Production, Hidden Action, and the Payment System*

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Abstract

In this paper, we study a model economy that can account for the distribution of payments within a day. In our model, debtors choose when to arrive at the settlement location. Concomitant with choosing their arrival, debtors are making a production decision. We assume there is a cost to arriving early; that is, late-arrival is associated with a technology that dominates early arrival/production. Second, we treat the debtor's choice as hidden from creditors. We derive conditions in which the planner allocate production to each type. In the decentralized setting, there is an arbitrage condition that is consistent with a positive intraday rate. The central bank may be able to implement the planner's allocation with a proper intraday interest rate. In some cases, the intraday rate is positive.

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1 Introduction

In the United States, the payment system settles a very large volume of transactions. In 2007, Fedwire and CHIPS settled $7.3 trillion in transactions. The distribution of payments is disproportionately settled in the afternoon. Armantier, Arnold and McAndrews (2008) present evidence that twenty percent of the volume of payments occur before 1 pm in the Eastern Time Zone, leaving 80 percent settled between 1 pm and 6:30 pm.

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Freeman (1996) wrote the seminal paper on payment system risk. In his model economy, there was a sequencing friction. He found that both inside and outside money could be valued in equilibrium. In particular, outside money was valued as the means of gross settlement while inside money was valued as a means of financing current consumption. Inside money was risky in the sense that creditors and debtors were not necessarily matched at the settlement location. Freeman specified a secondary market for inside money when the timing friction resulted in such mismatches. Freeman's friction was exogenously determined and involved arrival and departures at the settlement location. Because debtors may not arrive to settle when creditors needed their liquidity, old-age creditor's consumption is at risk. More specifically, Freeman showed that with insufficient liquidity, IOUs traded in the secondary market might sell at less than par. In Freeman's model economy, the planner's allocation called for consumption to be equal across all old-age creditors, leaving open the question: is there a policy in the decentralized economy that would implement the first-best allocation? Freeman showed that the planner's allocation could be implemented in a decentralized economy if the central bank offered zero-interest, intra-period discount window loans.

In subsequent papers, researchers focused on two issues. Green (1997) asks two questions. First, what is the role of outside money as a means of final settlement? Second, what is the essence of the liquidity problem. Green then specifies a model in which the clearinghouse can offer intra-day loans. This specification goes toward our understanding of the role of the central bank as the sole supplier of liquidity. He demonstrates that there is no compelling reason why the central bank is the necessary solution to the liquidity problem because his clearinghouse can implement the same risk-sharing equilibrium Freeman's central bank. Mills (2004) extends the question further, asking a mechanism design question; under what conditions do liquidity-providing institutions have a role? Mills demonstrates that spatially separated agents must have no memory. With memory, outside money is not necessary to settle debt.[1]

Kahn and Roberds (2008) seek to understand money's role as the means of final settlement. They characterize money's role as consisting of transferability and finality. Transferability addresses the risk borne by third parties as they accept the debt as a means of payment. If, for example, third-party acceptance of IOUs means that debt is observationally equivalent to money, then the IOUs possess transferability. Finality is the characteristic that debts can be settled with money. Freeman's model economy is setup so that outside money possesses finality.

Bech and Garrett (2003) have studied the distribution of payments within the day using a game theoretic approach. They specify a game in which a day is divided between morning and afternoon. Banks receive payment requests and strategically choose when the banks will release the funds to settle the payment request. In a collateralized credit regime when liquidity is costly, there arises a prisoners' dilemma where banks settle in the afternoon but the equilibrium is socially inefficient. In a priced credit regime, there can arise a stag hunt game in which either all banks settle in the morning or all in the afternoon. Delay is socially optimal when the payment flows are skewed towards the afternoon.

In this paper, we are interested in understanding how settlement is affected in a payment system in which returns play a meaningful role. We specify a modified version of Freeman's economy in which there are two production technologies. First, the choice of the production technology is tantamount to choosing when debtors arrive to settle their IOUs. Production decisions pin down the timing of the debtor's arrival in our model economy and the returns paid. In this setup, returns are higher for the late-arriving debtor than for the early-arriving debtor, thus creating an incentive to settle debts in the afternoon. We do not intend to apply production returns to the ultra-high-frequency observations associated with the morning and afternoon settlements. Rather, we offer relative returns as one way that would account for non-uniform distribution of settlements within the day. In

[1] See also Townsend (1980) and Kocherlakota (1998) for detailed analyses of the role that memory plays in implementing equilibrium in monetary economies.

addition, the return differential may mean that there are social costs associated with the early-arriving, low-return technology. Indeed, we demonstrate that these social costs bear on the desirability of implementing policy that keeps intra-day rates equal to zero.

Overall, our paper permits us to examine three questions. First, because Freeman specifies a sequencing friction that is completely determined by nature, our model permits one to ask how incentives can influence the timing of debtor's decisions and, ultimately, the settlement process. For instance, if the debtor has the option, under what conditions would the debtor make payments before the last possible instant in a payment system? The evidence is quite clear; the distribution of payments, perhaps surprisingly, are not concentrated in the last few seconds that the clearinghouse is open.

Second, we are capable of identifying conditions under which the demand for liquidity incurs a social cost. In particular, we show that the interpretation of the creditor's departure and their consumption timing are critical to whether the social cost is present or not. If, for example, the creditor's departure is not tied to the timing of their consumption, there is no social cost. We call a creditor a Freeman-type consumer if creditors depart and then consume at the end of old-age. In contrast, for a Diamond-Dybvig type of consumer – that is, a creditor that consumes immediately after leaving the settlement location – then the planner may allocate less consumption to early-leaving than to late-leaving creditors because there is a social cost.

Third, we can assess whether monetary policy is sufficient to implement the planner's allocation. In general, the answer is no. The intraday interest rate, measured by the difference between the par value and the resale value of IOUs, can not implement the planner's allocation because it is one price. Here, there are two dimensions upon which the resale value is operating: the tradeoff between first- and second-stage production and the tradeoff between consumption by the early-leaving creditors and all other agents. Some other policy must be used, such as contingent monetary policy directed at early-leaving creditors or a non-distortionary redistribution scheme.

In this paper, we relax the condition that nature selects both the measure of creditors departing the settlement site early and the measure of debtors arriving at the settlement site early. We characterize the debtor's arrival time as a decision problem, applying three key modifications. First, IOUs are production loans as opposed to consumption loans. The production loans constitute a one-to-one mapping to the debtor's arrival time; more concretely, the first-stage technology corresponds to debtor arriving early at the settlement site and the second-stage technology corresponds to the debtor arriving late at the settlement site. With intra-day loans priced at zero nominal interest, there is an incentive for debtors to wait as long as possible to settle IOUs.[2] Second, the debtor faces an intertemporal consumption choice problem; old debtors derive utility from production. If the returns to production are positively related to time of arrival – that is, late-arriving debtors are rewarded with higher rates of return – then there is further incentive to delay arrival for as long as possible. Third, the debtor's production decision is a hidden action. It is revealed at the central island so that moral hazard is not a problem. But, there is no way of knowing the production/settlement schedule at the time IOUs are issued.

In our decentralized economy, outside money is not necessarily the only means of settlement. As such, finality differs from the concept developed in Freeman's model economy. Specifically, IOUs can be settled either with outside money or through an exchange of goods. As old creditors learn what specific goods they derive utility from, it is possible that old debtors provide them with those goods. Even if all IOUs are settled with goods, outside money is still valued because old debtors trade with young debtors in the same spirit as in the overlapping generations tradition; that is, money possesses the finality property as the means to execute intergenerational transfers.[3]

Three major results are derived. The first major result is derived in the context of the decentralized economy. We demonstrate that there exists a mixed-strategy equilibrium in

[2] See Martin (2004) provides an explanation for pricing intra-day loans.

[3] As Green (1997) hints at, even with two means of settlement, old-age creditors continue to be at risk. Memory does not exist to get around the need for payment even when money and goods are available to redeem IOUs.

which some measure of debtors will choose to arrive at the settlement site early. In fact, we show that there are no pure strategy equilibria in this model economy. Two things drive the mixed-strategy equilibrium: essentially, there is an arbitrage condition. The debtors are indifferent between the return offered by the second-stage technology and the combination of the return on the first-stage technology and gains made from purchasing discounted IOUs in the first stage and selling them at par value in the second stage. Insofar as the IOU is held from stage one to stage two and then settled at par value, the intra-period return on IOUs is greater than zero. In the decentralized economy, the mixed-strategy equilibrium has one important implication; namely, early-leaving old creditors will consume less than the late-leaving old creditors when IOUs sell at a discounted price.

