# The Role of Adverse Selection and Liquidity in Financial Crisis

preliminary and incomplete

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**Abstract**

This paper develops a model that illustrates how adverse selection in asset market can lead to the increased asset price volatility and possibly to the breakdown of trade. The asymmetric information about the asset returns generates the Akerlof's "lemons" problem when buyers do not know whether the asset is sold because of its low quality or because the seller experienced a sudden need for liquidity. The adverse selection can lead to an equilibrium with no trade reflecting the buyers' belief that most assets that are offered for sale are of low quality. I analyze the role of market liquidity and beliefs about the likelihood of the crisis in amplifying the effect of adverse selection.

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# 1 Introduction

In the current crisis of 2007-2008, the market for securities backed by subprime mortgages was the first to suffer the sudden dry up in liquidity. Some of the possible explanations for "frozen" markets are increased uncertainty and information asymmetries about the true value of an asset.[1] In particular, the difficulty in assessing the fundamental value of securities may lead to the adverse selection issue.

Collateralized debt obligations (CDOs) written on subprime mortgages had skewed payoffs: they offered high expected return in most states of nature but suffered substantial losses in extremely bad states. When economy is in a normal state with strong fundamentals, the asymmetric information does not significantly affect the value of mortgage backed securities (MBS). However, when an economy is subject to a negative shock, the value of the security becomes more sensitive to private information and the adverse selection may influence the trading decisions. (Morris and Shin [10]) When subprime mortgage defaults had increased in February 2007,[2] a large fraction of CDOs have been downgraded[3]. The impact of declining housing prices on MBS depended on the exact composition of mortgages that backed the securities, some MBS were affected more than others. Due to the complexity of structured financial products and heterogeneity of the underlying asset pool, owners have an informational advantage in estimating how much those securities are worth. This asymmetric information about the true value of the asset generates the lemons problem[4]: a buyer does not know whether the seller is selling the security because of a sudden need for liquidity, or

[1] The junior equity tranches (also referred to as "toxic waste") were usually held by the issuing bank; they were traded infrequently and were therefore hard to value. Also, these structured finance products received overly optimistic ratings from the credit rating agencies. One of the reason the underlying securities default risks were underestimated is that the statistical models were based on the historically low mortgage default and delinquency rates.(Brunnermeier [5])

[2] This increase in subprime mortgage defaults triggered the liquidity crisis in February 2007. (Brunnermeier [5])

[3] "27 of the 30 tranches of asset-backed collateralized debt obligations underwritten by Merrill Lynch in 2007, saw their triple-A ratings downgraded to “junk” Overall, in 2007, Moody’s downgraded 31 percent of all tranches for asset-backed collateralized debt obligations it had rated and 14 percent of those initially rated AAA." (Coval, Jurek and Stafford [6])

[4] Akerlof (1970)

because the seller is trying to get rid of the toxic assets. This adverse selection issue can lead to the market illiquidity reflecting buyers' beliefs that most securities offered for sale are of low quality.[5]

As market condition worsened, investors' value for liquidity had increased which was reflected in the high spreads of MBS relative to Treasuries (Krishnamurthy [8]). The flight to liquidity can amplify the effect of adverse selection during the crisis leading to the increased asset price volatility and possibly to the complete breakdown of trading. As market liquidity falls, it becomes difficult to find trading partners which leads to the fire-sale pricing.[6] The deleveraging that accompanies the initial shock can further aggravate the adverse selection problem.[7] Because of the losses on their MBS, some banks became undercapitalized; however, their attempts to recapitalize push their market price further down. This reflects the investors' fear that any bank that issues new equity or debt may be overvalued, leading to the liquidity crunch.

In this paper, I develop a model that illustrates how adverse selection in an asset market can lead to an equilibrium with no trade during the crisis. Also, I analyze the role of market liquidity and expectations in amplifying the effect of adverse selection.

In my model, agents have the Diamond-Dybvig[8] type of preferences: they consume in period one or in period two, depending on whether they receive a liquidity shock in period one. In period zero, investors choose how much to invest into risky long-term assets which have idiosyncratic payoffs. In period one, liquidity shocks are realized and, subsequently, risky investments are traded in the financial market. The late consumers (who have not experienced a liquidity shock) are the buyers in the financial market.

I begin by examining the portfolio choice when investors have private information about

[5] Krishnamurthy [8] identifies adverse selection issue as one of the diagnosis of the current crisis: market participants may fear that if they transact they will be left with a "lemon".

[6] The haircut on ABSs increased from 3-5% in August 2007 to 50-60% in August 2008. The haircut on equities increased from 15% to 20% for the same period. (Gorton and Metrick (2009))

[7] "The large haircuts on some securities could be seen as a response by leveraged entitites to the potential drying up of trading possibilities in the asset-backed securities (ABS) market. The equity market, in contrast, is populated mainly with non-leveraged entities such as mutual funds, pension funds, insurance companies and households, and hence is less vulnerable to the drying up of trading partners." Morris and Shin [10]

[8] Diamond and Dybvig (1983)

their investment payoff and it is public information which investors have received a liquidity shock. Then I analyze the situation when the identity of investors hit by a liquidity shock is private information. In the latter case, investors can take advantage of their private information by selling the low-payoff investments and keeping the ones with high payoffs. This leads to the lemons problem. If market is liquid then informed investors can gain from trading on private information by pretending to be liquidity traders (investors who experienced a liquidity shock). However, if the fraction of low quality assets offered for sale is sufficiently large then the adverse selection can lead to the market illiquidity.

Following the Allen and Gale "cash-in-the-market" framework,[9] in my model liquidity depends on the amount of the safe asset held by the investors that is available to buy risky assets from liquidity traders. The market liquidity, defined as the demand for risky investments in the interim period, depends on the investors liquidity preference. Allen and Gale [3] show that the "cash-in-the-market" pricing[10] leads to the market prices below fundamentals if the preference for liquidity is high. I demonstrate that the presence of adverse selection in the market can further depress the market prices exacerbating asset price volatility. As a result, during the crisis the asset is priced below its expected payoff, which can lead to a no trade equilibrium.

I show that if a crisis is accompanied by the flight to liquidity (increase in investors' liquidity preference), the effect of adverse selection can be amplified leading to a fire-sale pricing or a breakdown of trade during the crisis. Furthermore, I show that underestimating the likelihood of the crisis can aggravate the adverse selection effect as well. Next, I analyze the investment choice from the central planner prospective. The central planner can improve upon the market allocation by reducing the lemons problem.

This paper is organized as follows. In the next section, I discuss the related literature. Section 3 describes the model environment, and Section 4 characterizes the equilibrium.

[9] The amount of cash in the market depends on the participants liquidity preference. The higher the average liquidity preference of investors in the market, the greater is the average level of the safe assets in portfolios and the greater is the market ability to absorb liquidity trading without large price changes. (Allen and Gale [1])

[10] The equilibrium price of the risky asset is equal to the lesser of two amounts: the discounted value of future dividends and the amount of cash available from buyers divided by the number of shares sold.

Section 5 concludes the paper and outlines the directions for future research. All results are proved in the Appendix.

## 2 Related Literature

Morris and Shin [10] show that adverse selection can lead to the failure of trade. They analyze a coordination game among differently informed traders. If the condition of approximate common knowledge of an upper bound on expected losses fails then traders withdraw from trade because they fear their uninformed partners may refrain from trade. Adverse selection reverberates throughout the information structure and gets amplified in the process, leading to a breakdown in trade.

Easley and O'Hara [7] show that uncertainty about the true value of an asset can lead to a no-trade equilibrium when investors have incomplete preferences over portfolios. They suggest alternatives for valuing assets in illiquid markets since mark-to-market accounting becomes problematic in an uncertain environment.

Krishnamurthy [9] examines two amplification mechanisms that operate during liquidity crises. The first mechanism involves asset prices and balance sheets: a negative shock to agents' balance sheets causes them to liquidate assets, lowering prices, further deteriorating balance sheets and amplifying the shock. The second mechanism involves investors' Knightian uncertainty: shocks to financial innovations increase agents' uncertainty about their investments, causing them to disengage from risk and seek liquid investments, which amplifies the crisis.

Allen and Gale ([1], [2], [3], [4]) developed a liquidity-based approach to study financial crises. When supply and demand for liquidity are inelastic in the short run, a small degree of aggregate uncertainty can have a large effect on asset prices and lead to financial instability.

My paper contributes to the literature by analyzing the interaction between adverse selection and liquidity, and their role during the crisis. I show that the adverse selection leads to lower market liquidity and asset price volatility even if there is no aggregate uncertainty about liquidity preferences. The aggregate uncertainty about liquidity amplifies the effect of adverse selection, potentially resulting in a breakdown of trade.

# 3 Model

I consider a model with three dates indexed by $t = 0, 1, 2$. There is a continuum of ex-ante identical agents with an aggregate Lebesgue measure of unity. There is only one good in the economy that can be used for consumption and investment. All agents are endowed with $\omega$ units of good at date $t = 0$, and nothing at the later dates.

## 3.1 Preferences

Agents consume at date one or two, depending on whether they receive a liquidity shock at date one. The probability of receiving a liquidity shock in period one is denoted by $\lambda$. So $\lambda$ is also a fraction of investors hit by a liquidity shock. Investors who receive a liquidity shock have to liquidate their risky long-term asset holdings and consume all their wealth in period one. So they are effectively early consumers who value consumption only at date $t = 1$. I will also refer to them as liquidity traders. The rest are the late consumers who value the consumption only at date $t = 2$.

Investors have Diamond-Dybvig type of preferences:

$$U(c_1, c_2) = \lambda u(c_1) + (1 - \lambda)u(c_2) \tag{1}$$

where $c_t$ is the consumption at dates $t = 1, 2$. In each period, investors have logarithmic utility: $u(c_t) = \log c_t$.

## 3.2 Investment technology

Agents have access to two types of constant returns investment technologies. One is a storage technology (also called the safe asset or cash), which has zero net return: one unit of safe asset pays out one unit of safe asset in the next period. Another type of technology is a long-term risky investment project (also called a risky asset). In period two, the risky investment in project $i$. Each investor $i$ has a choice of starting his own investment project $i$ by investing a fraction of his endowment. The investor can start only one project, and each project has only one owner. Each investment project $i$ has a random payoff of $R = R^m + R^i$ per unit of investment where $R^m$ represents the market (aggregate) productivity and $R^i$ is an idiosyncratic (investment specific) productivity. The idiosyncratic productivity realizations are independent across investments.

