Information revelation in the Diamond-Dybvig banking model*

Ed Nosal and Neil Wallace

September 22, 2009

1 Introduction

Green and Lin [4] (GL), Peck and Shell [7] (PS), and Andolfatto et al [1] (ANW) explore optima in similar, but not identical versions of the Diamond-Dybvig [2] (DD) model. The common ingredients are a finite number of agents, independent-across-agents determination of preference types, impatient or patient (and, hence, aggregate risk), and sequential service—early withdrawal requests have to be dealt with in order (on a first-come, first-serve basis) and the ordering is exogenous and random. The versions differ about what agents know when they turn up in order to make early withdrawals. In GL, each knows his place in the ordering; in ANW, each knows that and the announcements of those who preceded him; in PS, each knows nothing. Here, we show how to interpret these versions as special cases of a more general framework. In the more general framework, as in PS, each agent starts out knowing nothing. However, the planner—who deals with the agents in order and who, therefore, necessarily knows each agent's place in the ordering and the previous announcements—can choose how much to reveal. From that perspective, GL study optima under the restriction that the planner reveals place in the ordering, ANW study optima under the restriction that the planner reveals everything, while PS study optima under the restriction that the planner reveals nothing.

Two kinds of implementability constraints are common to all versions of DD. One is physical feasibility implied by the resource constraint. The other are incentive constraints that arise from the private information. They depend on what the planner reveals and on whether the planner is trying to solve a weak implementability problem or a strong implementability problem. (Recall

*This is a preliminary version. Please do not cite without permission of the authors.

that an allocation is weakly implementable if it is the outcome of *some* equilibrium; it is strongly implementable if it is the outcome of *every* equilibrium.) In terms of the new framework, the results in the above papers can be summarized as follows. For environments in which the incentive constraints implied by planner revelation of everything are nonbinding, GL show that if the planner reveals either place in the ordering or everything, then the first best (best subject only to physical feasibility) is strongly implemented. ANW show that if the planner reveals everything, then weak implementability implies strong implementability—whether or not incentive constraints bind. These strong implementability results imply that there are no bank runs. PS show that there exist settings such that if the planner reveals nothing, then the best weakly implementable allocation is not strongly implementable. In particular, there is another equilibrium that is a bank run.

Thus, in terms of the current framework, ANW fail to consider the possibility that the planner could achieve a better outcome by withholding information, while PS fail to consider the possibility that the planner could achieve a better outcome by revealing some information. These are the issues addressed here.[1] In section 2, we set out the model. In section 3, we set out a nesting result: the less the planner reveals, the larger the set of weakly implementable allocations. In section 4, we study two examples, including the pertinent PS example. We show that the PS conclusion survives in our more general framework. We end with comments about the model and its interpretation.

2 Environment

There are N ex ante identical agents, two dates, 1 and 2, and there is one good per date. The economy is endowed with an amount $Y > 0$ of date-1 good and has a constant returns-to-scale technology with gross rate-of-return $R > 1$. That is, if C_i denotes total date-i consumption, then $C_2 \leq R(Y - C_1)$.

The sequence of events and actions is as follows. Let $\mathbf{N} = \{1, 2, ..., N\}$ be the set of possible places in the ordering and let $T = \{impatient, patient\}$ be the set of possible preference types. First nature selects a queue, denoted $t^N = (t_1, t_2, ..., t_N) \in T^N$, where $t_k \in T$ is the type of the k-th agent in the ordering. Nature's draw is from a probability distribution $g : T^N$. (One example that shows up later is that all 2^N queues are equally probable.) If an agent

[1] The issue of how much to reveal does not arise in GL. Their result is general in one sense: because the first best is achieved, it follows that revealing the information is undominated. However, their result does not apply to the large class of environments with binding incentive constraints. See Lin [6] for the sense in which the general case has binding incentive constraints.

observed the realized queue, which is not the case, then the agent would know his own place in the ordering and preference type, his own $(k,t) \in \mathbf{N} \times T$, and would know the preference types of others by place in the ordering. The realized queue is observed by no one, not even the planner, but each agent privately observes his type in the set T.[2] Each agent maximizes expected utility. An agent of type $t \in T$ has utility function $u(\cdot,\cdot,t)$, where the first argument is date-1 consumption and the second is date-2 consumption. For a given t, u is increasing and concave. Then agents meet the planner in the order determined by the queue.[3] In a meeting, the planner knows the vector of announced types of the agents with earlier places in the ordering (and the agent's place in the ordering). The planner announces part of what he knows to the agent. Then the agent announces an element of T. The outcome of the meeting is the agent's date-1 consumption. After all the date-1 meetings occur, the planner simultaneously assigns date-2 consumption to each agent.[4]