The second major result deals with the planner's problem. Production timing dictates when goods are available for old-age creditors. Now settlement and consumption are potentially linked in a way not examined in Freeman's setup. Specifically, Freeman's studied a planner's problem in which consumption goods for old-age creditors were explicitly available from young-debtors only. In contrast, we propose a setup in which the supply of goods for old-age creditors is materially changed. The additional dimension lies between the first stage and second stage. We show that consumption smoothing across old-age creditors – that is, early leavers consume the same as late leavers – is the planner's allocation if there is no distinction between departure time and the treatment of the supply of debtor's goods. It follows that in this setup, the planner would choose only second-stage production to maximize social welfare. However, if one treats the creditor's departure time as akin to a Diamond-Dybvig consumption shock, we derive conditions in which the planner's allocation consists of both first- and second-stage production technologies in order to smooth consumption across the old-age creditors. Because early-leaving creditors must consume in the first stage, some first-stage production must be allocated in order to satisfy the planner's objective function, if the early-leaving creditors' consumption demand cannot be fully accommodated by an intergenerational transfer. Put more generally, the planner faces a trade-off along the consumption dimension and the production dimension: At the optimum, the planner equates the marginal cost of shifting production from second stage to the first stage with the marginal social gain of allocating more consumption goods to the early-leaving creditors.

The third major result pertains to the decentralized economy and the planner's allocation. In general, the decentralized economy will not implement the planner's allocation. In Freeman's setup, the central bank can temporarily expand the money supply to relax the liquidity constraint to restore the planner's allocation. The optimal interest rate in Freeman's economy is zero because the friction is purely due to timing; it does not affect the fundamentals. However, in an economy with Diamond-Dybvig type of consumption pattern, if the early-leaving creditors' consumption demand cannot be fully accommodated by the endowment of the young generation and the planner decides to allocate some resources to the first-stage technology, the planner faces a tradeoff: there exist marginal social costs because returns to first-stage technology are lower than returns to second-stage technology and marginal social gains of the additional consumption goods allocated to the early-leaving creditors. The planner determines the optimal sizes of investments in both technologies by evaluating the tradeoff. However, the decentralized economy fails to achieve the planner's allocation with any intraday rate for two reasons. First, the intraday rate affects both the creditors and the debtors' intertemporal consumption cost and thus their choice of trade volume and production quantity. Adjusting the intraday rate cannot correct the distortions in both agents' intertemporal choices. In other words, we have too few tools to achieve the multiple policy goals. Second, as the debtors choice of technology is unobservable, the unified market price of capital fails to reflect the social cost of allocating some resources to the low-return, first-stage technology.

The paper is organized as follows. In Section 2, we describe the physical environment. Equilibrium in the decentralized economy is characterized in Section 3. The planner's problem is specified and solved in Section 4. In Section 5, we characterize the role of the central bank, describing policies that would implement the planner's allocation. In Section 6, we briefly summary our findings and offer concluding remarks.

2 The physical environment

We consider a modified version of the payment system model developed by Freeman (1996). The key difference is that young debtors can create capital from goods borrowed from creditors. In doing so, debtors can finance their old-age consumption.

There is an infinite sequence of time periods. Dates are indexed by $t = 0, 1, 2, ...$ At each date t, there are two subperiods. The first subperiod is marked by the timing at which creditors and some debtors arrive at the settlement location. At the end of the first subperiod, some creditors depart and the remaining old debtors arrive. There are I outer islands where I is a large, even number. The outer islands are arranged in pairs around a central island. Within each pair, there is a creditor island and a debtor island. There is also a central island. Here, an enforcement authority costlessly enforces all contracts. In addition, a monetary authority exists on the central island.

At each date $t \geq 0$, two-period lived agents are born. There is a continuum of agents of measure one born at each date on each island. At date $t = 0$, there is a continuum of agents called the initial old that live for one period. Each island has a unique perishable consumption good. Hence, there are $i = 1, 2, ..., I$ types of debtor and creditor consumption goods.

On the creditor island, the young born at date $t \geq 0$ are endowed with y units of the island-specific consumption good. The creditor derives utility from their home-island creditor good when young and from a debtor-island good when old. We represent the creditor's lifetime utility as $u(cy_{1t}, cx_{2t+1})$, where u is strictly concave in both goods. Formally, $u_j > 0$, $u_{jj} < 0$, $\lim_{l \to 0} u_j = \infty$, and $\lim_{l \to \infty} u_j = 0$, $l = cy_{1t}$, cx_{2t+1}, where u_j is the derivative of the creditor's utility function with respect to the j^{th} argument. We further assume that utility is separable with $u_{12} = u_{21} = 0$.

On the debtor island, the young born at date $t \geq 0$ are endowed with x units of the island-specific consumption good. The debtor wants to consume during both periods of their lifetime. Here, we assume that the debtor only wants to consume units of their home-island good. Let preferences be represented as $v(dx_{1t}, dx_{2t+1})$. The debtor's utility function is strictly concave in each consumption good. Formally, we assume the debtor's utility function is characterized as follows: $v_j > 0$, $v_{jj} < 0$, $\lim_{l \to 0} v_j = \infty$, and $\lim_{l \to \infty} v_j = 0$, $l = dx_{1t}$, dx_{2t+1}, where v_j is the derivative of the debtor's utility function with respect to the j^{th} argument. We further assume that utility is separable with $v_{12} = v_{21} = 0$.

The young debtor can costlessly travel to the creditor island at the beginning of each period to meet with the young creditors. The young debtor can costlessly transform units of the creditor consumption good into capital, here denoted by k, at one-for-one rate. The capital can be used to produce home-island consumption good. There are two technologies, first-stage and second-stage, respectively, available to transform date-t capital to date-$t+1$ home-island consumption good. The production functions for the two technologies are $f(k)$ and $F(k)$, respectively. For a given level of capital, the return to the second-stage technology dominates the return to the first-stage technology. Formally, $f(k_0) < F(k_0)$ for any $k_0 > 0$. At date $t + 1$, the first-stage technology matures in the first-subperiod whereas the second-stage technology matures in the second subperiod. Physical capital fully depreciates after one period. We assume that the functions $f(k)$ and $F(k)$ are strictly increasing, strictly concave. After meeting with the young creditors, the young debtor returns to his home island, where he will meet the old creditors who are randomly relocated to his island and the old debtors who just return to the home island.

When old, the debtors arrive at the central island either in the first or second subperiod. The arrival time is determined by the maturation of the production technology. Upon maturation, the old debtor travels costlessly to the central island. If he has chosen the first-stage technology, he will arrive at the central island in the first subperiod. We refer to these as early producers. The counterparts are called late producers. Let λ denote the measure of early producers, while $1-\lambda$ is the measure of late producers.

When old, the creditor costlessly travels to the central island. With probability $1-\alpha$, the old creditors will be asked to leave the central island after the first subperiod is over. The likelihood that the old creditor will leave at the end of the second stage is α. When the old creditors leave the central island, each old creditor is randomly assigned preferences for the consumption good of a particular debtor's island. We assume that the distribution of the preferences as such that the demand for debtor good is uniform.

The initial old debtors are endowed with capital. The initial old creditor is endowed with m_0 units of fiat money. Let k_0 denote the endowment of physical capital that the initial old debtors possess.

3 The decentralized economy

In this physical environment, it is possible to illustrate how debtors and creditors might interact in a decentralized economy. For agents born at date $t \geq 0$, both debtors and creditors will consume some of their endowment. Debtors would like to acquire some of the young creditors goods to finance old-age consumption. However, young debtors do not have anything that young creditors want when they meet at the beginning of the period. Debtors offer an IOU, purchasing some of the young creditor's endowment, transforming the good into capital and waiting for old-age production. By accepting the IOU, the young creditor gives up some of his consumption good. The young debtor then returns to his home island, where he can sell some of his endowment to the old creditors who are relocated to his island. The old creditors pay the young debtor using money. The young debtor then starts to produce with his choice of technology.

When old, both creditors and debtors go to the central island to settle their accounts using either money or real goods. Following Freeman, the key friction is that arrivals and departures are non-synchronous. If not all debtors are early producers, then old creditors leave the central island, perhaps before meeting their debtors because of production delays.