There are two states of nature $s = 1$ and $s = 2$ that are revealed at $t = 1$. The state 1 is a normal state and the state 2 is a crisis state. These states are realized with ex-ante probabilities $(1 - q)$ and $q$. I will also use the notation $q_1 = 1 - q$ and $q_2 = q$.

The market payoff $R^m$ is a random variable that takes two values: $R_1^m$ with probability $q_1$ and $R_2^m$ with probability $q_2$ where $R_1^m \geq R_2^m$. The idiosyncratic payoff of each investment $i$ is an independent realization of a random variable $R^i$ that takes two values: a low value $R^L$ with probability $\pi_s$ and a high value $R^H$ with probability $(1 - \pi_s)$ where $s \in \{1, 2\}$. Denote the investment payoff with low idiosyncratic productivity in state $s$ as $R_L(s)$ and the investment payoff with high idiosyncratic productivity in state $s$ as $R_H(s)$.

The state 1 is a normal state where the fraction of low quality assets is small: $\pi = \pi_1$. The state 2 is a crisis state with a significantly larger fraction of low quality assets: $\pi = \pi_2 > \pi_1$ and a lower market productivity is $R_2^m \leq R_1^m$.

The expected payoff of each individual risky project in state $s$ is denoted by $\overline{R}_s = \pi_s R_L(s) + (1 - \pi_s) R_H(s)$ with $R_L(s = 1) < 1 < R_H(s = 2)$. The expected payoff is denoted by $\overline{R} = (1 - q)\overline{R}_1 + q\overline{R}_2$ with $\overline{R} > 1$. The long-term asset can be liquidated prematurely at date $t = 1$, in this case, one unit of the risky asset yields $r_s$ units of the good, where $R_L(s) < r_s < 1$. The holdings of the two-period risky asset can be traded in financial market at date $t = 1$. Figure 1 summarizes the payoff structure.

| time | 0 | 1 | 2 |
|---|---|---|---|
| safe asset | 1 | 1 | 1 |
| risky asset | 1 | $r_s$ | $R_i(s)$ |

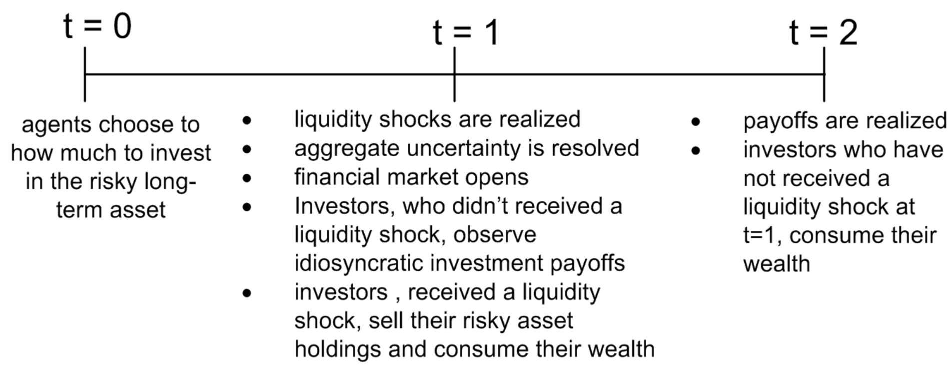
*Figure 1.* Payoff structure.

### 3.3 Information

At date $t = 0$, investors make investment choices between the two technologies, safe and risky, in proportion $x$ and $(1 - x)$ respectively. They choose their asset holdings to maximize their expected utility.

At date $t = 1$, the liquidity shocks and the aggregate state are realized, and the financial market opens. If investors have not received a liquidity shock, they privately observe the idiosyncratic component of the payoff on investment they own. The supply of the risky asset comes from the investors who have experienced a liquidity shock. The demand for risky asset comes from investors who have not received a liquidity shock.

The timeline of the model is summarized in the figure below.




I will consider two cases. In the first case, it is public information which investors have experienced a liquidity shock. If an investor gets a liquidity shock, he sells or liquidates his holdings of the risky asset in order to consume as much as possible in period one. If an investor is not hit by a liquidity shock and learns that his investment has low payoff, he can liquidate it, receiving $r$ units of the good per unit of investment.

In the second case, the identity of investors hit by a liquidity shock is private information. Therefore, after observing investment payoffs, agents can take advantage of this private information by selling low quality projects in the market at date $t = 1$. In this case, buyers are not able to distinguish whether an investor is selling his asset holdings because of its low payoff or because of the liquidity needs. This generates adverse selection problem, and leads to a discount on the investments sold before maturity.

# 4 Equilibrium

## 4.1 Equilibrium without Adverse Selection

First, I consider the case where the identity of investors hit by a liquidity shock is public information. Therefore, there is no adverse selection. All risky assets at t=1 are sold by liquidity traders who cannot wait for the maturity of their investments at date $t = 2$.

Since all the investments have idiosyncratic payoffs, the expected payoff of the risky asset sold in period one is $\overline{R}_s$ in state $s$. All risky assets sold at $t = 1$ are aggregated in the market, hence, the variance of the asset bought at date $t = 1$ is zero. Therefore, the return on risky asset bought in period one is $\overline{R}_s/p_s$, where $p_s$ is the market price in state $s$. The late consumers will be willing to buy risky asset at date $t = 1$ if the market price $p_s$ is less than the expected payoff $\overline{R}_s$. The earlier consumers will be willing to sell their projects if the market price $p_s$ is greater than the liquidation value $r_s$.

At date $t = 0$, investors choose the investment allocations between the risky and safe technologies, in proportion $x$ and $(1 - x)$ respectively, in order to maximize their expected utility.

$$\lambda \log c_1 + (1-\lambda) \sum_{s=1,2} q_s \left(\pi_s \log c_{2L}\left(s\right) + (1-\pi_s) \log c_{2H}\left(s\right)\right) \tag{2}$$

$$\begin{aligned}
s.t. \quad (i) \quad & c_1\left(s\right) = \begin{cases} 1 - x + p_s x & if \quad p_s \geq r_s \\ 1 - x + rx & if \quad p_s < r_s \end{cases} \\
(ii) \quad & c_{2H}\left(s\right) = \begin{cases} xR_H(s) + y_s \overline{R}_s & if \quad p_s \geq r_s \\ xR_H + (1-x) & if \quad p_s < r_s \end{cases} \\
(iii) \quad & c_{2L}\left(s\right) = \begin{cases} xr_s + y_s \overline{R}_s & if \quad p_s \geq r_s \\ xr_s + (1-x) & if \quad p_s < r_s \end{cases}
\end{aligned}$$

The consumption of early consumers in state $s$ is denoted by $c_1\left(s\right)$ and the consumption of late consumers in state $s$ is denoted by $c_{2j}\left(s\right)$ where $j = L, H$ refers to payoff of an investment project $i$.

The late consumers will be willing to buy risky assets at $t = 1$ if the market price $p$ is less than the expected payoff $\overline{R}$. Therefore, the demand for risky asset at $t = 1$ in state $s$ is given by

$$y\left(s\right) = \begin{cases} \frac{1-x}{p_s} & if \quad p_s \leq \overline{R}_s \\ 0 & if \quad p_s > \overline{R}_s \end{cases} \tag{3}$$

Therefore, the aggregate demand at $t = 1$ in state $s$ is given by

$$D\left(s\right) = \begin{cases} (1-\lambda) \frac{1-x}{p_s} & if \quad p_s \leq \overline{R}_s \\ 0 & if \quad p_s > \overline{R}_s \end{cases} \tag{4}$$

The earlier consumers will be willing to sell their projects if the market price $p$ is greater than the liquidation value $r$. Therefore, the aggregate supply at $t = 1$ in state $s$ is given by

$$S(s) = \begin{cases} \lambda x & if \quad p_s \geq r_s \\ 0 & if \quad p_s < r_s \end{cases} \tag{5}$$

The price in state is determined by the market clearing conditions:

$$\lambda x p_s = (1 - \lambda)(1 - x) \tag{6}$$

Since the investment allocations are determined at $t = 0$ and there are no aggregate uncertainty about the probability of a liquidity shock $\lambda$, the price of the risky asset sold at t=1 is the same in both states: $p_1 = p_2 \equiv p$ where

$$p = \frac{(1 - \lambda)}{\lambda} \frac{(1 - x)}{x}. \tag{7}$$

The following assumptions on asset returns parameters are maintained throughout.

This assumption ensures that a risky investment is always more productive than the safe asset.

Assumption 1. $r \geq \underline{r} : EU(p(\underline{r}), x(\underline{r})) \geq EU(p(\underline{r}), 1)$

This assumption rules out the situation when a risky asset dominates the safe asset at $t = 1$. If $r < \underline{r}$ then the market price at $t = 1$ is greater than one, therefore, no one will choose to hold the safe asset at $t = 0$. In particular, this assumption implies that $r > R_L(s)$.

Assumption 2. $r \leq \overline{r} : EU(p(\overline{r}), x(\overline{r})) \geq EU(p(\overline{r}), 0)$

This assumption rules out the situation when the safe asset dominates a risky asset at $t = 1$. If $r > \overline{r}$ then the return on the risky asset bought at $t = 1$ is higher that the return on investment made at $t = 0$, so no one will choose to invest in risky projects at $t = 0$. In particular, this assumption implies that $r < 1$.

