left hand side is negative since $\alpha^I = 1$ and $\alpha_{l,\theta}^L = 0$ cannot be optimal. A type-(l,θ) bank cannot invest everything into the illiquid asset, not liquidate any part of it and still lend in the interbank market. Hence, $\alpha_{l,\theta}^R = 0$ iff $p(1+r) \geq 1$ (we assume that a type-(l,θ) bank does not reinvest into the liquid asset when the condition holds as an equality).

Proof of Lemma 3

Let $\mu_5^{k,\theta}$ and $\mu_6^{k,\theta}$ be the Lagrange multipliers on $0 \leq \alpha_{k,\theta}^L \leq 1$. The first-order condition for a type-(k,θ) bank w.r.t. to $\alpha_{k,\theta}^L$ is:

$$-p_\theta R\alpha^I + \alpha^I l_\theta(\alpha_{k,\theta}^R + \mu^{k,\theta}(1-\alpha_{k,\theta}^R)) + \mu_5^{k,\theta} - \mu_6^{k,\theta} = 0. \tag{24}$$

Substituting $\mu^{h,\theta} = p_\theta(1+r)$ and $\mu^{l,\theta} = p_\theta p(1+r)$ (Lemma 1) and $\alpha_{k,\theta}^R = 0$ (Lemma 2 and the assumption that there is full participation in the interbank market so that a type-(l,θ) bank does not reinvest into the liquid asset) into (24) yields:

$$\begin{aligned} -p_\theta\alpha^I(-R+(1+r)l_\theta) &= -\mu_5^{h,\theta} + \mu_6^{h,\theta} \\ -p_\theta\alpha^I(-R+p(1+r)l_\theta) &= -\mu_5^{l,\theta} + \mu_6^{l,\theta}. \end{aligned}$$

Since it cannot be optimal to invest nothing into the illiquid asset (if $\alpha^I = 0$ then $\mu_5^{h,\theta} = \mu_6^{h,\theta} = 0$ since they cannot both strictly positive, and thus $\alpha_{k,\theta}^L \in (0,1)$, which contradicts that nothing was invested into the illiquid asset), we have $\alpha_{h,\theta}^L = 0$ iff $(1+r)l_\theta \geq R$ and $\alpha_{l,\theta}^L = 0$ iff $p(1+r)l_\theta \geq R$ (we assume that a bank does not convert illiquid investment into liquidity when the conditions hold as an equality).

Proof of Proposition 1

First, if there is full participation in the interbank market, then banks with excess liquidity must prefer to lend it out rather than reinvest it in the liquid asset. Lemma 2 gives the lower bound on the feasible interest rate such that this is the case. Moreover, banks with a liquidity shortage must prefer to obtain liquidity in the interbank market rather than by converting their illiquid investments into liquidity. Lemma 3 gives the upper bounds on the feasible interest rate where the lowest upper bound is given by a safer borrower since $l_s > l_r$.
To show sufficiency, assume that, contrary to the claim in the Proposition, $\frac{1}{p} \leq 1+r \leq \frac{R}{l_s}$ and participation in the market is not full. Since $1+r \leq \frac{R}{l_s} < \frac{R}{l_r}$ holds, we have, by Lemma 3, that both types of borrowers prefer to obtain liquidity in the market rather than by converting illiquid investments into liquidity. Since both types of borrowers are in the market, potential lenders face an average counterparty risk p. Since $p(1+r) \geq 1$, we have, by Lemma 2, that expected return on lending in the interbank market is higher than the return on reinvesting liquidity. Hence, both type of lenders also participate in the market. Contradiction.

Proof of Proposition 2

We need to check when the interest rate (equation (6)) is feasible under full participation (Proposition 1). The lower bound requires that $\frac{1}{p} \leq \frac{R}{\delta}$, or equivalently, $\delta \leq pR$, which is always satisfied since $\delta < 1$ and $pR > 1$. The upper bound is $\frac{R}{\delta} \leq \frac{R}{l_s}$, which simplifies to the condition in the Proposition.
To show sufficiency, assume that, contrary to the claim in the Proposition, $\frac{1}{\delta} \leq \frac{1}{l_s}$ and participation in the market is not full. By Proposition 1), the latter is equivalent to $p(1+r) < 1$ or $1+r > \frac{R}{l_s}$, where $1+r = \frac{R}{\delta}$. First inequality implies that $pR < \delta < 1$, a contradiction with the

assumption $pR > 1$. Second inequality implies that $\frac{R}{\delta} > \frac{R}{l_s}$, or, equivalently, $\frac{1}{\delta} > \frac{1}{l_s}$, a contradiction with the condition in the Proposition.