In other words, while old creditors arrive at the central island en masse, only λ measure of old arrive in the first subperiod. When the first subperiod is over, $1-\alpha$ measure of old creditors leave and the remaining $1-\lambda$ measure of old debtors arrive. Put more concretely, during first subperiod, there is a measure one of old creditors on the central island and λ measure of old debtors. During second subperiod, there are α measure of old creditors on the central island and measure one of old debtors on the central island. Whichever subperiod we are in, old creditors costlessly find their old debtors and the accounts are settled; that is, IOUs are redeemed.[4] The early-leaving creditors can sell the IOUs issued by the late producers in a loan resale market on the central island. The potential buyers of the IOUs are the late-leaving creditors and early producers. During the second subperiod, settlement is completed and both old debtors and old, late-leaving creditors leave the central island. Old debtors return home and consume. Old creditors, go the assigned debtor island, trade with the young debtors using money and then consume.

3.1 Debtor problem

We assume that all debtors born at date $t \geq 0$ are the same. Consequently, we can treat a young debtor's problem as identical, dropping the i subscript. Here, a young debtor chooses the production technology they will adopt, which determines when they will arrive at the central island. Note that young debtors are not permitted to diversify among their production choices. Young debtors always choose the technology that results in higher lifetime utility, taking all prices as given.

[4] Alternatively, we permit old debtors to trade amongst themselves, acquiring enough of the appropriate goods to redeem the IOUs with goods. The mix of outside money or goods used to settle IOUs is indeterminate. If old debtors use money to settle debt, then old creditors take money and acquire consumption from young debtors. If old debtors settle with goods, then we permit old debtors to costlessly travel in the second subperiod to a debtor island, taking money to trade with young debtors.

3.1.1 Early producer

If a debtor chooses to be an early producer, he has the following budget constraints when young:

$$p_{xt}x = p_{xt}dx_{1t} + m_t \quad (1)$$

$$p_{yt}k_t = h_t \quad (2)$$

where m_t is the demand for currency, and h_t is his nominal value of the debtor's indebtedness. h_t is also the nominal value of IOUs he issues to a creditor to acquire capital from young creditors. Thus, $p_{yt}k_t = h_t = p_{yt}dy_t$. The debtor will repay the creditor in the second period using either real goods or money.

An early producer arrives on the central island at the first stage in his second period of life, and he can trade with other creditors who needs credits or goods for IOUs. Thus, budget constraint for an early producer when old is

$$p_{xt+1}f(k_t) + m_t - h_t + b_{t+1}\left(1 - \rho_{t+1}\right) = p_{xt+1}dx_{2t+1} \quad (3)$$

where b is the par value of the debt purchased by the debtor, ρ is the discounted nominal value of one dollar of that debt. Note that under this constraint, debtors are capable of meeting their their old-age needs – consumption and settlement – through a combination of production, outside money and gains from IOU purchases. Money holdings by the old debtors need not be equal to IOU values.[5]

The life-time budget constraint for the early producer is

$$p_{xt+1}f(k_t) + p_{xt}\left(x - dx_{1t}\right) - p_{yt}k_t + b_{t+1}\left(1 - \rho_{t+1}\right) = p_{xt+1}dx_{2t+1} \quad (4)$$

$$x - dx_{1t} \geq 0 \quad (5)$$

[5] To illustrate, suppose that the old debtor used production returns to settle debt, taking the money to the home island to purchase goods from young debtors. Money is used only to execute intergenerational transfers. The key point is some combination the real bills doctrine and fiat money coexists.

The early producer faces a liquidity constraint in the loan resale market:

$$p_{xt+1}f(k_t) + p_{xt}(x - dx_{1t}) - p_{yt}k_t - \rho_{t+1}b_{t+1} \geq 0 \tag{6}$$

which says the early producer cannot borrow to purchase the IOUs.

An early producer thus solves the following maximization problem

$$\begin{array}{cc} \max_{dx_{1t},dx_{2t+1},k_t,b_t} & v(dx_{1t}, dx_{2t+1}) \\ s.t. & (4) \quad - (6) \,. \end{array}$$

The first-order conditions for the early producers are

$$\begin{aligned} (x - dx_{1t})\left(v_1(dx_{1t}) - v_2(dx_{2t+1})\frac{p_{xt}}{p_{xt+1}}\frac{1}{\rho_{t+1}}\right) &= 0 && (7)\\ f'(k_t) - \frac{p_{yt}}{p_{xt+1}} &= 0 && (8)\\ v_2(dx_{2t+1})\frac{1-\rho_{t+1}}{p_{xt+1}} - \mu_1\rho_{t+1} &= 0 && (9) \end{aligned}$$

where μ_1 is the Lagrangian multiplier associated with an early producer's liquidity constraint. If the liquidity constraint is nonbinding, then the complementary slack condition is satisfied if and only if $\mu_1 = 0$. With the nonbinding liquidity constraint, note that $v_2(dx_{2t+1})\frac{1-\rho_{t+1}}{p_{xt+1}} - \mu_1\rho_{t+1} = 0$ is satisfied if and only if $\rho = 1$. Alternatively, if the liquidity constraint is binding, we obtain a case in which $\rho < 1$.

3.1.2 Late producer

If a debtor chooses to be a late producer, his budget constraints when young are

$$\begin{aligned} p_{xt}x &= p_{xt}dx^*_{1t} + m^*_t && (10)\\ p_{yt}k^*_t &= h^*_t && (11) \end{aligned}$$

where the notation with superscript star means "late producers".

When old, the debtor will not be able to trade in the loan resale market as he arrives late. The late producer's budget constraint when old is

$$p_{xt+1}F(k_t^*) + m_t^* - h_t^* = p_{xt+1}dx_{2t+1}^* \tag{12}$$

The late producer's life-time budget constraint is

$$\begin{aligned} p_{xt+1}F(k_t^*) + p_{xt}\left(x - dx_{1t}^*\right) - p_{yt}k_t^* &= p_{xt+1}dx_{2t+1}^* && (13) \\ x - dx_{1t}^* &\geq 0 && (14) \end{aligned}$$

A late producer solves the following maximization problem:

$$\begin{array}{cc} \max_{dx_{1t}^*, dx_{2t+1}^*, k_t^*} & v(dx_{1t}^*, dx_{2t+1}^*) \\ s.t. & (13) - (14)\,. \end{array}$$

The first-order conditions for the late producer's problem are:

$$\begin{aligned} \left(x - dx_{1t}^*\right)\left(v_1\left(dx_{1t}^*\right) - v_2\left(dx_{2t+1}^*\right)\frac{p_{xt}}{p_{xt+1}}\right) &= 0 && (15) \\ F'\left(k_t^*\right) - \frac{p_{yt}}{p_{xt+1}} &= 0 && (16) \end{aligned}$$

A debtor compares the lifetime utilities from using the first-stage and the second-stage technologies given the prices of goods, the resale price of the loan, and the fractions of debtors who choose the first-stage and the second-stage technologies, respectively. If $v\left(dx_{1t}, dx_{2t+1}\right) > v\left(dx_{1t}^*, dx_{2t+1}^*\right)$, all debtors will choose to be early producers; whereas if $v\left(dx_{1t}, dx_{2t+1}\right) < v\left(dx_{1t}^*, dx_{2t+1}^*\right)$, all debtors will choose to be late producers. If $v\left(dx_{1t}, dx_{2t+1}\right) = v\left(dx_{1t}^*, dx_{2t+1}^*\right)$, a debtor is indifferent to either of the technologies. Each debtor is assumed to choose the first-stage technology with probability λ. By law of large numbers, a fraction of λ debtors use the first-stage technology and pay their debts early.