**Proposition 1.** *If assumption 1 and 2 are satisfied, then there exists a unique equilibrium, and the equilibrium allocation into long-term risky investment $x$ and the market price of investment sold at date one $p$ are given by*

$$p = \frac{\lambda + \sum_{s=1,2} q_s \left( \pi_s \frac{\overline{R}_s}{r_s + \overline{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{\overline{R}_s}{R_H(s) + \overline{R}_s \frac{\lambda}{(1-\lambda)}} \right)}{\lambda + \sum_{s=1,2} q_s \left( \pi_s \frac{r_s}{r_s + \overline{R}_s \frac{\lambda}{(1-\lambda)}} + (1 - \pi_s) \frac{R_H(s)}{R_H(s) + \overline{R}_s \frac{\lambda}{(1-\lambda)}} \right)} \tag{8}$$

$$x = \begin{cases} (1-\lambda)\left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r_s}{\left(r_s + \overline{R}_s \frac{\lambda}{1-\lambda}\right)} + (1-\pi_s) \frac{R_H(s)}{\left(R_H(s) + \overline{R}_s \frac{\lambda}{1-\lambda}\right)}\right)\right) & if \quad p \geq r_s \\ (1-\lambda)\left(1 - \sum_{s=1,2} q_s \pi_s\right) \frac{1}{(1-r_s)} + \left(\lambda + (1-\lambda) \sum_{s=1,2} q_s \pi_s\right) \frac{1}{(1-R_H(s))} & if \quad p < r_s \end{cases} \tag{9}$$

*Furthermore, the investment allocation and welfare are larger in the market equilibrium (when $p \geq r_s$) relative to an equilibrium with no trade (when $p < r_s$).*

The equilibrium consumption of early consumers is the same in both states and is given by:

$$c_1(s) = \begin{cases} \frac{(1-x)}{\lambda} & if \quad p \geq r_s \\ \left(\lambda + (1-\lambda) \sum_{s=1,2} q_s \pi_s\right) \frac{R_H(s) - r_s}{R_H(s) - 1} & if \quad p < r_s \end{cases} \tag{10}$$

The consumption of late consumers with low payoff investment in state $s$ is given by

$$c_{2L}(s) = \begin{cases} x\left(r_s + \overline{R}_s \frac{\lambda}{1-\lambda}\right) & if \quad p \geq r_s \\ \left(\lambda + (1-\lambda) \sum_{s=1,2} q_s \pi_s\right) \frac{R_H(s) - r_s}{R_H(s) - 1} & if \quad p < r_s \end{cases} \tag{11}$$

The consumption of late consumers with high payoff investment in state $s$ is given by

$$c_{2H}(s) = \begin{cases} x\left(R_H(s) + \overline{R}_s \frac{\lambda}{1-\lambda}\right) & if \quad p \geq r_s \\ (1-\lambda)\left(1 - \sum_{s=1,2} q_s \pi_s\right) \frac{R_H(s) - r_s}{1 - r_s} & if \quad p < r_s \end{cases} \tag{12}$$

## 4.2 Equilibrium with Adverse Selection

Now suppose the identity of investors who have received a liquidity shock is private information. Therefore, after observing investment payoff, agents can take advantage of this private information by selling low productive investments in the market at date t=1. This generates the adverse selection problem and therefore, leads to the discount on the price of risky assets sold at $t = 1$. Investors always can choose to liquidate the project if it yield a low payoff.

The investor who buys a risky asset at date $t = 1$, does not know whether it is sold due to the liquidity shock or because of its low payoff. The buyers believe that with probability $\lambda$ investment is sold due to a liquidity shock, and with probability $(1-\lambda)(1-\pi_s)$ it sold because of the low payoff. Hence, buyers believe that the payoff of the prematurely sold risky assets in state $s$ is $\widehat{R}_s$ such that

$$\widehat{R}_s = \frac{\lambda}{\lambda + (1-\lambda)\pi_s}\overline{R}_s + \frac{(1-\lambda)\pi_s}{\lambda + (1-\lambda)\pi_s} R_L \tag{13}$$

The late consumers will be willing to buy risky asset at t=1 if the market price $p$ is less than the expected payoff $\widehat{R}$. Therefore, the demand for risky asset at $t = 1$ is given by

$$y_s = \begin{cases} \frac{1-x}{p_s} & if \quad p_s \leq \widehat{R}_s \\ 0 & if \quad p_s > \widehat{R}_s \end{cases} \tag{14}$$

The earlier consumers will be willing to sell their projects if the market price $p_s$ is greater than the liquidation value $r$.

The price in state $s$ is determined by market clearing conditions:

$$(\lambda + (1-\lambda)\pi_s) x p_s = (1-\lambda)(1-x) \tag{15}$$

Therefore, the market price in state $s$ can be expressed as

$$p_s = \min\left\{\frac{(1-\lambda)}{(\lambda + (1-\lambda)\pi_s)}\frac{(1-x)}{x}, \widehat{R}_s\right\} \tag{16}$$

Note, that the price is no longer the same in both states since the fraction of low productive investments is larger in a crisis state: $\pi_2 > \pi_1$. Therefore, the price in the crisis state is lower than the price in the normal state: $p_2 < p_1$.

Investors choose their asset holdings $(x, 1-x)$ to maximize their expected utility:

$$\lambda \log c_1 + (1-\lambda)\sum_{s=1,2} q_s \left(\pi_s \log c_{2L}(s) + (1-\pi_s)\log c_{2H}(s)\right) \tag{17}$$

$$s.t. \quad (i) \quad c_1(s) = \begin{cases} 1 - x + p_s x & if \quad p_s \geq r_s \\ 1 - x + r_s x & if \quad p_s < r_s \end{cases}$$

$$(ii) \quad c_{2H}(s) = \begin{cases} xR_H(s) + (1-x)\widehat{R}_s/p_s & if \quad p_s \geq r_s \\ xR_H + (1-x) & if \quad p_s < r_s \end{cases}$$

$$(iii) \quad c_{2L}(s) = \begin{cases} xp_s + (1-x)\widehat{R}_s/p_s & if \quad p_s \geq r_s \\ xr_s + (1-x) & if \quad p_s < r_s \end{cases}$$

**Proposition 2.** *If assumptions 1 and 2 are satisfied then there exists a unique equilibrium. There are three possible equilibrium types:*

*I. equilibrium with market trading in both states;*

*II equilibrium with market trading in normal state $s = 1$ and no trade in a crisis state $s = 2$;*

*III. equilibrium with no trade in both states.*

*Furthermore, the presence of adverse selection leads to a lower level of investments $x$, and lower welfare relative to an equilibrium without adverse selection.*

The presence of adverse selection leads to the lower price level and price volatility across states. The market price in a crisis state is lower relative to the normal state since the fraction of low quality assets is larger. As a result, assets offered for sale at $t = 1$ have lower expected return. Informed investors benefit from the private information at the expense of liquidity traders. Furthermore, adverse selection leads to a loss in aggregate welfare since informed investors sell low productive investments instead of liquidating them.

Consider the special case when the equilibrium price in a normal state is equal to the expected payoff of risky asset: $p_1 = \widehat{R}_1$. The adverse selection generates asset price volatility, leading to the equilibrium price below expected payoff or to a no trade equilibrium in a crisis state. If the fraction of low quality asset $\pi_1$ is small then the effect of adverse selection is also small: $\frac{\overline{R}_1 - p_1}{\overline{R}_1} = \frac{(1-\lambda)\pi_1(1-\pi_1)}{\lambda+(1-\lambda)\pi_1}\frac{(R_H(s)-R_L(s))}{\overline{R}_1}$. The price of the asset in a crisis state is $p_2 = \frac{(\lambda+(1-\lambda)\pi_1)}{(\lambda+(1-\lambda)\pi_2)}\widehat{R}_1$ if there is trade. Then the effect of adverse selection on the asset price is given by

$$\frac{\overline{R}_2 - p_2}{\overline{R}_2} = \frac{(1-\lambda)\pi_2(1-\pi_2)}{\lambda+(1-\lambda)\pi_2}\frac{(R_H(s)-R_L(s))}{\overline{R}_2} + \frac{(\pi_2-\pi_1)}{\lambda+(1-\lambda)\pi_2}\frac{(R_L(s)-\lambda R_H(s))}{\overline{R}_2}; \tag{18}$$

$$\frac{\widehat{R}_2 - p_2}{\widehat{R}_2} = \frac{(\pi_2-\pi_1)\,(R_L(s)-\lambda R_H(s))}{\lambda(1-\pi_2)R_H(s)+\pi_2 R_L(s)}. \tag{19}$$

If the fraction of low quality asset $\pi_2$ is sufficiently large:

$$\pi_2 > \frac{\lambda}{(1-\lambda)}\frac{\widehat{R}_1 - r_1}{r_1} + \pi_1\frac{\widehat{R}_1}{r_1} \tag{20}$$

then there is no trading in a crisis state.

The equilibrium investment allocations is given by

$$x = \frac{(1-\lambda)}{(\lambda(1-\pi_1)R_H(s) + \pi_1 R_L(s)) + (1-\lambda)}. \tag{21}$$

#### 4.2.1 Properties of Equilibrium

**Probability of a crisis state** The probability of a crisis state $q$ reflects the investors' beliefs about the likelihood of a crisis. In this section, I examine how the equilibrium changes with respect to changes in $q$.

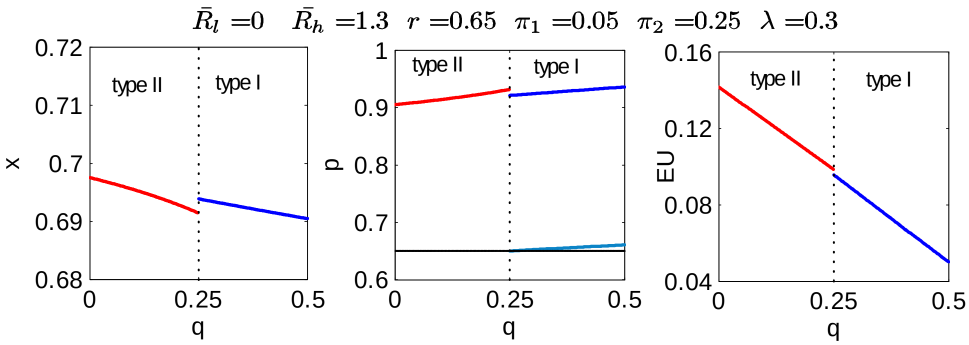
**Corollary 1.** *If investors believe a crisis state is more likely to occur ($q$ is larger) then (i) investment allocation is smaller; (ii) market prices are higher; (iii) expected utility is lower. If the economy is in a type II equilibrium with market trading in normal state and no trade in a crisis state then increase in $q$ may lead to shift a type I equilibrium with market trading in both states.*

The higher probability of a crisis state $q$ implies a higher probability of the asset becoming a lemon, which makes asset ex-ante less less profitable. Therefore, the increase in probability of a crisis state $q$ leads to a lower level of investment allocation and lower expected utility. The smaller investment at $t = 0$ implies a smaller supply and a larger demand for risky assets at $t = 1$. This leads to higher market prices (in both type I and II equilibria).