When the planner meets the k-th agent in the ordering, he knows a history $h^{k-1} \in H^{k-1} = T^{k-1}$, the set of vectors of *announced* types of the first $k-1$ agents in the ordering. Let $H = \{H^0, H^1, ..., H^{N-1}\}$, where $H^0 \equiv \varnothing$ is the null history when the planner meets the first agent in the ordering. When the planner meets the k-th agent in the ordering, his announcement is a *partition* of H, denoted P_k, a partition consistent with having observed an element in H^{k-1}. That is, the planner's announcement, determined by P_k, is a subset of H that contains the *actual* history of the first $k-1$ agent announcements. Let $P = (P_1, P_2, ..., P_N)$. (For example, if the planner tells the k-th agent for $k = 2, 3, ..., N$ only that he is not the first agent, then he says that his history is in the set $H \setminus H^0$. And if he also tells what the first agent announced, then his partition distinguishes between two subsets of $H \setminus H^0$ according to the content of h^1.) The partition P' is said to be *coarser* than the partition P'' if $P''_k \subseteq P'_k$ for all $k = 1, 2 \ldots, N$ with strictness for some k.

In PS, the planner uses the *coarsest* partition of H, which is no partition at all. In this case, agents learn nothing from the planner. In ANW, the planner uses the least *coarse* partition of H that is consistent with having observed an

[2] Thus, *queue* here is used not in the sense of a line of people, each of whom is in touch with those nearby. Instead, our queue is like the order in which people arrive at a drive-up window.

[3] We are studying a model in which all types meet the planner at date 1. This is the version that ANW study and that PS study in one of their appendices. We prefer this version because in a version with many types, as described in Lin [6], almost everyone would have to meet the planner.

[4] Notice that we give the planner control of all the resources and that the only decision that an agent makes is an announcement of type in the set T. This suffices for our purposes. There are generalizations of the model in which each agent starts out owning Y/N and can, before he learns his type in T, defect to autarky. That option is not relevant for what we do.

element in H^{k-1}. That partition distinguishes among all events in H^{k-1}. In this case, agent k, $k = 2\dots, N$, learns the announcements made by the previous $k-1$ agents and the first agent learns that he is first. In GL, the planner uses a partition that distinguishes only among the H^{k-1} for different k.

3 Weak and strong implementability

For a given P, a strategy for the k-th agent in the ordering is $\boldsymbol{\sigma}_k : H \times T \to T$, where the first argument is what the planner reveals and the second is the true type of the agent. We let $\boldsymbol{\sigma}^k = (\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \dots, \boldsymbol{\sigma}_k)$. A mechanism is (P, c), where $c = (c^1, c^2, ..., c^N)$, $c^k = (c_1^k, c_2^k)$, and $c_1^k : T^k \to \mathbb{R}_+$ is date-1 consumption of the k-th agent in the ordering and $c_2^k : T^N \to \mathbb{R}_+$ is date-2 consumption of that agent. The domain of c is announcements. We say that c is feasible if for all $t^N \in T^N$, $R(Y - \sum_{k=1}^N c_1^k) \geq \sum_{k=1}^N c_2^k$. Let C denote the set of all feasible c. Given (P, c) and $\boldsymbol{\sigma}^N$, we let $g_k(t)$ be the conditional distribution over T^N of the k-th agent in the ordering who is type t. Here an element of T^N is a queue as defined above and $g_k(t)$ is derived from g, P, and $\boldsymbol{\sigma}^N$ via Bayes' rule. Now we can define equilibrium.

Definition 1 *Given (P, c) with $c \in C$, the strategy $\boldsymbol{\sigma}^N$ and the associated belief given by $g_k(t)$ is a perfect Bayesian equilibrium if for each $(k, h^{k-1}, t) \in (\mathbf{N} \times H^{k-1} \times T)$,*

$$\begin{aligned} & E_{g_k(t)} u[c_1^n(\boldsymbol{\sigma}^{n-1}, \sigma_n), c_2^n(\boldsymbol{\sigma}^{n-1}, \sigma_n, \boldsymbol{\sigma}_{n+1}^N), t] \\ \geq \quad & E_{g_k(t)} u[c_1^n(\boldsymbol{\sigma}^{n-1}, \tilde{\sigma}_n), c_2^n(\boldsymbol{\sigma}^{n-1}, \tilde{\sigma}_n, \boldsymbol{\sigma}_{n+1}^N), t], \end{aligned} \tag{1}$$

where $E_{g_k(t)}$ denotes expectation with respect to the distribution $g_k(t)$ and where $\tilde{\sigma}_n \neq \sigma_n$.