3.2 Creditor problem

Here, we formalize the problem solved by a creditor. For all creditors born at date $t \geq 0$, the objective is to maximize lifetime utility. When young, the budget constraint is

$$p_{yt}y = p_{yt}cy_{1t} + l_t. \tag{17}$$

In other words, the nominal value of the creditor's endowment is equal to the nominal value consumed of his own good plus any loans offered to young debtors. We assume the debtors can buy from as many creditors as they wish, and they can buy arbitrary amounts of capital from a debtor. A creditor thus cannot distinguish whether a buyer is a late producer or an early producer from the quantity of capital that a buyer purchases from him. When old, creditors are randomly assigned to either leave the central island early or late. The creditor's old-age departure can affect the trading outcomes. If the old creditor leaves the central island early, there is a chance that the old creditor and old debtors are not matched temporally; that is, some of the old debtors will not have arrived. Consequently, the early-leaving creditor faces the following old-age budget constraint:

$$\rho_{t+1}\left(1 - a_t\right) l_t + a_t l_t = p_{xt+1}cx_{2t+1} \tag{18}$$

where ρ is the price that the old creditor sells IOUs and a is the proportion of the IOUs that is repaid early. **In this way, the old creditor can sell the IOUs that are not redeemed and recoup at least some of the value.** The proportion of the IOUs that needs to be sold on the resale market is $1 - a_t$. The rest of the IOUs will be paid in full. With these resources, the old-age creditor can purchase units of the consumption good from debtors (either young or old or both). We assume that creditors can trade debtor island-specific goods with each other on the central island. So if the IOUs are paid in goods, the creditors can convert it to his preferred debtor island-specific goods.

If, in contrast, the old creditor leaves late, there is no risk that creditor and **his debtors** will not meet on the central island. In this event, the old-age budget constraint for the creditor is written as:

$$l_t + \left(1 - \rho_{t+1}\right) q_{t+1} = p_{xt+1} cx^*_{2t+1} \tag{19}$$

where the late-leaving old creditor has the value of the IOUs, the value represented by the IOUs purchased in the secondary market, denoted by q. Together, these resources are used to purchase units of the consumption good. Let cx^*_2 denote the consumption by the late leaving old creditor.

The liquidity constraint for a late-leaving creditor is

$$a_t l_t - \rho_{t+1} q_{t+1} \geq 0 \tag{20}$$

Thus, the creditor solves the following life-time maximization problem:

$$\begin{array}{cc} \max_{l_t, q_t, cy_{1t}, cx_{2t+1}, cx^*_{2t+1}} & (1-\alpha) u\left(cy_{1t}, cx_{2t+1}\right) + \alpha u\left(cy_{1t}, cx^*_{2t+1}\right) \\ s.t. & (17) - (20). \end{array}$$

Substitute cy_{1t}, cx_{2t+1}, cx^*_{2t+1} using l_t and q_{t+1}, the first-order conditions with respect to l_t and q_t for this problem are:

$$-u_1\left(cy_{1t}\right) + \left(\begin{array}{c} (1-\alpha) u_2\left(cx_{2+1}\right)\left(a_t + \rho_{t+1}\left(1 - a_t\right)\right) + \\ \alpha u_2\left(cx^*_{2t+1}\right) \end{array}\right) \frac{p_{yt}}{p_{xt+1}} + p_{yt}\mu_2\lambda = 0 \tag{21}$$

$$\alpha u_2\left(cx^*_{2t+1}\right) \frac{1 - \rho_{t+1}}{p_{xt+1}} - \mu_2 \rho_{t+1} = 0, \tag{22}$$

where μ_2 is the Lagrangian multiplier associated with creditor's liquidity constraint. Combining the two FOCs, we get,

$$-\frac{u_1\left(cy_{1t}\right)}{p_{yt}}+\left(a_t+\rho_{t+1}\left(1-a_t\right)\right)\left(\left(1-\alpha\right)\frac{u_2\left(cx_{2t+1}\right)}{p_{xt+1}}+\alpha\frac{1}{\rho_{t+1}}\frac{u_2\left(cx^*_{2t+1}\right)}{p_{xt+1}}\right)=0. \quad (23)$$

If the liquidity constraint is not binding, then by (22) we have $\rho_{t+1}=1$ and $cx_{2t+1}=cx^*_{2t+1}$.

3.3 Equilibrium

All markets clear in equilibrium. The goods market clearing conditions at date t are

$$\begin{aligned} y &= cy_{1t}+\lambda k_t+\left(1-\lambda\right)k^*_t, & (24)\\ x+\lambda f\left(k_{t-1}\right)+\left(1-\lambda\right)F\left(k^*_{t-1}\right) &= \lambda\left(dx_{1t}+dx_{2t}\right)+\left(1-\lambda\right)\left(dx^*_{1t}+dx^*_{2t}\right)+ & (25)\\ &\quad\left(1-\alpha\right)cx_{2t}+\alpha cx^*_{2t}. \end{aligned}$$

The money market clearing condition is

$$\lambda m_t+\left(1-\lambda\right)m^*_t=m_0 \quad (26)$$

The loan market clearing condition at date t is

$$l_t=\lambda h_t+\left(1-\lambda\right)h^*_t. \quad (27)$$

The equilibrium proportion of loans that is repaid early is

$$a_t=\frac{\lambda h_t}{\lambda h_t+\left(1-\lambda\right)h^*_t}. \quad (28)$$

The loan resale market clearing condition at date t is

$$\lambda b_t + \alpha q_t = (1-\alpha)\,(1-a_{t-1})\,l_{t-1}. \tag{29}$$

The first-order conditions and the market clearing conditions pin down the equilibrium amount of consumption goods for both agents, the relative prices of goods, the equilibrium ratio of early producers, and the equilibrium price of loans in the resale market. Proposition 1 shows that in the equilibrium, there will be a fraction of, but not all, debtors choosing the first-stage technology. The resale price of loan on the central island is always less than 1, that is, the liquidity constraint is always binding in the resale market.

Proposition 1 *There exists a mixed strategy Nash equilibrium,* $\lambda \in (0,1)$ *and* $\rho \in (0,1)$*. Moreover, no pure-strategy Nash equilibrium exist.*

Proof. Suppose $\lambda = 1$. Thus, all debtors choose the first-stage technology. From equation (28), it follows that $a_t = 1$. By the market clearing condition (29), we have $b_{t+1} = q_{t+1} = 0$. The early producers will not gain at all from the resale market. Compare the life-time budget constraints of an early producer and of a late producer, a late producer can consume more goods when old. Thus, we reach a contradiction.

Suppose all debtors choose the second-stage technology; that is, $\lambda = 0$, then $a_t = 0$. The liquidity constraint of a late creditor implies that $q_{t+1} = 0$. The resale market will not clear. Again, we reach a contradiction. Thus, neither $\lambda = 1$ nor $\lambda = 0$ – the pure strategies – are equilibrium.

That λ is strictly between 0 and 1 implies that a debtor is indifferent to the technologies given the equilibrium λ. That is, a debtor cannot benefit from technology arbitrage in equilibrium.

$$v(dx_{1t}, dx_{2t+1}) = v(dx^*_{1t}, dx^*_{2t+1}). \tag{30}$$

Now let us prove that $0 < \rho_{t+1} < 1$. Note that if $\rho = 1$, the early producers do not gain from the loan resale market. Compare the lifetime budget constraints of the early producers and late producers and by the first-order conditions that $f'(k_t) = F'(k^*_t) = \frac{p_{yt}}{p_{xt+1}}$, the late producers' budget is greater than the early producers' budget when $\rho = 1$. So all debtors will choose to be late producers, which contradicts the fact that $\lambda < 1$. If $\rho = 0$, the early producers and late-leaving creditors have infinitely large demand for the loans on the resale market, so the market does not clear. Therefore, the resale price of the loan is strictly between 0 and 1. ■

Intuitively, debtors follow a mixed strategy in order to satisfy the arbitrage condition. Once at the settlement site, they can find creditors leaving early and offer payment at less than par for the IOUs. Because there is no default risk, the return to early production plus intra-period gains from the resale market equal the return paid to late producers.

Note there is payment-form indeterminacy in the gross settlement. Debtors are free to settle IOUs with goods, outside money, or any combination of the two. If a debtor settles IOUs with goods, he can buy home-island goods from the young debtors using outside money. Mills (2004) offered a mechanism design explanation for why outside money is valued in Freeman's model economy; namely, that with memory creditors would accept goods at settlement from debtors. In our setup, outside money is valued for settlement purposes and for executing intergenerational transfers. We share the same absence-of-memory feature as Freeman exhibits. The key difference is that memory-loss does not coincide with finality of money. In Mills, the absence of memory was sufficient to render outside money, and by association the central bank, useless. In our setup, money is not necessary to settle IOUs and thus money does not satisfy Kahn-Roberds finality condition. Despite the absence of finality, outside money is valued because it serves two simultaneous roles: it can serve as the means of final settlement and it is the means of executing intergenerational transfers.[6]

In the following sections, we will focus on the stationary equilibrium, or the steady state. In the steady state, prices and allocations are invariant to time. So we will drop the

[6] There is nothing that pins down the method used by debtors to settle debt. They can use either outside money or goods. One would need an additional friction in the model economy to pin down this feature. We thank David Mills for pointing out this issue in a previous draft.

time subscripts.[7]

4 Planner's allocation

We now turn to the discussion of the unconstrained first-best allocation, or a planner's allocation. We consider a planner's problem in two slightly different settings. The settings are distinguished in terms of how we model the timing of consumption needs of the early-leaving creditors. In the first setting, we follow Freeman's approach; that is, early-leaving creditors do not have to consume in the first stage. We refer to this consumption pattern as "Freeman type creditor." In the second setting, the early-leaving creditors have to consume in the first stage. They do not value the consumption in the second stage. Such a setting captures features of consumption timing that may be present in the payment system. For example, one can imagine situations in which creditors require payment to finance a consumption shock during the day. Because this consumption timing is similar to the ex post consumption shock studied in Diamond and Dybvig (1983), we refer to it as the "Diamond-Dybvig type creditor."