The fact that the market price is increasing in the probability of a crisis state $q$ makes it is possibility to move from one type of equilibrium to another. Suppose an economy is in type II equilibrium where there is no trade in a crisis state. Suppose the probability of a crisis state $q$ increased, i.e., investors believe a crisis is now more likely to occur. Then it is possible that the price in a crisis state will increase sufficiently to switch to type I equilibrium with market trading in both states. (If an economy is initially in type I equilibrium then the type of the equilibrium will not change if $q$ is increased. If an economy is in type II equilibrium and the probability $q$ is decreased then the equilibrium type will not change either.)

Consider the following numerical example. The asset return parameters are given $R_L(s) = 0$, $R_H(s) = 1.3$, $r_s = 0.65$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$ and in a crisis state $\pi_2 = 0.25$, and probability of a liquidity shock $\lambda = 0.3$.

In this example, 5% of assets become lemons (with zero payoff) in a normal state, and in a crisis state, the quarter of all assets are lemons. The figures below depicts the equilibrium values of investment, prices and expected utility as a function of probability of a crisis state $q$. At $q = 0.25$, there is a switch from an equilibrium with no trade in a crisis state to an equilibrium with trading in both state.




Therefore, the initial expectation can affect the type of equilibrium. Underestimating the likelihood of a crisis may result in a no-trade outcome if the crisis state is realized.

Suppose a probability of a crisis $q$ depends on the previously realized state. So that conditional probability of transition from a normal state to a crisis state is smaller than the conditional probability of remaining in a crisis state. The transition matrix is given by $\begin{bmatrix} 1-q_{12} & q_{12} \\ 1-q_{22} & q_{22} \end{bmatrix}$ where $q_{22} > q_{12}$ and $q_{jk} = \Pr(s = s_k | s = s_j)$. Then we can compare equilibria sequentially.

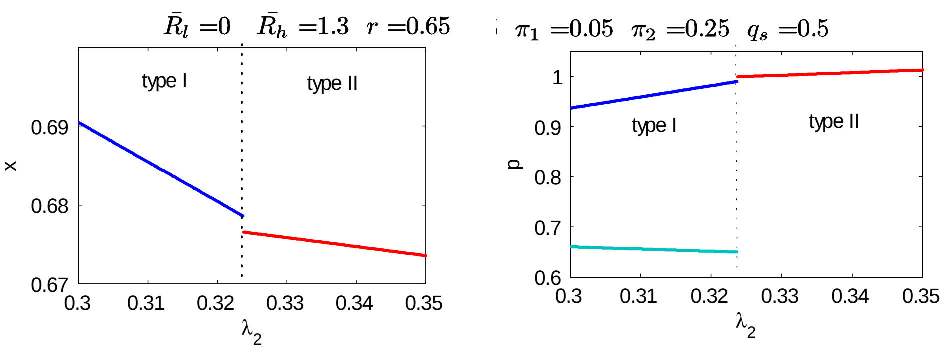
Let's look again at the numerical example considered before. Suppose $q_{11} = 0.1$ and $q_{22} = 0.5$. If an economy is in a normal state then it is in type II equilibrium: if the crisis is realized, there is no trading. Once an economy is in a crisis state, the beliefs are revised and investment allocation are adjusted, and an economy moves to the type I equilibrium. So, the market trading is resumed next period even if the crisis state persists.

**Liquidity preference** Now consider the situation when a crisis is accompanied by an exogenous increase in liquidity preference $\lambda$ in addition to a larger fraction of low quality assets.

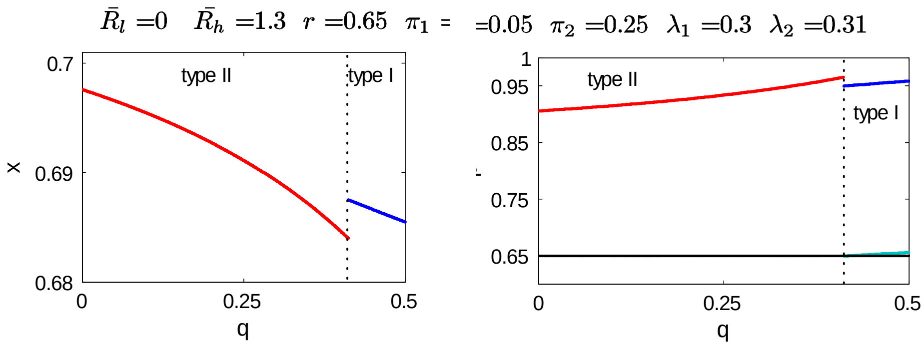
**Corollary 2.** *Suppose the economy is type I equilibrium with market trading in both states. The increase in liquidity preference $\lambda$ in a crisis state may lead to shift a type II equilibrium with market trading in normal state and no trade in a crisis state.*

The price is a decreasing function of preference for liquidity $\lambda$. Therefore, the higher preference for liquidity $\lambda$ in a crisis state results in the further decrease of the market price relative to a normal state. Hence, a lack of liquidity during the crisis may amplify the adverse selection problem pushing the asset prices further down and possibly leading to a complete breakdown of trade. This reflects the fire-sale phenomenon when depressed prices reflect the difficulty of finding buyers during the crisis.

Again consider the numerical example: $R_L(s) = 0$, $R_H(s) = 1.3$, $r_s = 0.65$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$ and in a crisis state $\pi_2 = 0.25$, and probability of a liquidity shock $\lambda = 0.3$. The figure below illustrates the effect of an increase in the liquidity preference in a crisis state $\lambda_2$ from 0.3 to 0.35 on the equilibrium investment and prices. When preference for liquidity is the same in both states $\lambda_1 = \lambda_2 = 0.3$, there is trading in both states. However, if $\lambda_2 > 0.325$ then there is no trade in a crisis state.




The next figure depicts the equilibrium investment and prices as a function of probability of a crisis state $q$ when the preference for liquidity in a crisis state is higher: $\lambda_1 = 0.3$ and $\lambda_2 = 0.31$. The threshold value of a crisis likelihood (where economy switches from type II to type I equilibrium) is larger relative to the case when the liquidity preference in both states are the same $\lambda_1 = \lambda_2 = 0.3$. If a crisis is accompanied by flight to liquidity, the adverse selection effect is magnified exacerbating the asset price volatility.




## 4.3 Central Planner *[incomplete]*

In this section, I analyze the equilibrium from the central planner prospective.

First, consider the case when it is public information which investor has received a liquidity shock. Then the central planner solves the following maximization problem:

$$\max_x \{\lambda \log c_1 + (1-\lambda) \sum_{s=1,2} q_s \left(\pi_s \log c_{2L}(s) + (1-\pi_s) \log c_{2H}(s)\right)\} \tag{22}$$

$$\begin{aligned} s.t. \quad (i) \quad & c_1 = \tfrac{1-x}{\lambda} \\ (ii) \quad & c_{2L}(s) = x\left(r_s + \overline{R}_s \tfrac{\lambda}{1-\lambda}\right) \\ (iii) \quad & c_{2H}(s) = x\left(R_H(s) + \overline{R}_s \tfrac{\lambda}{1-\lambda}\right) \end{aligned}$$

The optimal investment allocation is $x = (1-\lambda)$ and the consumption allocations are given by

$$\begin{aligned} c_1 &= 1 \\ c_{2L}(s) &= (1-\lambda) r_s + \lambda \overline{R}_s \\ c_{2H}(s) &= (1-\lambda) R_H(s) + \lambda \overline{R}_s \end{aligned} \tag{23}$$

Next, suppose that the identity of investors hit by a liquidity shock is private information. This adds incentive compatibility constraints to the maximization problem: the period one consumption $c_1$ has to be less than any of the consumptions in period two. The smallest period two consumption is attained in state 2 with low productive investment: $c_{2L}(s_2)$. Therefore,

$$(iv) \; c_1 \leq c_{2L}(s) \tag{24}$$

If an equilibrium $\left(c_1^o, c_{2j}^o(s) : j = L, H\right)$ satisfies the incentive compatibility constraint $(iv)$ then it remains an equilibrium.

If $c_1 \geq c_{2L}(s=2)$, i.e.$(1-\lambda) r_2 + \lambda \overline{R}_2 \leq 1$, then the equilibrium investment allocation $x$ is given by

$$x = \frac{1-\lambda}{(1-\lambda) + \lambda \left((1-\lambda) r_2 + \lambda \overline{R}_2\right)} \tag{25}$$

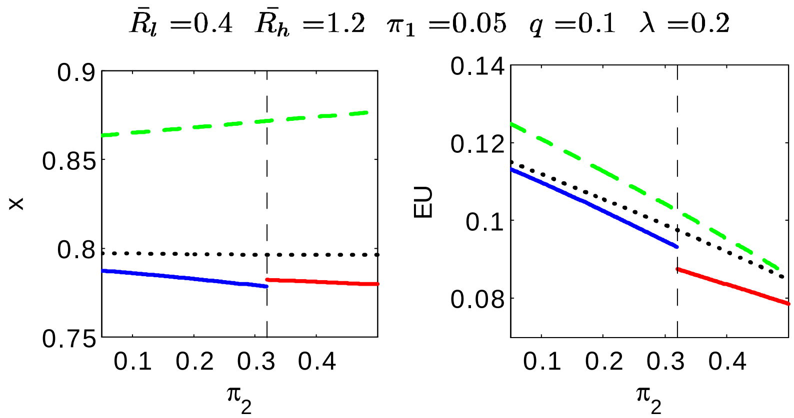
The consumption allocations are given by