Proof of Corollary 1

Without counterparty risk, $p = 1$, there is no risk premium, $\delta = 1$. The Corollary follows immediately from propositions 2.

Proof of Lemma 4

See the proof of Lemma 1 and replacing r and p with r_r and p_r in (1) and (2). Type-h, s banks do not participate in the interbank market.

Proof of Lemma 5 and Lemma 6

See the proofs of Lemma 2 and Lemma 3 using $\mu^{k,\theta}$ from Lemma 4.

Proof of Proposition 3

The lower bound on the feasible interest rate in the interbank market is given by Lemma 5. The upper bound is given by Lemma 6.
To show sufficiency, assume that, contrary to the claim in the Proposition, the interbank interest rate is smaller or equal to $\frac{R}{l_r}$ and yet there is no adverse selection in market. Then, there are two possible cases: 1) either both types of borrowers participate in the market; or 2) none does. By Lemma 3, both types of borrowers are in the market if and only if $1 + r \leq \frac{R}{l_s}$. The interbank interest rate is given by $1 + r = \frac{R}{\delta}$. Since we are considering the range of parameters such that $\frac{1}{\delta} > \frac{1}{l_s}$ we have that $\frac{R}{\delta} > \frac{R}{l_s}$, a contradiction. By Lemma 6, there are no borrowers in the market if and only if the interbank interest rate is above $\frac{R}{l_r}$, a contradiction with the condition in the Proposition.

Proof of Proposition 4

We need to check when the interest rate (equation (11)) is feasible under adverse selection (Proposition 3). The lower bound requires that $\frac{1}{p_r} \leq \frac{R}{\delta_r}$ and the upper bound is $\frac{R}{\delta_r} \leq \frac{R}{l_r}$, which simplify to the conditions in the Proposition.

Proof of Proposition 5

At $t = 0$, banks choose their portfolio to maximize:

$$\begin{aligned}
\max_{0 \leq \alpha_{tr}^I \leq 1} \quad & \pi_l p[R\alpha_{tr}^I + qp_s(1 + r_{tr}^s)L_l + (1 - q)p_r(1 + r_{tr}^r)L_l - (1 - \lambda_l)d_2] \\
+ \quad & \pi_h qp_s[R\alpha_{tr}^I - (1 + r_{tr}^s)L_h - (1 - \lambda_h)d_2] \\
+ \quad & \pi_h(1 - q)p_r[R\alpha_{tr}^I - (1 + r_{tr}^r)L_h - (1 - \lambda_h)d_2] \text{ s.t.}
\end{aligned}$$

$$
\begin{aligned}
L_l &= (1-\alpha_{tr}^I) - \lambda_l d_1 \\
L_h &= \lambda_h d_1 - (1-\alpha_{tr}^I).
\end{aligned}
$$

The first-order condition with respect to α_{tr}^I yields:

$$\pi_l p[qp_s(1+r_{tr}^s) + (1-q)p_r(1+r_{tr}^r)] + \pi_h[qp_s(1+r_{tr}^s) + (1-q)p_r(1+r_{tr}^r)] = pR. \tag{25}$$

In equilibrium, lenders must be indifferent between lending to safer or riskier borrowers:

$$p_s(1+r_{tr}^s) = p_r(1+r_{tr}^r). \tag{26}$$

Combining (25) and (26) results in the interest rates given in the proposition.

Proof of Proposition 6

Comparing (19) and (18), we see that the cost of partial guarantees exceeds the cost of full guarantees if and only if:

$$l_s > \frac{\hat{p}-p}{1-p}.$$

Since the participation constraint of safe borrowers is binding at the interest rate r_{PG}, we know that $l_s = \hat{p}\pi_l + \pi_h$ (see Proposition 2). Thus, the condition above can be written as:

$$\hat{p}\pi_l + \pi_h > \frac{\hat{p}-p}{1-p},$$

which simplifies to $\hat{p} < 1$ and hence the claim in the Proposition follows.