4.1 Freeman type creditor

Following Mills (2004), we set up a social planner's problem as follows.

$$\max_{cy_1,cx_2,cx_2^*,dx_1,dx_2,k,k^*,\lambda} \quad \beta \begin{pmatrix} \lambda v\left(dx_1, dx_2\right) + \\ \left(1-\lambda\right) v\left(dx_1^*, dx_2^*\right) \end{pmatrix} + \left(1-\beta\right) \begin{pmatrix} \alpha u\left(cy_1, cx_2^*\right) + \\ \left(1-\alpha\right) u\left(cy_1, cx_2\right) \end{pmatrix}$$

[7] The uniqueness of the steady state is not guaranteed. But the uniquess exists in some classes of utility functions and production functions. For example, log linear utility function and Cobb-Douglas production functions.

$$
\begin{aligned}
s.t. \quad & x + \lambda f(k) + (1-\lambda) F(k^*) = (1-\alpha) c x_2 + \alpha c x_2^* + \lambda (d x_1 + d x_2) + (1-\lambda)(d x_1^* + d x_2^*) \\
& y = c y_1 + \lambda k + (1-\lambda) k^* \\
& 0 \leq \lambda \leq 1
\end{aligned}
$$

where β is the weight of debtors in the planner's welfare function. The first two constraints are the resource constraints for goods x and y, respectively.

We denote the solution to the planner's allocation by $\left(c\hat{y}_1, c\hat{x}_2, c\hat{x}_2^*, d\hat{x}_1, d\hat{x}_2, \hat{k}, \hat{k}^*, \hat{\lambda}\right)$. It is easy to see that the planner devotes all resources to the second-stage technology because it produces more goods than the first-stage technology. That is, we have $\hat{k} = 0$ and $\hat{\lambda} = 0$. The solution to the planner's problem satisfies the following first-order conditions that describe the consumptions for each type of agents in both periods:

$$
\begin{aligned}
v_1(d\hat{x}_1^*) &= v_2(d\hat{x}_2^*), && (31) \\
v_2(d\hat{x}_2^*) &= \frac{1-\beta}{\beta} u_2(c\hat{x}_2^*), && (32) \\
u_1(c\hat{y}_1) &= u_2(c\hat{x}_2^*) F'\left(\hat{k}^*\right), && (33) \\
c\hat{x}_2 &= c\hat{x}_2^*. && (34)
\end{aligned}
$$

The planner treats the early-leaving and late-leaving creditors equally. That is, $c\hat{x}_2 = c\hat{x}_2^*$. An early-leaving creditor's consumption is not affected by his obligation to leave the central island early as the planner can transfer him some of the produced goods at the second stage when production is completed. A planner can always use tax-transfer policy to reallocate the income to achieve the desired wealth distribution that satisfies equation (32).

4.2 Diamond-Dybvig type creditor

In this alternative setting, the planner has an additional resource constraint in the first stage of a period: the amount of good x available at the first stage must be large enough to accommodate the consumption demanded by the early-leaving creditors. That is,

$$x + \lambda f\left(k\right) \geq \left(1-\alpha\right) c x_2 \tag{35}$$

With the additional constraint, the planner's solution is in one of the following three cases.

Case 1: The resource constraint (35) is unbinding, which means $x > \left(1-\alpha\right) c\hat{x}_2$ and $\hat{\lambda} = 0$. Then the planner's solution is the same as in the Freeman setting. The solution is described by equations $(31) - (34)$.

Case 2: The resource constraint (35) is binding, and the planner's solution requires $\hat{\lambda} = 0$ and $x = \left(1-\alpha\right) c\hat{x}_2$. In this case, the planner still invests all resources in the second-stage technology. He rations the endowments of the young debtors among the early-leaving creditors. The first-order conditions are equations $(31)-(33)$. In particular, the first-order conditions implies

$$u_2\left(c\hat{x}_2\right) = u_2\left(c\hat{x}_2^*\right) + \mu_3 \tag{36}$$

where μ_3 is the Lagragian multiplier associated with (35). Because (35) is binding, we have $\mu_3 > 0$ and, by concavity, $c\hat{x}_2^* > c\hat{x}_2$.

Case 3: The resource constraint (35) is binding, and the planner's solution requires $\hat{\lambda} > 0$ and $x < \left(1-\alpha\right) c\hat{x}_2$. The planner invests some resources in the first-stage production technology, and allocates the endowments of the young debtors and produced goods from the first-stage production. The first-order conditions are

$$\begin{aligned}
d\hat{x}_1 &= d\hat{x}_1^*, && (37)\\
d\hat{x}_2 &= d\hat{x}_2^*, && (38)\\
u_1\left(c\hat{y}_1\right) &= u_2\left(c\hat{x}_2\right) f'\left(\hat{k}\right), && (39)\\
u_2\left(c\hat{x}_2\right) f\left(\hat{k}\right) - u_1\left(c\hat{y}_1\right)\hat{k} &= u_2\left(cx_2^*\right) F\left(\hat{k}^*\right) - u_1\left(c\hat{y}_1\right)\hat{k}^*. && (40)
\end{aligned}$$

and equations $(31)-(33)$. (36) still hold in this case. Equation (40) is the first-order condition with respect to the measure of first-stage producing debtors; that is, the fraction of λ. The left-hand side of (40) is the net welfare gain from having an additional unit of first-stage production; in other words, the marginal utility to early-arriving creditors from the extra production less the marginal utility of less consumption when young. The right-hand side is the welfare loss, which is the difference between the marginal value of smaller second-stage production to all other agents less the marginal value of creditors' consumption when young. (Note that the debtors' weighted marginal utility when young and old, $\frac{\beta}{1-\beta}v_1\left(dx_1^*\right)$ and $\frac{\beta}{1-\beta}v_2\left(dx_2^*\right)$, respectively, are equal to $u_2\left(cx_2^*\right)$. $F\left(k^*\right)$ is the total consumption reduced by shifting one unit of production from the second-stage to the first-stage. The loss of $F\left(k^*\right)$ is borne by the debtors and the late-leaving creditors.) The optimal λ equates the welfare gain to the welfare loss. Also note that by equations (31), (37), and (38), we have the following equation for an early producer's intertemporal consumption decision:

$$v_1\left(d\hat{x}_1\right)=v_2\left(d\hat{x}_2\right) \tag{41}$$

In the planner's allocation, early producers and late producers are treated equally. In contrast, early-leaving creditors and late-leaving creditors may be treated differently. To illustrate this point, suppose consumption allocated to the early-leaving creditors is strictly less than the endowments of the young debtors – that is, $x>(1-\alpha)\,c\hat{x}_2$ – then consumption of the early-leaving creditors can be completely accommodated by a resource transfer from the young debtors to the early-leaving creditors. Under this condition, consumption allocated to the early-leaving creditors will be the same as that allocated to the late-leaving creditors. In contrast, if the early-leaving creditors are rationed to consume the endowments of the young debtors or if production needs to be allocated to the first-stage technology to accommodate the early-leaving creditors' consumption, then we have $c\hat{x}_2^*>c\hat{x}_2$. Thus, early-leaving creditors will be treated less favorably by the planner.

Compare the first-order conditions for the decentralized economy and planner's allocation. The following proposition summarizes the comparison between the planner's allocation and the decentralized equilibrium. We consider both the Freeman-type creditor and all three Diamond-Dybvig type creditors.