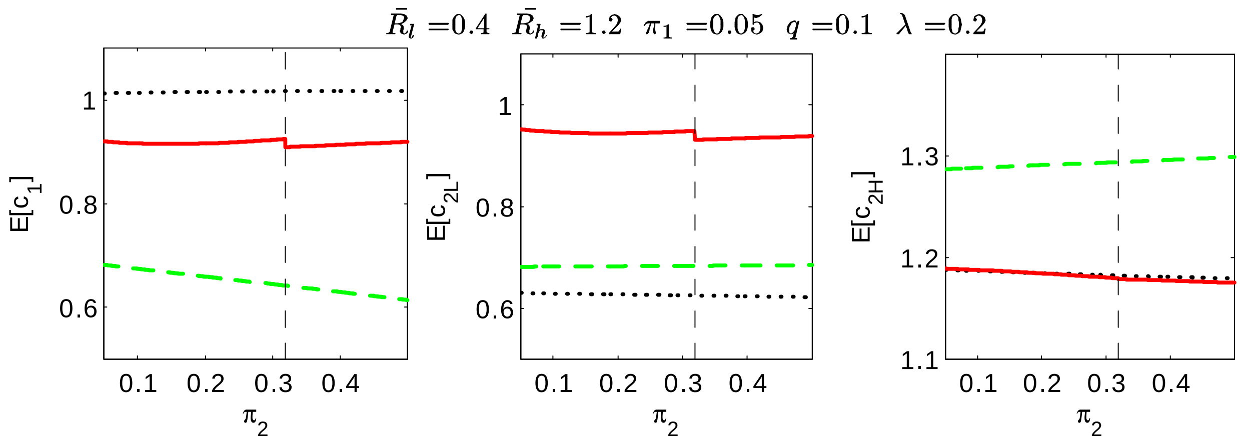
$$\begin{aligned} c_1 &= \frac{(1-\lambda) r_s + \lambda \overline{R}_2}{(1-\lambda) + \lambda \left((1-\lambda) r_2 + \lambda \overline{R}_2\right)} \\ c_{2L}(s) &= \frac{(1-\lambda) r_s + \lambda \overline{R}_s}{(1-\lambda) + \lambda \left((1-\lambda) r_2 + \lambda \overline{R}_2\right)} \\ c_{2H}(s) &= \frac{(1-\lambda) R_H(s) + \lambda \overline{R}_s}{(1-\lambda) + \lambda \left((1-\lambda) r_2 + \lambda \overline{R}_2\right)} \end{aligned} \tag{26}$$

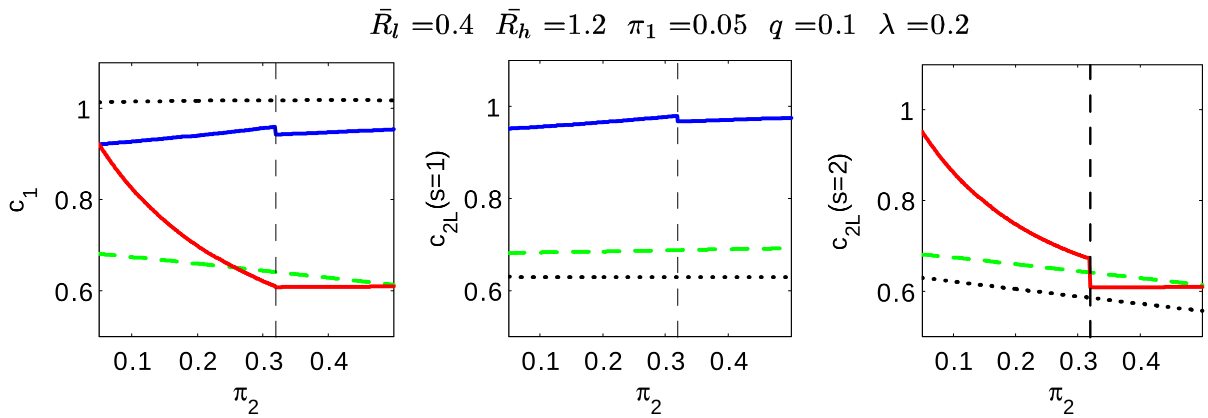
Note, that the investment allocation in the new incentive compatible equilibrium is larger than in the previous one. This benefits late consumers with high productive investments at the expense of early consumers and late consumers with low productive investments. Furthermore, the second period consumption depends on which state is realized, however, it does not depend on the probability of a crisis.

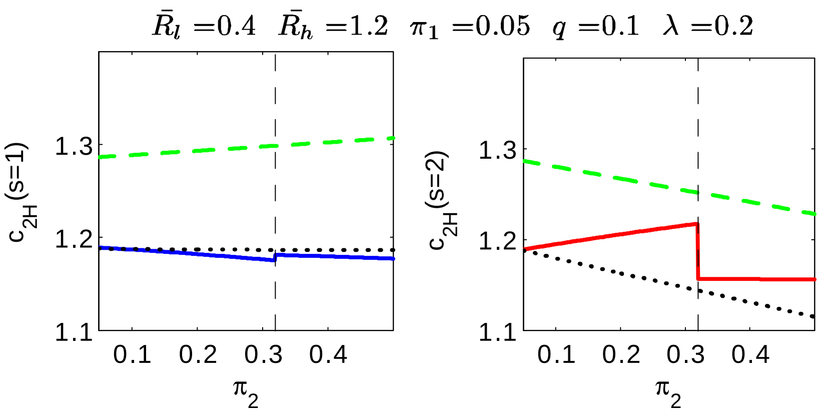
Now we can compare the market vs the central planner equilibrium. The investment allocation is larger in the central planner solution than in any of market equilibria. The consumption of early is the largest in the equilibrium without adverse selection. The late consumers with low productive investments consume the most in the market equilibrium with adverse selection. They benefit at the expense of liquidity traders. The late consumers with low productive investments consume the most in the central planner allocation. (See figure below for an example) The market equilibrium is optimal when $p = 1$.[11]

[11] See section A.3 of the Appendix for the proof.


$\bar{R}_l = 0.4$ $\bar{R}_h = 1.2$ $\pi_1 = 0.05$ $q = 0.1$ $\lambda = 0.2$
x
EU
$\pi_2$
0.1 0.2 0.3 0.4
0.75 0.8 0.85 0.9
0.08 0.1 0.12 0.14


$\bar{R}_l = 0.4$ $\bar{R}_h = 1.2$ $\pi_1 = 0.05$ $q = 0.1$ $\lambda = 0.2$
E[$c_1$]
E[$c_{2L}$]
E[$c_{2H}$]
$\pi_2$
0.1 0.2 0.3 0.4
0.6 0.8 1
1.1 1.2 1.3




The adverse selection results in a lower consumption for both early and late consumers in each state. However, the late consumers with low productive investment benefit from adverse selection and get a higher level of consumption in a normal state relative to the central planner allocation. The rest of the investors consume less. The market equilibrium with adverse selection is not optimal. The central planner can improve upon the market equilibrium by reducing the adverse selection.

## 5 Conclusion.

I analyze the effect of adverse selection in the asset market. The asymmetric information about asset returns generates the lemons problem when buyers do not know whether the asset is sold because of its low quality or because the seller's sudden need for liquidity. This adverse selection can lead to market illiquidity reflecting the buyers' belief that most assets that are offered for sale are of low quality. The lack of market liquidity and underestimating the likelihood of a crisis can amplify the effect of adverse selection leading to increased asset price volatility and possibly to a breakdown of trade during the crisis.

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# 6 Appendix

## 6.1 Assumptions

Assumption 3: $r : p\,(r) \leq 1$

$$\Longrightarrow \quad (r - R_L(s)) - (1-\lambda)\,(1-\pi_s)\,\frac{(R_H(s) - r)\,(R_H(s) - R_L(s))}{(1-\lambda)\,R_H(s) + \lambda\overline{R}_s} > 0$$

$$\Longrightarrow \quad r > \frac{R_L(s)\left((1-\lambda)\,R_H(s) + \lambda\overline{R}_s\right) + R_H(s)\,(1-\lambda)\,(1-\pi_s)\,(R_H(s) - R_L(s))}{\left((1-\lambda)\,R_H(s) + \lambda\overline{R}_s + (1-\lambda)\,(1-\pi_s)\,(R_H(s) - R_L(s))\right)} > R_L(s)$$

Assumption 4:

$$EU(p\,(r)\,, x\,(r)) \geq (1-\lambda) \sum_{s=1,2} q_s \log\left(\overline{R}_s/p\,(r)\right)$$

## 6.2 Private Information Equilibrium

### 6.2.1 Proof of Proposition 1.

**Proof.** The market clearing in state s is given by $\lambda x p_s = (1-\lambda)\,(1-x)$. Therefore, $p_1 = p_2 = p$ since $x$ is decided at $t = 0$. Hence, an investor's maximization problem becomes

$$EU_s\,(x, p) = \lambda \log(1 - x + px) + (1-\lambda) \sum_{s=1,2} q_s \left(\pi_s \log\left(xr + (1-x)\overline{R}_s/p\right) + (1-\pi_s)\log\left(xR_H(s) + (1-x)\overline{R}_s/p\right)\right)$$

The equilibrium price and investment allocation $(x, p)$ are determined by the following system of equations:

$$\lambda \tfrac{p-1}{x(p-1)+1} + (1-\lambda) \sum_{s=1,2} q_s \left(\pi_s \tfrac{r - \overline{R}_s/p}{x\left(r - \overline{R}_s/p\right) + \overline{R}_s/p} + (1-\pi_1)\,\tfrac{R_H(s) - \overline{R}_s/p}{x\left(R_H(s) - \overline{R}_s/p\right) + \overline{R}_s/p}\right) = 0$$

$$\lambda x p - (1-\lambda)\,(1-x) = 0$$

Therefore, the equilibrium price is given by

$$p_a^* = \frac{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{\overline{R}_s}{r + \overline{R}_s \frac{\lambda}{(1-\lambda)}} + (1-\pi_s)\,\frac{\overline{R}_s}{R_H(s) + \overline{R}_s \frac{\lambda}{(1-\lambda)}}\right)}{\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{r + \overline{R}_s \frac{\lambda}{(1-\lambda)}} + (1-\pi_s)\,\frac{R_H(s)}{R_H(s) + \overline{R}_s \frac{\lambda}{(1-\lambda)}}\right)}$$

By assumption 3 and 4, the equilibrium price $p$ satisfies the dynamic consistency conditions. Assumption 3 rules out the situation that a risky asset dominates the safe asset at $t = 1$. If the market price $p \geq 1$, then no one will choose to hold the safe asset at $t = 0$. Assumption 4 rules out the situation that the safe asset dominates a risky asset at $t = 1$. If the market price $p < p\,(\overline{r})$ such that $EU(p\,(\overline{r})\,, x\,(\overline{r})) = (1-\lambda) \sum_{s=1,2} q_s \log\left(\overline{R}_s/p\,(\overline{r})\right)$ then the return on the risky asset bought at $t = 1$ is higher that the return on investment made at $t = 0$, hence, no one will choose to invest in risky projects at $t = 0$.