Appendix B

In this Appendix, we show that the three regimes in the interbank market described above also arise in a model in which the two types of long-term investments can be converted into liquidity at the same rate but differ in their long-run returns. That is, we assume that $R_s < R < R_r$ and $l_s = l_r = l < 1$. We retain the assumption that safer investments are more likely succeed: $p_s > p > p_r$ where $p = qp_s + (1-q)\,p_r$. Hence, safer investments are characterized by a lower long-run return but a higher success probability than riskier investments. We let R denote the expected $t = 2$ return on the illiquid investments: $R \equiv qp_sR_s + (1-q)\,p_rR_r$.
One possible interpretation of the liquidation technology is the ability to recover some of the original investment made at $t = 0$. This is costly in that the liquidation return is lower per unit than the original investment: l <1. By contrast, in the set-up developed above we interpret the liquidation technology as the ability to gather some of the *future* returns on the long-term investments already at $t = 1$. Since the long-run returns are not yet realized, this is costly: $l_s < 1$ and $l_r < 1$.
We derive different regimes in the interbank market: 1) full participation and no impairment to market functioning; 2) adverse selection and higher interest rates; and 3) market breakdown. We follow the same solution technique as before. We start with the decisions taken by the banks at $t = 1$ and derive the price of liquidity from banks' portfolio allocation at t =0.

Interbank market regimes

Having received liquidity shocks, $k = l, h$, and being privately informed about the risk of their illiquid investment, $\theta = s, r$, banks need to manage their liquidity at $t = 1$ in order to maximize profits at $t = 2$. At $t = 1$, type-$l_{s,r}$ banks solve the following problem:

$$\begin{array}{cr}
\max\limits_{\alpha_l^L,\alpha_l^R,L_l} p_{s,r}[R_{s,r}(1-\alpha_{l,\theta}^L)\alpha^I + \alpha_{l,\theta}^R\left(1-\alpha^I\left(1-\alpha_{l,\theta}^L l\right)\right) + \tilde{p}(1+r)L_l - (1-\lambda_l)d_2] & \\
\lambda_l d_1 + \alpha_{l,\theta}^R\left[(1-\alpha^I) + \alpha_{l,\theta}^L\alpha^I l\right] + L_l \leq (1-\alpha^I) + \alpha_{l,\theta}^L\alpha^I l \qquad [\mu_1^l] & \\
L_l \geq 0 \qquad [\mu_2^l] & \\
0 \leq \alpha_{l,\theta}^L \leq 1 \qquad [\mu_3^l, \mu_4^l] & \\
0 \leq \alpha_{l,\theta}^R \leq 1 \qquad [\mu_5^l, \mu_6^l] &
\end{array} \tag{27}$$

Similarly, type-$h_{s,r}$ banks solve:

$$\begin{array}{cr}
\max\limits_{\alpha_h^L,\alpha_h^R,L_h} p_{s,r}[R_{s,r}(1-\alpha_{h,\theta}^L)\alpha^I + \alpha_{h,\theta}^R\left(1-\alpha^I\left(1-\alpha_{h,\theta}^L l\right)\right) - (1+r)L_h - (1-\lambda_h)d_2] & \\
\lambda_h d_1 + \alpha_{h,\theta}^R\left[(1-\alpha^I) + \alpha_{h,\theta}^L\alpha^I l\right] \leq (1-\alpha^I) + \alpha_{h,\theta}^L\alpha^I l + L_h \qquad [\mu_1^h] & \\
L_h \geq 0 \qquad [\mu_2^h] & \\
0 \leq \alpha_{h,\theta}^L \leq 1 \qquad [\mu_3^h, \mu_4^h] & \\
0 \leq \alpha_{h,\theta}^R \leq 1 \qquad [\mu_5^h, \mu_6^h] &
\end{array} \tag{28}$$