Proposition 2 *The decentralized economy does not achieve the planner's allocation.*

Proof. The comparison between the Freeman-type creditor and the decentralized equilibrium is straightforward. In the planner's allocation, $\hat{\lambda} = 0$ whereas the equilibrium value has λ lying between zero and one. For Diamond-Dybvig type creditors, the planner will allocate zero measure to first-stage production in both Cases 1 and 2. Clearly, the decentralized economy will not implement the planner's allocation in these two cases.

In Case 3, the resource constraint for the early-leaving creditors is binding and the planner's allocation calls for $\hat{\lambda} > 0$. Note that the planner's allocation calls for $v_1 (d\hat{x}_1) = v_2 (d\hat{x}_2)$; that is, the debtor's intertemporal marginal rate of substitution is equal to one. However, in a stationary equilibrium in the decentralized economy, debtors choosing first-stage production will satisfy the following first-order condition: $(x - dx_1) \left(v_1 (dx_1) - v_2 (dx_2) \frac{1}{\rho} \right) = 0$. With $0 < \rho < 1$, $v_1 (dx_1) > v_2 (dx_2)$, in other words, the early-arriving debtor's intertemporal marginal rate of substitution is less than one. Hence, there is no equilibrium value of ρ that implements the planner's allocation in any of the Diamond-Dybvig economies, including Case 3. ■

There are two reasons that account for why the equilibrium in the decentralized economy is not identical to the planner's solution. First, in a decentralized economy, IOUs must be discounted to attract some of the debtors to arrive early, thus providing liquidity to those creditors who need to consume immediately. As a result, excessive resources are used in low-return technology in a decentralized economy. Second, in a planner's economy, the planner can transfer goods from the young debtors to the early-leaving creditors and compensate the young debtors when the second-stage production is completed. In a decentralized economy, the old creditors cannot buy more than what the young debtors want to sell, unless there exists a fiscal policy that taxes the young debtors at the first stage at time t, and then transfers goods from the old debtors to the young at the second stage.

4.3 Social cost and the planner's allocation

In a planner's economy, the early-leaving creditors' consumption demand might incur a social cost in the form of lost output. These social costs are observed in Cases 2 and 3 with Diamond-Dybvig type creditors. To illustrate this point, consider an economy populated with Freeman-type creditors. Because consumption takes place after the high-return, second-stage production completes, there is no lost output in this economy. No production needs to be sacrificed in this version of the planner's problem. Similarly, in an economy populated with Diamond-Dybvig type of creditors, there is no lost output if the early-leaving creditor's consumption needs can be completely accommodated by transferring endowments of the young to the early-leaving creditors (i.e. $x > (1-\alpha)\, c\hat{x}_2$). The planner allocates all capital at desired level to the second-stage production.

Consider a case in which $x \leq (1-\alpha)\, c\hat{x}_2$. The early-leaving creditors' consumption demand results in a reduction in production through two channels. First, with early-leaving creditors being allocated less consumption than the late-leaving creditors – $c\hat{x}_2 < c\hat{x}_2^*$ – when the resource constraint is binding, the planner partially compensates the creditor for the consumption uncertainty by letting them consume more when young. With greater consumption by young creditors, there are fewer resources for capital by young debtors, implying that investment decreases and so does output. Second, if the planner shifts some resource from the second-stage technology to the first-stage to produce goods for the early-leaving creditors, then the total output is reduced as the first-stage production offers a lower return than the second-stage production. Hence, some output is lost.

5 Central bank policies

Here, we consider a modified version of the decentralized economy in which there is a central bank on the central island. The central bank provides a discount window service that provides unlimited loans at the interest rate of $1 + r$ at the first subperiod of each period. The loans have to be repaid at the end of the second subperiod when the late-arriving debtors arrive in the central island. The bank maintains a constant stock of outside money in the economy in each period.[8] The late-leaving creditors and early producers trade between the central bank and early-leaving creditors as "commercial banks." The competitive market will result in a non-arbitrage condition in which $\rho = \frac{1}{1+r}$.

The central bank also can use a lump-sum tax and transfer scheme (i) to reallocate the wealth between creditors and debtors to achieve the desired welfare weights in the objective function; (ii) to transfer goods from the young debtors to the early-leaving old creditors at the first stage and compensate the young debtor with the same amount at the second stage; and (iii) to attract some debtors to choose the first-stage technology by setting relative size of T_d to T_{d^*} along with the choice of ρ. We use the vector $(T_d, T_{d^*}, T_c, T_{c^*})$ to denote the tax/transfer scheme, where $T_d, T_{d^*}, T_c, T_{c^*}$ are the net life-time transfers to the early-producer, late-producer, early-leaving creditor, and late-leaving creditors, respectively.

Our task is to answer the following question: What central bank policies will implement the planner's allocation? Specifically, we are interested in determining if one particular set of central bank policies – those with the intraday rate equal to zero – can implement the planner's allocation. We examine how different central bank policies will affect the equilibrium outcomes as the follows: The creditors and the debtors take ρ, taxes, and transfers as given, maximizing their expected utility by choosing the volume of trades in each period and the production technology; the central bank chooses interest rate ρ and a tax-transfer scheme to maximize the weighted-average expected utility of the creditors and

[8] Central bank loans can be considered outside money. The constant stock of outside money referred to here then is "unbacked" outside money. Any outside money created through the discount window is "backed" outside money, where backing refers to the loan itself.

the debtors. The central bank reallocates the profits made through the discount window to the creditors and debtors as lump-sum transfers. The balanced-budget constraint is

$$(1-\lambda)\,T_d + (1-\alpha)\,T_c + \lambda T_{d^*} + \alpha T_{c^*} = (1-\rho)\left[(1-\alpha)(1-\lambda)\,l - \lambda b - \alpha q\right]$$

We look for an optimal discount rate on the resale loan that can maximize the social welfare in the decentralized economy. We will discuss both the Freeman type of economy and all cases in Diamond-Dybvig type of economy.

5.1 Freeman type of creditor

With a Freeman-type creditor, there exists a discount window policy with $\rho = 1$ that implements the planner's allocation. Consider the central bank that sets $T_{d^*} > T_d$. Debtors will choose the second-stage technology. Thus, with $\lambda = 0$ and $\rho = 1$, the central bank's balanced budget constraint becomes

$$\alpha T_c + (1-\alpha)\,T_{c*} + T_{d^*} = 0.$$

With $T_c = T_{c^*}$, the central bank's policy can smooth consumption across creditors, achieving $cx_2 = cx_2^*$. Moreover, the creditor's first-order condition $u_1(cy_1) = u_2(cx_2^*)\,F'(k^*)$ is satisfied. Thus, with the zero-interest rate policy, the discount rate of the loan and the ratios of intertemporal marginal utilities for both debtors and creditors are identical to that in the planner's allocation. The value of T_{d^*} and T_{c^*} can be chosen in such a way that the marginal utility between two types of agents satisfies $v_2(dx_2^*) = \frac{1-\beta}{\beta}u_2(cx_2^*)$.

5.2 Diamond-Dybvig type of creditor

We discuss three cases in the economy with Diamond-Dybvig type of creditors, corresponding to the three separate planner's solutions.

Case 1: $(1-\alpha)\,c\hat{x}_2 < x$

The planner's solution has an unbinding resource constraint at the first stage. The solution is characterized by the same system of equations as in the economy with Freeman type of creditors. The optimal discount rate is $\rho = 1$, and the policy in the decentralized economy can implement the planner's allocation.

Case 2: $(1-\alpha)\, c\hat{x}_2 = x$ and $\hat{\lambda} = 0$.

Setting $\rho = 1$ does not implement the planner's allocation in this case. To see this, suppose the discount rate policy $\rho = 1$ and the tax-transfer scheme yield an equilibrium with $\lambda = 0$. The creditor's first-order condition with $\rho = 1$ in the decentralized economy is

$$u_1\left(cy_1\right) = F'\left(k^*\right)\left[(1-\alpha)\, u_2\left(cx_2\right) + \alpha u_2\left(cx_2^*\right)\right],$$

which is different than equation (33) for the planner's solution unless $cx_2 = cx_2^*$. But with $cx_2 = cx_2^*$, the equilibrium solution contradicts the planner's allocation with $c\hat{x}_2 < c\hat{x}_2^*$.