If the market price $p \geq r$ then the equilibrium investment allocation $x$ is given by

$$x_a^* = (1-\lambda)\left(\lambda + \sum_{s=1,2} q_s \left(\pi_s \frac{r}{\left(r + \overline{R}_s \frac{\lambda}{1-\lambda}\right)} + (1-\pi_s)\frac{R_H(s)}{\left(R_H(s) + \overline{R}_s \frac{\lambda}{1-\lambda}\right)}\right)\right)$$

If the market price $p < r$ then an investor's maximization problem becomes

$$EU_{no\ trade}(x) = \lambda \log(1 - x + rx) + (1-\lambda)\sum_{s=1,2} q_s\left[\pi_s \log(xr + (1-x)) + (1-\pi_s)\log(xR_H(s) + (1-x))\right]$$

Therefore, the equilibrium investment allocation $x$ is given by

$$x_a^{**} = \frac{(\lambda + (1-\lambda)\overline{\pi})(r-1) + (1-\lambda)(1-\overline{\pi})(R_H(s)-1)}{(1-r)(R_H(s)-1)}$$

In both cases, the corner solutions: x=0 and x=1 are dominated by the interior solution. If all endowments is invested in risky assets: $x = 1$, then the consumption at date 1 $c_1 = 0$, which implies the utility equal to negative infinity. If all endowment is kept in the safe asset then the expected utility is zero while interior solution yields the positive utility since $\overline{R}_s > 1$.

If it exists, the market equilibrium always dominates the no trade equilibrium since it provides a higher consumption in each state in both dates. Suppose not, let $x_a^{**}$ be a solution to the investor maximization problem even if $p \geq r$. The expected utility in the market equilibrium is larger than the $EU_{market}(x_a^{**}, p) > EU_{no\ trade}(x_a^{**})$ since $\overline{R}_s/p > 1$ and $p \geq r$,

$$\begin{aligned} EU_{market}(x_a^{**}, p) &= \lambda \log(1 - x_a^{**} + px_a^{**}) + (1-\lambda)\sum_{s=1,2} q_s\left(\pi_s \log\left(x_a^{**}p + (1-x_a^{**})\overline{R}_s/p\right) + (1-\pi_s)\log\left(x_a^{**}R_H(s) + (1-x_a^{**})\overline{R}_s/\right.\right. \\ &> \lambda \log(1 - x_a^{**} + rx_a^{**}) + (1-\lambda)\sum_{s=1,2} q_s\left(\pi_s \log(x_a^{**}r + (1-x_a^{**})) + (1-\pi_s)\log(x_a^{**}R_H(s) + (1-x_a^{**}))\right) = EU_n \end{aligned}$$

and $\forall x : EU_{market}(x_a^*, p) \geq EU_{market}(x, p)$. Contradiction. It is impossible to have market equilibrium in one state and no trade equilibrium in another state Since the market price is the same in both states.

Furthermore, the investment allocation is larger in the market equilibrium relative to no trade equilibrium: $x_a^* > x_a^{**}$

$$\begin{aligned} x_a^* - x_a^{**} &= \lambda\left((1-\lambda) + \frac{1}{(R_H(s)-1)}\right) + (1-\lambda)\sum_{s=1,2} q_s\left(\pi_s\left(\frac{r}{\left(r + \overline{R}_s\frac{\lambda}{1-\lambda}\right)} + \frac{1}{(R_H(s)-1)}\right) + (1-\pi_s)\left(\frac{R_H(s)}{\left(R_H(s) + \overline{R}_s\frac{\lambda}{1-\lambda}\right)} - \right.\right. \\ &= \lambda\left((1-\lambda) + \frac{1}{(R_H(s)-1)}\right) + (1-\lambda)\sum_{s=1,2} q_s\left(\pi_s\left(\frac{R_H(s)r + \overline{R}_s\frac{\lambda}{1-\lambda}}{r + \overline{R}_s\frac{\lambda}{1-\lambda}}\right) + (1-\pi_s)\left(\frac{R_H(s)r + \overline{R}_s\frac{\lambda}{1-\lambda}}{R_H(s) + \overline{R}_s\frac{\lambda}{1-\lambda}}\right)\right) > 0 \end{aligned}$$

■

The market equilibrium consumption:

$$\begin{aligned} c_1 &= \frac{(1-x)}{\lambda} \\ c_1 &= \left(\lambda + (1-\lambda)\sum_{s=1,2} q_s\left(\pi_s\frac{\overline{R}_s}{(1-\lambda)r + \lambda\overline{R}_s} + (1-\pi_s)\frac{\overline{R}_s}{(1-\lambda)R_H(s) + \lambda\overline{R}_s}\right)\right) \end{aligned}$$

$$
\begin{aligned}
c_{2i}(s) &= x\left(R_i+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\\
c_{2i}(s) &= (1-\lambda)\left(R_i+\frac{\lambda}{1-\lambda}\overline{R}_s\right)\left(\lambda+\sum_{s=1,2}q_s\left(\pi_s\frac{r}{\left(r+\overline{R}_s\frac{\lambda}{1-\lambda}\right)}+(1-\pi_s)\frac{R_H(s)}{\left(R_H(s)+\overline{R}_s\frac{\lambda}{1-\lambda}\right)}\right)\right)
\end{aligned}
$$

Note, $c_1\leq 1$ since $\sum_{s=1,2}q_s\left(\pi_s\frac{(1-\lambda)\left(\overline{R}_s-r\right)}{(1-\lambda)r+\lambda\overline{R}_s}+(1-\pi_s)\frac{(1-\lambda)\left(\overline{R}_s-R_H(s)\right)}{(1-\lambda)R_H(s)+\lambda\overline{R}_s}\right)\leq 0$ which is implied by $p\leq 1$.

The no trade equilibrium consumption:

$$
\begin{aligned}
c_1 &= 1-x+rx\\
c_1 &= (\lambda+(1-\lambda)\overline{\pi})\frac{R_H(s)-r}{R_H(s)-1}
\end{aligned}
$$

$$
\begin{aligned}
c_{2i}(s) &= xR_i+(1-x)\\
c_{2H}(s) &= (1-\lambda)(1-\overline{\pi})\frac{(R_H(s)-r)}{(1-r)}\\
c_{2L}(s) &= (\lambda+(1-\lambda)\overline{\pi})\frac{R_H(s)-r}{R_H(s)-1}
\end{aligned}
$$

## 6.3 Equilibrium with adverse selection

### 6.3.1 Proof of Proposition 2.

**Proof.** Similarly to equilibrium without adverse selection, if the market equilibrium exist in a state s then it will dominate an equilibrium with no trade. Consider type (1) equilibrium:

$$
\begin{aligned}
\max_x \quad & \lambda\log(1-x+p_sx)+(1-\lambda)\sum_{s=1,2}q_s\left(\pi_s\log\left(xp_s+(1-x)\widehat{R}_s/p_s\right)+(1-\pi_s)\log\left(xR_H(s)+(1-x)\widehat{R}_s/p_s\right)\right)\\
s.t \quad & (i)\ \ 0\leq x\leq 1\\
& (ii)\ p_s\geq r\ \forall s
\end{aligned}
$$

Therefore, the type 1 equilibrium investment allocation and market prices are determined by the following equations:

$$
\sum_{s=1,2}q_s\left(\lambda\frac{p_s-1}{1-x+p_sx}+(1-\lambda)\left(\pi\frac{p_s-\widehat{R}_s/p_s}{xp_s+(1-x)\widehat{R}_s/p_s}+(1-\pi)\frac{R_H(s)-\widehat{R}_s/p_s}{xR_H(s)+(1-x)\widehat{R}_s/p_s}\right)\right)=0
$$

$$
(\lambda+(1-\lambda)\pi_s)p_sx=(1-\lambda)(1-x)
$$

Substituting prices $p_s$, we can get

$$
F_b(x)\equiv\sum_{s=1,2}q_s\left(\lambda\frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_s\right)}+(1-\lambda)\pi\frac{(1-x)}{(1-x)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)^2x}+(1-\lambda)(1-\pi_s)\frac{R_H(s)}{R_H(s)+\widehat{R}_1\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)}\right)-x=0
$$

This is a monotonically decreasing function of $x$. At $x = 0$, $F_b$ is greater than 0 and at $x = 1$, $F_1$ is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique $x^*$ such that at $F_1(x^*) = 0$ The $x^*$ can be derived as a root to a cubic equation:$a_1x^3 + a_2x^2 + a_3x + a_4 = 0$, where

$$
\begin{aligned}
a_1 &= -d_1d_2 \\
a_2 &= d_1d_2d_3 - ((1-\lambda) q_1\pi_1 + 1) d_2 - ((1-\lambda) q_2\pi_2 + 1) d_1 \\
a_3 &= ((d_1 + d_2) d_3 - 1) + ((1-\lambda) q_1\pi_1 (d_2 - 1) + (1-\lambda) q_2\pi_2 (d_1 - 1)) \\
a_4 &= d_3 + (1-\lambda) q_1\pi_1 + (1-\lambda) q_2\pi_2 \\
d_1 &= \left( \widehat{R}_1 \left( \frac{\lambda}{(1-\lambda)} + \pi_1 \right)^2 - 1 \right) \\
d_2 &= \left( \widehat{R}_2 \left( \frac{\lambda}{(1-\lambda)} + \pi_2 \right)^2 - 1 \right) \\
d_3 &= \lambda \sum_{s=1,2} q_s \frac{1}{\left( \frac{1}{(1-\lambda)} + \pi_s \right)} + (1-\lambda) \sum_{s=1,2} q_s \frac{(1-\pi_s) R_H(s)}{R_H(s) + \widehat{R}_s \left( \frac{\lambda}{(1-\lambda)} + \pi_s \right)}
\end{aligned}
$$

Denote the solution as $x_b^*$, then the prices are given by

$$p_b^*(s) = \frac{(1-\lambda)}{(\lambda + (1-\lambda)\pi_s)} \frac{(1-x_b^*)}{x_b^*}$$

If $p_b^*(s_2) \geq r$ then this is the equilibrium of type (1).