The next definition relies on the revelation principle.

Definition 2 *Given P, $c \in C$ is weakly implementable (by truth-telling), if for each $(k, h^{k-1}, t) \in (\mathbf{N} \times H^{k-1} \times T)$,*

$$\begin{aligned} & E_{g_k(t)} u[c_1^n(t^{n-1}, t), c_2^n(t^{n-1}, t, t_{n+1}^N), t] \\ \geq \quad & E_{g_k(t)} u[c_1^n(t^{n-1}, \tilde{t}), c_2^n(t^{n-1}, \tilde{t}, t_{n+1}^N), t] \end{aligned} \tag{2}$$

where $\tilde{t} \neq t$.

In (2), t_{n+1}^N is a random variable because it is the part of the queue that has not been revealed when the planner encounters the k-th person. In addition, part of (t^{n-1}, n) may be random. Its distribution depends on how much the planner reveals.

We can also define strong implementability while making use of the revelation principle.

Definition 3 *Given P, $c \in C$ is strongly implementable if truth-telling is the unique equilibrium for (P, c).*

Motivated in part by the comparisons between PS, GL and ANW, we compare different P's according to *coarseness*. The following nesting result is an immediate consequence of the law of iterated expectations.

Claim 1 *Let P' be coarser than P''. If $c \in C$ satisfies definition 2 for $P = P''$, then c satisfies definition 2 for $P = P'$.*

The planner's objective in this model is ex ante expected utility; namely

$$W(\boldsymbol{\sigma}^N; c, P) = E_g u[c_1^n(\boldsymbol{\sigma}^{n-1}, \boldsymbol{\sigma}_n), c_2^n(\boldsymbol{\sigma}^{n-1}, \boldsymbol{\sigma}_n, \boldsymbol{\sigma}_{n+1}^N), t]\}, \tag{3}$$

where g is the ex ante distribution over queues. The planner's *weak implementability problem* is to choose (P, c) subject to $c \in C$ and satisfaction of definition 2. The planner's *strong implementability problem* adds satisfaction of definition 3 as an additional constraint.[5]

[5] ANW mistakenly claim that weak implementability implies strong implementability if the planner reveals only place in the ordering. Their mistake can be explained using the above formulation.

Suppose that $N = 3$ and that π is the (independently and identically distributed) probability that a person is patient. Let c satisfy definition 2 when agents learn only their place in the ordering. In a truth-telling equilibrium, the second agent knows that with probability π the first agent announced patient and with probability $1 - \pi$ announced impatient. Our nesting implies that (2) is a weighted average of two "underlying" incentive conditions; one associated with the first agent announcing patient and the other associated with him announcing impatient, with weights π and $1 - \pi$, respectively.

Consider now a candidate (run) equilibrium where the first two agents announce impatient independent of type and the last agent announces truthfully—the only possibility for a run equilibrium with $N = 3$ when the ordering is revealed by the planner. Will the second agent defect? If patient, then that agent will announce patient only if the underlying incentive condition associated with the first agent announcing impatient is slack, which, of course,.is not implied by (2). Finally, inequality (2) says nothing about what the first agent does if he believes that the second will always announce impatient.

More generally, if there are N agents and each learns only his place in the ordering, then the incentive constraint (for truthful revelation) for the n-th agent is an average of 2^{n-1} underlying incentive conditions, one condition for each possible history of the previous $n - 1$ agent announcements. Again, satisfaction of the average does not imply satisfaction of the 2^{n-1} separate constraints. In particular, the incentive condition for agent n associated with

4 Two examples

For each of two examples, we describe the best weakly implementable allocation and the best strongly implementable allocation. One example is the PS appendix B example and the other is a slight variant of it. The PS example has $N = 2$, $Y = 6$, $R = 1.05$, equally likely queues, and $u\left(c_1, c_2, \mathit{impatient}\right) = 10v(c_1)$ and $u\left(c_1, c_2, \mathit{patient}\right) = v(c_1 + c_2)$, with $v(x) = -x^{-1}$. The alternative is identical except that $v(x) = \ln x$. We denote by w^i date-1 consumption for the first person to announce *impatient* whose position in the ordering is $i \in \{1, 2\}$. Given (w^1, w^2), the other components of the allocation are determined residually from the resource constraint.