However, there exists a discount rate less than 1 that can implement the planner's allocation. To achieve $\lambda = 0$ in the decentralized economy, T_d can be set sufficiently low to induce all debtors to be late producers. The late producer's decision is independent of ρ; ρ affects only the creditor's decision. Next, set

$$\rho = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(c\hat{x}_2\right)} = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(\frac{x}{1-\alpha}\right)},$$

so that the creditor's first-order condition in the decentralized economy becomes

$$u_1\left(cy_1\right) = F'\left(k^*\right)\left[(1-\alpha)\frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(c\hat{x}_2\right)} u_2\left(cx_2\right) + \alpha u_2\left(cx_2^*\right)\right]. \tag{42}$$

It is easy to see that $\left(c\hat{y}_1, \hat{k}^*, c\hat{x}_2^*, c\hat{x}_2\right)$ is the solution to (42), which is identical to (33). Hence, with $\rho = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(c\hat{x}_2\right)}$, the economy can produce the planner's amount of output $F\left(\hat{k}^*\right)$.

The central bank's balanced budget constraint in this case is

$$(1-\alpha)\,T_c + \alpha T_{c^*} + T_{d^*} = (1-\rho)\,(1-\alpha)\,l$$

The left-hand-side is the total transfer, whereas the right-hand-side is the profits made from the discount window. The tax-transfer scheme can be designed to achieve the desirable consumption allocation among all agents. Therefore, $\rho = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(\frac{x}{1-\alpha}\right)}$ can restore the economy to the planner's allocation.

Case 3: $(1-\alpha)\,c\hat{x}_2 > x$ and $\hat{\lambda} > 0$.

In this case, there does not exist a discount rate that can implement the planner's allocation. To see this, note that ρ appears both in the early-producer's and the creditor's intertemporal consumption decisions, equations (7) and (23). Equation (7) is not identical to equation (41) in the planner's first-order condition, unless ρ is set to be 1. With $\rho = 1$, equation (23) in steady state becomes

$$u_1\left(cy_1\right) = (1-\alpha)\,u_2\left(cx_2\right) f'\left(k\right) + \alpha u_2\left(cx_2^*\right) F'\left(k^*\right),$$

which is not identical to the intertemporal consumption decisions in the planner's solution (equations (33) and (39)) unless we have $cx_2 = cx_2^*$. But it contradicts the fact that $c\hat{x}_2 < c\hat{x}_2^*$ in the planner's solution.

We summarize the discussion to the following proposition.

Proposition 3 *(I)It is possible to implement the planner's allocation in economies with Freeman-type creditors and with Diamond-Dybvig type creditors and a nonbinding resource constraint (Case 1). Moreover, there exists a lump-sum tax-and-transfer policy combined with $\rho = 1$(or $r = 0$) that implements the planner's allocation in these two economies. (ii) For economies with Diamond-Dybvig type creditors and with the endowments equal to consumption by old, early-leaving creditors – that is, $x = (1-\alpha)\,c\hat{x}_2$, (i.e., Case 2) – there exists a lump sum-tax-and-transfer policy combined with $\rho = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(c\hat{x}_2\right)}$ that implements* *the planner's allocation. (III) For economies with Diamond-Dybvig type creditors and with endowments strictly less than consumption by old, early-leaving creditors – that is,* $x < (1-\alpha)\, c\hat{x}_2$ *(i.e., Case 3) – there is no discount window policy that will implement the planner's allocation.*

The intuition behind proposition 3 is as follows. With a zero-interest rate policy, the central bank can implement the planner's allocation in both the Freeman-type creditor and the Diamond-Dybvig type creditor in which the early-leaving resource constraint is not binding. The reason is straightforward: in these two settings, the departure of the early-leaving creditors is a pure liquidity issue. In other words, the timing of consumption demand does not impose a social cost in terms of reduced total production. In order to provide timely consumption goods for the early-leaving creditors, the central bank needs, at most, to transfer goods to the early-leaving creditors. The optimal price of liquidity in the decentralized economy, r, is thus zero, as in the Freeman's original analysis.

In contrast, consider a case in which the planner's allocation requires $x \leq (1-\alpha)\, c\hat{x}_2$. In this case, the early-leaving creditors' consumption demand imposes a cost in the form of less output. The social cost owes to two separate channels. First, as the early-leaving creditors are treated less favorably by the planner ($c\hat{x}_2 < c\hat{x}_2^*$) when the resource constraint is binding, the planner partially compensates the creditors for the consumption uncertainty by letting them consume more when young, which implies that the investment in the economy decreases, reducing output. We refer to this as the intertemporal consumption channel. Second, if the planner shifts some resource from the second-stage technology to the first-stage to produce goods for the early-leaving creditors, then the total output is reduced due to the low return on the first-stage technology. Here, we refer to this as production channel.

If the planner decides not to allocate the resource in the first-stage technology but ration the early-leaving creditors to consume the endowment of the young debtors, he shuts down the production channel. The cost in production is incurred only through the intertemporal consumption channel associated with smaller investment. The planner can optimally determine the size of the output cost by dictating the desirable investment level, $\hat{k}^*$. In the decentralized economy, the central bank can set T_d to be extremely low relative to T_{d^*} to discourage debtors from becoming first-stage producers, thereby inducing the production channel to vanish. The central bank then can use the policy tool, ρ, to implement the planner's allocation; that is, hitting $\hat{k}^*$. It turns out that the optimal discount rate guarantees that the ratio between the marginal utilities of being a late type and an early type are equal to the efficiency conditions in the planner's problem.

Lastly, consider the case in which the planner's allocation consists of strictly positive values of first-stage production. In this case, the production channel is operational. The social cost is increasing in the amount of capital placed into the first-stage technology. In the planner's allocation, the efficient values are allocated to λ, k, and k^*. However, in the decentralized economy, only two policy tools, ρ and the lump-sum tax-transfer scheme, are available. The number of policy tools are too few; in other words, there is no way for the central bank to apply two policy tools to implement the planner's allocation along three separate dimensions. The lump-sum tax/transfer scheme does not change the prices in the economy; it changes the debtors' incentive, affecting value of λ. The discount rate ρ appears in the early-producer's and the creditor's intertemporal consumption first-order conditions; that is, equations (7) and (23). These two equations are significantly different from those of the planner's (equations (41) , (33) and (39)). It is impossible to restore both intertemporal consumption decision functions to the planner's by adjusting ρ only. Thus, the equilibrium values of λ, k, and k^* are different from the planner's allocation in Case 3.

A secondary and subtle reason accounts for why central bank policies cannot implement the planner's allocation in Case 3. Hidden action plays an important role in terms of the relative price of capital. Debtors' production choices are not observable ex ante. As creditors cannot distinguish between early- and late-arriving debtors, they cannot sell capital at different prices to debtors who invest in different production technologies. Thus, there is one price of capital despite two types of production, and the choices of k, and k^* are both distorted.[9]

When is the production channel operational in the planner's allocation? The planner compares the welfare loss of having the early-leaving creditors consume rationed amount of endowment of the young debtors (i.e. $x = (1-\alpha) c\hat{x}_2$) with the welfare gain of having all other agents consume the output from the second-stage technology. If the welfare loss is small enough, the planner will shut down the production channel. Mathematically, the planner's allocation has a corner solution with respect to λ. Otherwise, the production channel is operational and the planner's allocation includes a positive value of λ, increasing first-stage production and early-leaving creditor's consumption with $\hat{\lambda}$ set to equate the welfare loss to the welfare gain.

Although in the economy with Diamond-Dybvig type of creditors and $x < (1-\alpha) c\hat{x}_2$, there is no discount rate that can implement the planner's allocation, a proper discount rate policy can improve the decentralized economy. The following numeric example illustrates the welfare in the decentralized economy, the planner's allocation, and the decentralized economy with central bank policies.

5.3 A numeric example (Diamond-Dybvig type of creditor)

In this example, utility functions are: $v(dx_{1t}, dx_{2t+1}) = \ln dx_{1t} + \ln dx_{2t+1}$, and $u(cy_{1t}, cx_{2+1}) = \ln cy_{1t} + \ln cx_{2t+1}$. Creditors and debtors are equally weighted. That is, $\beta = 0.5$.

Endowments: $x = 0.2$, $y = 1$.

Production functions: $f(k) = k^{1/3}$, $F(k) = 2^{2/3}k^{1/3}$.