If $p_b^*(s_2) < r$ and $p_b^*(s_1) \geq r$ then consider type (2) equilibrium:type (1) equilibrium:

$$
\begin{aligned}
\max_x \quad & \left\{ \begin{array}{l} \lambda \log(1 - x + p_s x) + (1-\lambda)(1-q) \left( \pi_1 \log \left( xp + (1-x)\widehat{R}_1/p_1 \right) + (1-\pi_s) \log \left( xR_H(s) + (1-x)\widehat{R}_1/p_1 \right) \right) + \\ +\lambda \log(1 - x + rx) + (1-\lambda) + q (\pi_2 \log(xr + (1-x)) + (1-\pi_2) \log(xR_H(s) + (1-x))) \end{array} \right\} \\
s.t \quad & (i) \;\; 0 \leq x \leq 1 \\
& (ii) \; p_1 \geq r
\end{aligned}
$$

Therefore, the type 1 equilibrium investment allocation and market prices are determined by the following equations:

$$
\begin{array}{l}
\left. \begin{array}{l} (1-q) \left( \lambda \frac{p_1 - 1}{1 - x + p_1 x} + (1-\lambda) \left( \pi \frac{p_1 - \widehat{R}_1/p_1}{xp_1 + (1-x)\widehat{R}_1/p_1} + (1-\pi) \frac{R_H(s) - \widehat{R}_1/p_1}{xR_H(s) + (1-x)\widehat{R}_1/p_1} \right) \right) \\ +q_2 \left( (\lambda + (1-\lambda)\pi_2) \frac{r-1}{xr + (1-x)} + (1-\pi_2) \frac{R_H(s) - 1}{xR_H(s) + (1-x)} \right) \end{array} \right. = 0 \\
(\lambda + (1-\lambda)\pi_1) p_1 x = (1-\lambda)(1-x)
\end{array}
$$

Substituting $p_1$, we can get

$$
G_b(x) \equiv \left( \begin{array}{l} q_1 \left( \lambda \frac{1}{\left( \frac{1}{(1-\lambda)} + \pi_1 \right)} + (1-\lambda) \pi \frac{(1-x)}{(1-x) + \widehat{R}_1 \left( \frac{\lambda}{(1-\lambda)} + \pi_s \right)^2 x} + (1-\lambda)(1-\pi_1) \frac{R_H(s)}{R_H(s) + \widehat{R}_1 \left( \frac{\lambda}{(1-\lambda)} + \pi_1 \right)} - x \right) \\ +q_2 \left( (\lambda + (1-\lambda)\pi_2) \frac{r-1}{xr + (1-x)} + (1-\pi_2) \frac{R_H(s) - 1}{xR_H(s) + (1-x)} \right) \end{array} \right) = 0
$$

$G_b$ is also a decreasing function in $x$, and it is positive at $x = 0$ and negative at $x = 1$. Therefore, the solution exists and it unique. Let $x_b^{**}$ denote the solution, then the market price in state $s_1$ is given by

$$p_b^{**}(s_1) = \frac{(1-\lambda)}{(\lambda + (1-\lambda)\pi_1)} \frac{(1-x_b^{**})}{x_b^{**}}$$

If $p_b^*(s_1) \geq r$ then this is the equilibrium of type (2). Note, $x_b^{**} < x_b^*$ since $F_1(x_b^{**}) > F_2(x_b^{**}) = 0$ and $F_1$ is decreasing in $x$.

If $p_b^*(s_1) < r$ then the equilibrium is of type (3). The type (3) no trade equilibrium is the same as no trade equilibrium considered in Proposition 1, and the equilibrium investment allocation is given by

$$x_a^{**} = \frac{(\lambda + (1-\lambda)\overline{\pi})(r-1) + (1-\lambda)(1-\overline{\pi})(R_H(s)-1)}{(1-r)(R_H(s)-1)}$$

Furthermore, the investment allocation in the market equilibrium without adverse selection is larger than the investment allocation when adverse selection is present: $x_b^* < x_a^*$.

Let $(x_a^*, p_a^*)$ be the equilibrium without adverse selection. Now consider the solution to maximization problem in Proposition 1 but with prices $p_b(s) = \frac{1}{\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)} \frac{1-x}{x}$ instead of $p_a = \frac{(1-\lambda)}{\lambda} \frac{(1-x)}{x}$. Denote the solution as $(x_a^o, p_b^o(s))$

$$x_a^o = \lambda \frac{1}{\left(\frac{\lambda}{(1-\lambda)} + \pi\right) + 1} + (1-\lambda) \sum_{s=1,2} q_s \left( \pi_s \frac{r}{r + \overline{R}_s\left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} + (1-\pi_s) \frac{R_H(s)}{R_H(s) + \overline{R}_s\left(\frac{\lambda}{(1-\lambda)} + \pi_s\right)} \right) < x_a^*$$

$$\begin{aligned} 0 &= \sum_{s=1,2} q_s \left( \lambda \frac{p_b^o(s) - 1}{1 - x_a^o + p_b^o(s)\, x_a^o} + (1-\lambda) \left( \pi_s \frac{r - \overline{R}_s / p_b^o(s)}{x_a^o r + (1-x_a^o)\overline{R}_s / p_b^o(s)} + (1-\pi_s) \frac{R_H(s) - \overline{R}_s/p_b^o(s)}{x_a^o R_H(s) + (1-x_a^o)\overline{R}_s / p_b^o(s)} \right) \right) < \\ &< \lambda \frac{p_b^o(s) - 1}{1 - x_a^o + p_b^o(s)\, x_a^o} + (1-\lambda) \sum_{s=1,2} q_s \left( \pi_s \frac{p_b^o(s) - \widehat{R}_s/p_b^o(s)}{x_a^o p_b^o(s) + (1-x_a^o)\overline{R}/p_b^o(s)} + (1-\pi_s) \frac{R_H(s) - \widehat{R}_s/p_b^o(s)}{x_a^o R_H(s) + (1-x_a^o)\overline{R}_s/p_b^o(s)} \right) = F_1(x_a^o) \end{aligned}$$

Therefore, $x_b^* < x_a^o$ such that $F_1(x_b^*) = 0$. Hence, $x_b^* < x_a^o < x_a^*$.

Also, adverse selection lead to a lower expected price: $p_a > p_b((1-q)s_1 + qs_2) \geq (1-q)p_b(s_1) + qp_b(s_2)$

$$p_a > (1-q)p_b(s_1) + qp_b(s_2)$$

In the presence of adverse selection, the highest utility is attained when there a market trading in equilibrium in both states. The equilibrium consumption of early and late consumers are given by

$$\begin{aligned} c_1^b(s) &= 1 - x_b + p_b(s)\, x_b \\ c_{2H}^b(s) &= x_b R_H(s) + (1-x_b)\, \widehat{R}_s / p_b(s) \\ c_{2L}^b(s) &= x_b p_b + (1-x_b)\, \widehat{R}_s / p_b(s) \end{aligned}$$

The expected consumption at both dates in the equilibrium with adverse selection are lower than the expected consumption at both dates without adverse selection. Therefore, expected utility is lower. In case of adverse selection the low quality projects do not get liquidated by informed investors. This results in the losses of total welfare. ■

### 6.3.2 Special Case: $p_1 = \widehat{R}_1$

In equilibrium, the investment allocation $x$ should satisfy the following condition:

**Proof.**

$$\sum_{s=1,2} q_s \left[ \lambda \frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_s\right)} + (1-\lambda)\pi_s \frac{(1-x)}{1+\left(\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)^2-1\right)x} + (1-\lambda)(1-\pi_s)\frac{R_H(s)}{R_H(s)+\widetilde{R}_1\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)} - x \right] = 0$$

If $p_1 = \widehat{R}_1$ in the equilibrium then $x = \frac{1}{\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)}\widehat{R}_1+1}$.This implies that the payoff parameters must satisfy the following condition:

$$\sum_{s=1,2} q_s \left[ \lambda \frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_s\right)} + (1-\lambda)\pi_s \frac{1}{1+\left(\frac{\lambda}{(1-\lambda)}+\pi_1\right)\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)^2\frac{\widehat{R}_s}{\widehat{R}_1}} + (1-\lambda)(1-\pi_s)\frac{R_H(s)}{R_H(s)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)} - \frac{1}{\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)}\widehat{R}}\right.$$

■

## 6.4 Comparative Static

### 6.4.1 Proof of Corollary 1

**Proof.** First consider an equilibrium with trade in both states.

The equilibrium investment allocation is determined from the following equation: $F_b(x) = 0$ ($F_b(x)$ is defined in the proof of Proposition 2, it is derived by substituting market clearing conditions into the FOC condition.) Denote by $F_{b1}(s)$ the following expression,

$$F_{b1}(s,x) \equiv \lambda \frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_s\right)} + (1-\lambda)\pi_s \frac{(1-x)}{(1-x)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)^2 x} + (1-\lambda)(1-\pi_s)\frac{R_H(s)}{R_H(s)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)} - x$$

$F_{b1}(s) = 0$ provides the solution for the problem with one state. $F_{b1}(s)$ is decreasing in $\pi_s$.Therefore, $F_b(x)$ is decreasing in $q$. Also, $F_b(x)$ is decreasing in $x$. Hence, $x$ is decreasing in $q$. The prices are determined by $p_s = \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)}\frac{(1-x)}{x}$. Therefore, $p_s$ are increasing in $q$. The one-state expected utility is decreasing in $\pi_s$. Therefore, as $q$ becomes larger the expected utility decreases.

Now consider an equilibrium with a no trade state 2.Denote by $G_{b2}(x) = \left(\left(\lambda + (1-\lambda)\pi_2\right)\frac{r-1}{xr+(1-x)} + (1-\pi_2)\frac{R_H(s)-1}{xR_H(s)+(1-x)}\right)$. If we compute $x^*$ such that $G_{b2}(x^*) = 0$ and $x^{**}$ such that $F_{b1}(s=1,x^*) = 0$ then $x^{**} > x^*$. The equilibrium x in a two-state problem is determined by $G_b(x) = (1-q)F_{b1}(s=1,x) + qG_{b2}(x) = 0$. Since $G_b(x)$ is decreasing in $x$ then the optimal $x$ is decreasing in $q$. Therefore, $p_1$ are increasing in $q$ since it negatively depends on $x$. The one-state expected utility is lower in a no-trade state vs the one with trade. Therefore, as $q$ becomes larger the expected utility decreases. The no-trade outcome arises since the price in the crisis state falls below liquidation value. The increase in q may increase the price in the crisis state sufficiently to restore the trading.