The first-order conditions for a type-$l_{s,r}$ with respect to L_l, α_l^L and α_l^R are:

$$\begin{aligned} p_{s,r}\tilde{p}(1+r) - \mu_1^l + \mu_2^l &= 0 \\ -p_{s,r}R_{s,r}\alpha^I + \mu_1^l \alpha^I l + \mu_3^l - \mu_4^l &= 0 \\ p_{s,r}(1-\alpha^I) - \mu_1^l(1-\alpha^I) + \mu_5^l - \mu_6^l &= 0 \end{aligned}$$

Similarly, the first-order conditions for a type-$h_{s,r}$ bank are given by:

$$\begin{aligned} -p_{s,r}(1+r) + \mu_1^h + \mu_2^h &= 0 \\ -p_{s,r}R_{s,r}\alpha^I + \mu_1^h \alpha^I l + \mu_3^h - \mu_4^h &= 0 \\ p_{s,r}(1-\alpha^I) - \mu_1^h(1-\alpha^I) + \mu_5^h - \mu_6^h &= 0 \end{aligned}$$

Regime I: Full participation of borrowers and lenders

Suppose there is full participation of borrowers and lenders in the interbank market so that $\tilde{p} = p$. It is easy to state the counterparts of the Lemma 1 and Lemma 2 in this case. We have that the marginal value of liquidity is $\mu_1^l = p_{s,r}p(1+r)$ for a lender and $\mu_1^h = p_{s,r}(1+r)$ for a borrower. Also, a borrowers does not reinvest in the liquid asset: $\alpha_{h,\theta}^R = 0$. A lender does not reinvest in the liquid asset if and only if $p\,(1+r) \geq 1$.

Next, we consider the decisions to convert long-term investments into liquidity. Conditions are:

$$1 + r > \frac{R_{s,r}}{l} \text{ and } p\,(1+r) > \frac{R_{s,r}}{l}$$

for banks in need of liquidity and those with excess liquidity, respectively (compare with Lemma 3). Note that $\frac{R_r}{l} > \frac{R_s}{l}$ and $\frac{R_r}{pl} > \frac{R_s}{pl} > \frac{R_s}{l}$ holds so that safer banks in need of liquidity would be the first to convert their illiquid investments.

An interval of feasible interbank interest rates is thus:

$$\frac{1}{p} \leq 1 + r \leq \frac{R_s}{l}, \tag{29}$$

where the lower bound is given by the participation constraint of the lender and the upper bound is given by the participation constraint of the safer borrower (see also Proposition 1).

Turning to the banks' optimization problem at $t = 0$, we have that:

$$\begin{aligned} \max_{\alpha^I} \quad & \pi_l R\alpha^I + \pi_l p\,[p(1+r)L_l - (1-\lambda_l)d_2] \\ & + \pi_h R\alpha^I - \pi_h p[(1+r)L_h + (1-\lambda_h)d_2] \end{aligned} \tag{30}$$

where $L_l = (1-\alpha^I) - \lambda_l d_1$ and $L_h = \lambda_h d_1 - (1-\alpha^I)$. Taking first-order condition with respect to α^I yields:

$$(1+r)\delta = \frac{R}{p}$$

where $\delta = p\pi_l + \pi_h$.

Combining the expression with the condition $1 + r < \frac{R_s}{l}$ yields that $\frac{R}{p\delta} < \frac{R_s}{l}$ must hold in Regime I so that the opportunity cost of liquidation for safer borrowers are higher than the cost of borrowing in the unsecured interbank market. Since $R_s < R$, the necessary condition is $l < p\delta$. Note that with full participation in the interbank market, the participation constraint of lenders never binds since: $\frac{R}{p\delta} > \frac{R}{p} > \frac{1}{p}$ (compare to Proposition 2).

Regime II: Adverse selection of borrowers in the interbank market
This is the case in which only riskier banks in need of liquidity borrow in the interbank market since safer banks find the interest rate in the unsecured interbank market too high to pay. Hence, we have that $L_{h,s} = 0$ and $\alpha^L_{h,s} = 1$. Moreover, it must be that $0 < \alpha^R_{h,s} < 1$, otherwise type-h, s banks either do not pay out d_1 at $t = 1$ or d_2 at $t = 2$. Thus, $\mu_5^{l,s} = \mu_6^{l,s} = 0$. It follows that $\mu_1^{h,s} = p_s$ and $\mu_2^{h,s} = p_s r$ and the resource constraint of the type-h, s bank binds.
Next, we consider decisions to convert the long-term investments into liquidity. These are given by:

$$1 + r > \frac{R_r}{l} \text{ and } p_r (1 + r) > \frac{R_{s,r}}{l}.$$

It follows that an interval of feasible interbank interest rates under adverse selection is:

$$\frac{1}{p_r} \leq 1 + r \leq \frac{R_r}{l}. \tag{31}$$

This is a counterpart of Proposition 3.
As in the analysis in the main text, there can be two cases in the adverse selection regime: 1) a case in which none of the lenders convert illiquid investments into liquidity, $1 + r \leq \frac{R_s}{p_r l} < \frac{R_r}{p_r l}$; and 2) a case in which safer lenders choose to convert their illiquid investments and to lend excess liquidity in the interbank market, $1 + r > \frac{R_s}{p_r l}$. We will focus on the former case as the other case does not add any new features to the results. Moreover, it did not seem to play a central role in the interbank market developments in the 2007-09 crisis. This is because liquidity hoarding, which we document above, cannot occur in this case: $p_r (1 + r) > \frac{R_s}{l} > 1$. We therefore proceed under the assumption that $p_r (1 + r) \leq \frac{R_s}{l}$. A sufficient condition for this to hold is $R_s > p_r R_r$, i.e. getting the safer return R_s for sure is more profitable the expected return on the riskier investment, $p_r R_r$.
At $t = 0$, banks' optimization problem is:

$$\begin{aligned} \max_{\alpha^I} \qquad & \pi_l R \alpha^I + \pi_l p \left(p_r (1 + r_r) L_l - (1 - \lambda_l) d_2\right) \\ & \pi_h (1 - q)\, p_r \left[R_r \alpha^I - (1 + r_r) L_{h,r} - (1 - \lambda_h) d_2\right] \end{aligned} \tag{32}$$

where $L_l = (1 - \alpha^I) - \lambda_l d_1$ and $L_{h,r} = \lambda_h d_1 - (1 - \alpha^I)$. Taking the derivative with respect to α^I and re-arranging, we get:

$$1 + r_r = \frac{\pi_l R + \pi_h (1 - q)\, p_r R_r}{p_r \left[q \pi_l p_s + (1 - q)(\pi_l p_r + \pi_h)\right]}.$$

We now check under which conditions this is an equilibrium, i.e. $\frac{1}{p_r} \leq 1 + r_r \leq \frac{R_r}{l}$ is satisfied. We have that $1 + r_r \geq \frac{1}{p_r}$ if and only if:

$$\pi_l \left[q p_s (R_s - 1) + p_r (1 - q)(R_r - 1)\right] \geq \pi_h (1 - q)(1 - p_r R_r).$$

Note that a sufficient condition is $p_r R_r > 1$ since then the right-hand side is negative while the left-hand side is positive. We have that $1 + r_r \leq \frac{R_r}{l}$ if and only if:

$$\frac{\pi_l R + \pi_h (1-q) p_r R_r}{q\pi_l p_s + (1-q)(\pi_l p_r + \pi_h)} \leq \frac{p_r R_r}{l}.$$

Regime III : Market breakdown
Lenders drop out: Lenders prefer to hoard liquidity by reinvesting it in the liquid asset when the lower bound in (31) is violated, i.e.

$$p_r (1 + r_r) < 1,$$

or, equivalently,

$$\pi_l [q p_s (R_s - 1) + p_r (1-q)(R_r - 1)] < \pi_h (1-q)(1 - p_r R_r).$$

Note that the necessary condition for this inequality to hold is that the expected return on the riskier illiquid investment is ex ante unprofitable: $p_r R_r < 1$. This is the same necessary condition for liquidity hoarding to occur as the one obtained in the set-up considered in the main text.
Borrowers drop out: Risky borrowers choose to leave the unsecured interbank market if adverse selection drives the interest rate up too much. The upper bound on the interest rate in (31) is violated when:

$$1 + r_r > \frac{R_r}{l},$$

or, equivalently,

$$\frac{\pi_l R + \pi_h (1-q) p_r R_r}{q\pi_l p_s + (1-q)(\pi_l p_r + \pi_h)} > \frac{p_r R_r}{l}.$$