Figure 1 shows the weighted average expected utilities of the debtors and creditors (social welfare) in a decentralized economy, a planner's economy, and a decentralized economy

[9] Hidden action is sufficient, but not necessary. Suppose that model were modified so that debtors' production choices are observable. Capital sold to the late producers will be charged a higher price to compensate for the uncertainty that affects old-age consumption. In equilibrium, the relative price of capital sold to the early producers and late producers is ρ, which will appear in both the creditors and the early-producers's intertemporal consumption decisions. Again, we do not have enough policy tools to achieve the first-best.

with central bank policies. The blue line represents the utility in a decentralized economy, the red line indicates the utility in a planner's economy, and the green line indicates the utility under the central bank policies when the planner's allocation is not achievable.

If the fraction of late-leaving creditors, α, is greater than fifty percent, then the first-stage resource constraint is not binding in the planner's allocation. That is, $x > (1 - \alpha)\, c\hat{x}_2$. In the decentralized economy, the central bank will want to choose $\rho = 1$ to implement the planner's allocation. This is why the central bank policy welfare line coincides with the planner's welfare line for $\alpha \geq 0.5$.

Figure 1 further shows what happens when $0.36 \leq \alpha < 0.50$. Here, the first-stage resource constraint is binding in the planner's problem so that $x = (1 - \alpha)\, c\hat{x}_2$. The optimal discount rate that the central bank sets is $\rho = \frac{u_2\left(c\hat{x}_2^*\right)}{u_2\left(c\hat{x}_2\right)} < 1$. This central bank policy implements the planner's allocation. The central bank policy welfare line also coincides with the planner's welfare line for $0.36 \leq \alpha < 0.50$. For $\alpha < 0.36$, no discount rate can implement the planner's allocation, and the welfare is strictly less than that under the planner's allocation. However, the allocation is better than the decentralized economy without the central bank policies.

The decentralized economy performs inferior to that with the central bank policies at all values of α because there is not a tax-transfer scheme in the decentralized economy that can redistribute consumption goods among the agents to reach the social optimum.

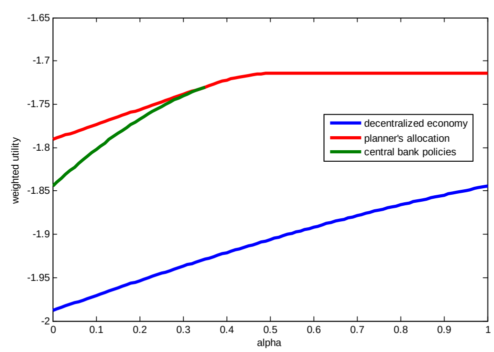

Figure1: An example of the social welfare under the planner's allocation, decentralized economy, and the decentralized economy with the central bank policies.

Table 1 lists the details of the decentralized economy, planner's allocation, and the decentralized economy with central bank policies for $\alpha = 0$. The consumptions and utilities of the debtors and creditors, and the weighted average aggregate utility, ω, are as follows:

Table 1: An example of the allocation in a decentralized economy, planner's allocation, and central bank policies ($\alpha = 0$)

	v	dx_1	dx_2	k	dx_1^*	dx_2^*	k^*
Decentralized economy	-2.0999	0.2000	0.6124	0.3750	0.2000	0.6124	0.7500
Planner's allocation	-1.6372	0.4411	0.4411	0.5411	0.4411	0.4411	0.5411
Central bank policies	-1.3863	0.3162	0.7906	0.2500	0.5000	0.5000	0.5000
	u	cy_1	cx_2	cx_2^*	λ	ρ	ω
Decentralized economy	-1.8767	0.5000	0.3062	0.6124	0.6667	0.5000	-1.9883
Planner's allocation	-1.9442	0.4589	0.3119	0.4411	0.1521	$N.A.$	-1.7907
Central bank policies	-2.3026	0.5000	0.2000	0.5000	0.0000	0.4000	-1.8444

When $\alpha = 0$, the zero-interest rate policy is not optimal in the decentralized economy with central bank policies. As Table 1 shows, The central bank policy equilibrium requires positive interest rate policy and a tax-transfer policy ($T_d/p_x = 0.0479$, $T_{d^*}/p_x = 0.1094, T_c/p_x = 0.0000, T_{c^*}/p_x = -0.1094$). Note that given the discount rate $\rho = 0.4000$ and the tax-transfer scheme, no debtors will choose the first-stage production. The chance of being treated unfavorably when old adversely affects the creditor's consumption decision when young. The young creditors choose to consume more and sell less capital, which reduce the total production and thus the consumption of other agents.

6 Conclusion

In this paper, we examine a model economy in which the arrival time at the settlement location is endogenous. Our setup can account for the distribution of daily payments observed at Fedwire and CHIPs. Given the size of the payment system, it is important to be able to understand why settlements occur mostly in the last half of the day. In our case, the rate of return offered by assets paying in the last half of the day will induce most debtors to settle in the last half of the day in our model economy. Because there are arbitrage opportunities, some debtors will settle in the morning.

We build on the work by Freeman to account for the within-day distribution of settlements. Our modification to his model economy raises additional issues about the role of the central bank and its policies. For instance, we derive conditions under which there is no intraday interest rate that will implement the planner's allocation. As such, these findings are in direct contrast to Freeman's result in which the central bank can implement the planner's allocation by setting the intra-day rate equal to zero. In Freeman's model, there is a transfer from early-leaving creditors to late-leaving creditors when no central bank is present and there is a binding liquidity constraint. The central bank can undo this transfer by providing liquidity, thereby rendering the liquidity constraint nonbinding. In our paper, the measure of debtors that arrive early is a decision that is commensurate to production timing. In the decentralized economy, the intra-period gains from the resale market are critical to induce some measure of debtors to arrive/produce early.

Our findings depend critically on the interpretation of when the consumer eats in relation to their departure from the settlement location. If, for example, creditors consume at the end of old-age, then our decentralized economy yields results identical to Freeman's; that is, central bank can implement the planner's allocation by setting the intra-day interest rate to zero. In this case, the planner's production decision is separable from the provision of consumption and the central can smooth consumption across old-age creditors through liquidity provision.

However, if early-leaving creditors are treated as if they must consume during the first-stage, then we have a kind of Diamond-Dybvig consumer. In the Diamond-Dybvig version of the model economy, the provision of timely consumption goods can be costly. To be more precise, if young debtors have a large enough endowment, then the planner' allocation consists of transferring the young debtor's endowment to early-leaving creditors. If the endowment is large enough relative to old-age creditor's consumption, there is no reduction in output and the early-leaving creditors and the late-leaving creditors will be treated equally. Otherwise, either the early-leaving creditors need be rationed to consume the young debtors' endowment or some low-return, first-stage production is needed to smooth old creditors' consumption. In the former case, the reduction in output results from the reduced investment; whereas in the latter, the output is reduced because investment is reduced and further because some investment is allocated to the low-return, first-stage technology. In both cases, the planner solves the problem by allocating less consumption to early-leaving creditors than to late-leaving creditors. Regardless, we show that the decentralized economy does not implement the planner's allocation.

The resource constraint in the planner's allocation also plays a critical role in deciding whether there exists an intraday interest rate that can implement the planner's allocation. If the first-stage resource constraint is non-binding, then a zero intra-day interest-rate policy will implement the planner's allocation. Basically, early consumption demand does not change the amount of resource allocated in the high-return production as in the planner's economy. We also demonstrate that if the first-stage resource constraint is binding and the planner rations the endowment of the young among the early-leaving creditors, then setting the intra-day interest rate equal to the ratio between the marginal utilities of being a late type and an early type in the planner's solution can implement the planner's allocation. With a binding first-stage resource constraint and if some resources are invested in the low-return, first-stage technology, we show that there is no discount rate policy that can implement the planner's allocation. The discount rate policy alone cannot achieve the multi-goals in the planner's allocation, namely, the optimal amounts of production at both stages.

The driving force in this model is twofold. First, the link between production decisions and debtor arrival is one way to endogenize the distribution of timing in the payment system. Second, there are two types of production, differentiated by returns. In this way, there is an opportunity cost to society associated with production (and arrival times) that is pertinent to the settlement process. We show that the wedge that exists because of the two production types is critical for what happens in the IOU resale market as well as the impact on the timing of debtor's arrival. With an IOU resale market, there is an arbitrage opportunity that can be exploited by early-arriving debtors. Our results suggest that intraday rates should not be zero if there exists social cost for the liquidity needs. At a fundamental level, we derive conditions under which zero intraday rates are inefficient. If these conditions are relevant, and we think they are, we should rethink the pricing of intraday credit.

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