Table 1 reports (w^1, w^2) and ex ante welfare of the best weakly and strongly implementable allocations under the different information revelation possibilities. (The full-information benchmark, in which the planner can observe the agent's type, is reported in the first row and other expected utilities are expressed relative to the normalized expected utility for it.) With $N = 2$, there are only three such schemes, which correspond to those studied by PS, GL, and ANW. Moreover, with $N = 2$, weak implementability implies strong implementability under reveal-place-in-the-ordering and reveal-everything.[6] Hence, there is only one allocation described for each of those schemes.

the previous $n - 1$ agents announcing impatient may not be satisfied. If not, then a bank run is possible.

GL get strong implementability when the planner reveals only place in the ordering because their preferences imply slack underlying truth-telling constraints.

[6] To understand uniqueness when $N = 2$ and only place in the ordering is revealed, suppose allocation $\left(w^1, w^2\right)$ is weakly implementable. If the candidate equilibrium has the first agent announcing "impatient" independent of type, then a patient second agent will announce "patient" because $R\left(Y - w^1\right) > Y - w^1$. Then, weak implementability implies that a patient first agent will defect from proposed equilibrium play and will announce truthfully. As explained in the last footnote, this result does not hold for $N > 2$.

Table 1. Optimal allocations: (w^1, w^2) and ex ante welfare		
Information Assumption	The PS example $v(x) = -x^{-1}$	The alternative $v(x) = \ln x$
No private information	(3.45,4.59) 1.000	(3.87,5.45) 1.0000
reveal nothing (weak solution)	(3.09,3.20) 0.9447	(3.10,3.20) 0.9121
reveal nothing (strong solution)	(3.15,3.15) 0.9444	(3.31,3.00) 0.9106
reveal place in the ordering	(2.98,3.31) 0.9435	(3.31,3.00) 0.9117
reveal everything	(2.99,3.15) 0.9346	(3.00,3.15) 0.9044
Autarky	(3.00,3.00) 0.9247	(3.00,3.00) 0.8966

For the kind of preferences in these examples, which are the same as those originally studied by DD, truth-telling is potentially binding only for patient people. Therefore, under reveal-nothing, there is one weak implementability (definition 2) constraint. For the examples, it takes the particular form

$$(1-\pi)(.5)\left\{v\left[\left(Y-w^1\right)R\right]+v\left[\left(Y-w^2\right)R\right]\right\}+\pi v\left(\frac{Y}{2}R\right) \quad (4)$$

$$\geq \quad .5\{v(w^1)+\left[(1-\pi)\,v\left(Y-w^1\right)+\pi v(w^2)\right]\},$$

where π is the (independently and identically distributed) probability that a person is patient, which is 0.5 in the examples. (When nothing is revealed, the patient person believes that the other person is impatient with probability $1-\pi$ and patient with probability π; and that with probability 0.5 he is either first or second in the ordering.) For the strong implementability problem under reveal-nothing, there is one additional constraint; namely,

$$v\left[\left(Y-w^1\right)R\right]+v\left[\left(Y-w^2\right)R\right] \geq v(w^1)+v\left(Y-w^1\right). \quad (5)$$

This says that a patient person prefers to announce truthfully when everyone else reports that they are impatient, given that it is equally likely that the person is first or second in the ordering.

Under reveal nothing, the weak and strong solutions differ for the PS example, as they report. They also differ for the alternative example. However, from the perspective of our model, a comparison between rows 2 and 3 is not enough. What would suffice is to confirm that the best weakly implementable allocation under the other information schemes gives lower ex ante welfare. And given nesting, it is enough to show that the solution to the weak implementability problem under reveal-place-in-the ordering is worse than under reveal-nothing.

Under reveal-place-in-the-ordering, there are two definition-2 implementability constraints for a patient agent:

$$(1-\pi)\, v\left[\left(Y-w^2\right)R\right]+\pi v\left(\frac{Y}{2}R\right)\geq v(w^1) \tag{6}$$

and

$$(1-\pi)\, v\left[\left(Y-w^1\right)R\right]+\pi v\left(\frac{Y}{2}R\right)\geq (1-\pi)\, v\left(Y-w^1\right)+\pi v(w^2). \tag{7}$$

Constraint (6) applies to a patient agent who is first in the ordering, while constraint (7) applies to one who is second in the ordering.