Consider some $q$ such that $p_2 = r - \varepsilon$ with $\varepsilon > 0$. Then there is no trading in state 2.

$$F_{b1}(s=1,x) = \lambda\frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_1\right)} + (1-\lambda)\pi_1\frac{1}{1+\widehat{R}_1\left(\frac{\lambda}{(1-\lambda)}+\pi_1\right)^2(r-\varepsilon)/\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)} + (1-\lambda)(1-\pi_1)\frac{R_H(s)}{R_H(s)+\widehat{R}_1\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)} - \frac{1}{\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r-\right.}$$

$$F_{b1}(s=2,x) = \lambda\frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_2\right)} + (1-\lambda)\pi_1\frac{1}{1+\widehat{R}_2\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r-\varepsilon)} + (1-\lambda)(1-\pi_1)\frac{R_H(s)}{R_H(s)+\widehat{R}_2\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)} - \frac{1}{\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r+\varepsilon)\right)}$$

Therefore, $F_{b1}(s=1,x) > F_{b1}(s=1,x)$. If $q$ increases sufficiently so that x goes down by more than $\frac{\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)\varepsilon}{\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r+\varepsilon)\right)\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)r\right)}$ then the trading in a crisis state restores. ■

### 6.4.2 Proof of Corollary 2

**Proof.** Suppose now the economy is parametrized by state 1: $(\lambda_1, \pi_1)$ and state 2 : $(\lambda_2, \pi_2)$ such that $\lambda_1 < \lambda_2$ and $\pi_1 < \pi_2$.

First consider an equilibrium with trade in both states. The equilibrium investment allocation is determined from the following equation: $F_b(x) = 0$

Denote by $F_{b1}(s)$ the following expression,

$$F_{b1}(s,x) \equiv \lambda\frac{1}{\left(\frac{1}{(1-\lambda)}+\pi_s\right)} + (1-\lambda)\pi_s\frac{(1-x)}{(1-x)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)^2 x} + (1-\lambda)(1-\pi_s)\frac{R_H(s)}{R_H(s)+\widehat{R}_s\left(\frac{\lambda}{(1-\lambda)}+\pi_s\right)} - x$$

$F_{b1}(s) = 0$ provides the solution for the problem with one state. $F_{b1}(s)$ is decreasing in $\lambda$. Also, $F_b(x)$ is decreasing in $x$. Hence, $x$ is decreasing in $\lambda$.

The effect of increase in $\lambda_2$ on the price in state 2 is determined by

$$\frac{\partial p_2}{\partial \lambda_2} = -\frac{\frac{1}{(1-\lambda_2)^2}}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)^2}\frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)}\frac{1}{x^2}\frac{\partial x}{\partial \lambda_2}$$

Therefore, increase in $\lambda_2$ can lead to the decrease in $p_2$, potentially resulting in $p_2 < r$. ■

### 6.5 Central Planner

#### 6.5.1 Liquidity shock is public information

$$\begin{aligned}
EU\left(c_1, c_2\right) &= \lambda \log c_1 + (1-\lambda) \sum_s q_s \left(\pi_s \log\left(c_{2Ls}\right) + \left(1-\pi_s\right) \log\left(c_{2Hs}\right)\right) \\
s.t. &: c_1 = \frac{1-x}{\lambda} \\
&: c_{2L} = x\left(r + \overline{R}\frac{\lambda}{1-\lambda}\right) \\
&: c_{2H} = x\left(R_H(s) + \overline{R}\frac{\lambda}{1-\lambda}\right) \\
IC &: x\overline{R}\frac{\lambda}{1-\lambda} \geq 1-x
\end{aligned}$$

$$\begin{aligned}
EU(x) &= \lambda \log\left(\frac{1-x}{\lambda}\right) + (1-\lambda)\sum_s q_s \left(\pi_s \log\left(x\left(r + \overline{R}_s\frac{\lambda}{1-\lambda}\right)\right) + \left(1-\pi_s\right)\log\left(x\left(R_H(s) + \overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)\right) \\
FOC &: -\lambda\frac{1}{1-x} + (1-\lambda)\left(\frac{1}{x}\right) = 0 \\
&\Longrightarrow x^o = (1-\lambda)
\end{aligned}$$

$$IC = (1-\lambda) + \lambda\overline{R} \geq 1$$

$$\begin{aligned}
\pi c_{2L} + (1-\pi)\, c_{2H} &> 1 \\
(1-\lambda)\left(\pi r + (1-\pi) R_H(s)\right) + \lambda\overline{R} &> 1
\end{aligned}$$

$$\begin{aligned}
c_1 &= 1 \\
c_{2L}(s) &= (1-\lambda)\left(r + \overline{R}_s\frac{\lambda}{1-\lambda}\right) \\
c_{2H}(s) &= (1-\lambda)\left(R_H(s) + \overline{R}_s\frac{\lambda}{1-\lambda}\right)
\end{aligned}$$

#### 6.5.2 Liquidity shock is private information

additional constraint:

$$c_{2L2} \geq c_1$$

$$(1-\lambda)\, r + \lambda\overline{R} \geq 1$$

if $(1-\lambda)\, r + \lambda\overline{R} \geq 1$ then no late consumer has incentive to pretend to be an early one $\Longrightarrow x^o = (1-\lambda)$

if $(1-\lambda)r+\lambda\overline{R}<1$ then $x$ is determined by $c_1=c_{2L2}$, hence,

$$\begin{aligned}
\frac{1-x}{\lambda} &= x\left(r+\overline{R}\frac{\lambda}{1-\lambda}\right)\\
x^{oo} &= \frac{1-\lambda}{\left((1-\lambda)+\lambda\left((1-\lambda)r+\lambda\overline{R}_2\right)\right)}=x\\
x^{oo} &= \frac{1-\lambda}{\left(1-\lambda\left[1-(1-\lambda)r+\lambda\overline{R}_2\right]\right)}>1-\lambda
\end{aligned}$$

Note, $x^{oo}<x^{o}$

$$\begin{aligned}
\frac{1}{\left(1+\frac{\lambda}{1-\lambda}\left((1-\lambda)r+\lambda\overline{R}_2\right)\right)} &< (1-\lambda)\\
1 &< (1-\lambda)+\lambda\left((1-\lambda)r+\lambda\overline{R}_2\right)
\end{aligned}$$

Therefore,

$$c_1^{oo}=c_{2L}^{oo}=\frac{\frac{\lambda}{1-\lambda}\left((1-\lambda)r+\lambda\overline{R}_2\right)}{1+\frac{\lambda}{1-\lambda}\left((1-\lambda)r+\lambda\overline{R}_2\right)}$$

Comparing the CP solution to the market solution.

**Claim 1** $EU(x^{o})>EU(x_a)$

**Proof.**

$$\begin{aligned}
EU(x^{o}) &= (1-\lambda)\sum_s q_s\left(\pi_s\log\left((1-\lambda)\left(r+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)+(1-\pi_s)\log\left((1-\lambda)\left(R_H(s)+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)\right)=\\
&= (1-\lambda)\log(1-\lambda)+(1-\lambda)\sum_s q_s\left(\pi_s\log\left(r+\overline{R}_s\frac{\lambda}{1-\lambda}\right)+(1-\pi_s)\log\left(R_H(s)+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)
\end{aligned}$$

$$\begin{aligned}
EU(x_a) &= \lambda\log\frac{(1-x)}{\lambda}+(1-\lambda)\sum_s q_s\left(\pi_s\log\left(x\left(R_i+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)+(1-\pi_s)\log\left(x\left(R_i+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)\right)=\\
&= \left(\begin{array}{l}\lambda\log\frac{(1-x)}{\lambda}+(1-\lambda)\log\frac{x}{(1-\lambda)}\\ +(1-\lambda)\log(1-\lambda)+(1-\lambda)\sum_s q_s\left(\pi_s\log\left(R_i+\overline{R}_s\frac{\lambda}{1-\lambda}\right)+(1-\pi_s)\log\left(R_i+\overline{R}_s\frac{\lambda}{1-\lambda}\right)\right)\end{array}\right)
\end{aligned}$$

therefore, $EU(x_a)-EU(x^{o})=\lambda\log\frac{(1-x)}{\lambda}+(1-\lambda)\log\frac{x}{(1-\lambda)}$

Claim: $x_a\geq x^{o}=1-\lambda$

$$x_a=\lambda(1-\lambda)+(1-\lambda)\left[\sum_{s=1,2}q_s\left(\pi_s\frac{r}{\left(r+\overline{R}_s\frac{\lambda}{1-\lambda}\right)}+(1-\pi_s)\frac{R_H(s)}{\left(R_H(s)+\overline{R}_s\frac{\lambda}{1-\lambda}\right)}\right)\right]\leq 1-\lambda$$

$$
\begin{aligned}
\lambda + \left[ \sum_{s=1,2} q_s \left( \pi_s \frac{r}{\left( r + \overline{R}_s \frac{\lambda}{1-\lambda} \right)} + (1-\pi_s) \frac{R_H(s)}{\left( R_H(s) + \overline{R}_s \frac{\lambda}{1-\lambda} \right)} \right) \right] &\geq 1 \\
\lambda + (1-\lambda) \left[ \sum_{s=1,2} q_s \left( \pi_s \frac{r}{(1-\lambda) r + \lambda \overline{R}_s} + (1-\pi_s) \frac{R_H(s)}{(1-\lambda) R_H(s) + \lambda \overline{R}_s} \right) \right] &\geq 1 \\
\pi_s \frac{r}{(1-\lambda) r + \lambda \overline{R}_s} + (1-\pi_s) \frac{R_H(s)}{(1-\lambda) R_H(s) + \lambda \overline{R}_s} - 1 &\geq 0 \\
\sum_{s=1,2} q_s \left[ \left( r - \overline{R}_l \right) - (1-\lambda)(1-\pi_s) \frac{(R_H(s) - R_L(s))(R_H(s) - r)}{(1-\lambda) R_H(s) + \lambda \overline{R}_s} \right] &\geq 0
\end{aligned}
$$

Hence, $EU(x_a) - EU(x^o) = \lambda \log \frac{(1-x)}{\lambda} + (1-\lambda) \log \frac{x}{(1-\lambda)} \geq 0$ since $x_a \geq (1-\lambda)$. ■