As shown in the fourth row of the table, both examples accomplish what PS set out to show—even from the broader perspective taken here. That is, even when we search over alternative information-revelation schemes by the planner, it remains true that there are settings in which the best weakly implementable allocation is not strongly implementable.[7]

It is also pertinent to study information schemes other than reveal-nothing if the goal is strong implementability. The solution to the strong implementability problem differs between the two examples. For the PS example, it is reveal-nothing. However, for the alternative example, the solution is reveal-place-in-the-ordering, but not everything.[8] This shows that solving the strong implementability problem can involve a nontrivial choice about what the planner should reveal.

5 Concluding remarks

The model above contains two extreme assumptions about the queue, assumptions that are best discussed against the background of the following related specification. Suppose that each agent gets a random draw, (t, τ), where, as above, $t \in T$ and where $\tau \in [0, 1]$ and is the *time* (of day) at which the agent

[7] This observation does not generalize to alternative economic environments. For example, Ennis and Keister [3] provide an example in which the first best allocation is weakly implementable under both reveal-nothing and reveal-place-in-the-ordering. However, it is strongly implementable under the latter and not under the former.

[8] Under reveal-everything, there are also two potentially binding definition-2 implementability constraints for a patient agent. When first, the constraint is (6) because the planner can only inform this person that he is first. When second and when following a person who announced *patient*, the constraint is $\frac{Y}{2}R \geq w^2$. (The constraint for a patient agent who is second and who follows a person who announced impatient is $\left(Y-w^1\right)R \geq Y-w^1$, which is always satisfied.)

will encounter the planner. One extreme assumption is that each agent does not observe his τ. (As PS say, the agent does not have a clock.) If the agent has a clock and privately observes (t, τ), then the agents have different signals about ordering in the queue even if the planner reveals nothing. A second extreme assumption is that agents cannot take actions, presumably costly actions, to influence the time at which they meet the planner. If they could, then we suspect that optima would display less dependence on ordering in the queue—dependence that we tend not to see.

Despite those extreme assumptions, the model has some nice features. Objects in the model match up in a simple and appealing way to objects in the actual economy. The agents are consumers and everything else in the model can be regarded as a consolidated banking-business sector. Consumers, who initially own all of the resource, exchange the resource for a bank deposit before learning their types. The optimal deposit contracts are consistent with the *demand* (self-selection) aspect of deposits and with low interest rates on deposit accounts if the interest rate is computed from the date-2 payoffs on deposits observed in the actual economy. (Given the kind of preferences assumed by DD, agents who acquire deposits rather than stay in autarky accept lower long-run (date 2) returns in exchange for better short-term returns.) The banking sector lends the resource in the form of callable loans to the business sector, which invests in the commonly available, constant returns-to-scale, intertemporal technology. The pattern of early withdrawal of deposits is necessarily matched by calls on loans and real investment liquidation. In the version studied here, it is information about that pattern that can be withheld by the planner in order to achieve better outcomes.

References

[1] D. Andolfatto, E. Nosal, N. Wallace, The role of independence in the Green-Lin Diamond-Dybvig model, J. of Econ. Theory, 137 (2007) 709-715.

[2] D. Diamond, P. Dybvig, Bank runs, deposit insurance, and liquidity, J. of Polit. Econ., 91 (1983) 401-19.

[3] H. Ennis, T. Keister, Run equilibria in the Green-Lin model of financial intermediation, J. of Econ. Theory, 144 (2009) 1996-2020.

[4] E. Green, P. Lin, Implementing efficient mechanisms in a model of financial intermediation, J. of Econ. Theory, 109 (2003) 1-23.

[5] E. Green, P. Lin, Diamond and Dybvig's Classic Theory of Financial Intermediation: What's missing?, F. R. B. Minneapolis Q. R., (Winter 2000) 3-13.

[6] P. Lin, Equivalence between the Diamond-Dybvig Banking Model and the Optimal Income Taxation Model, Economics Letters, Vol. 79 (2), May 2003, 193-198.

[7] J. Peck, K. Shell, Equilibrium bank runs, J. of Polit. Econ., 111 (2003) 103-123.