An attractive monetary model with surprising implications for optima: two examples*

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Abstract

Ex ante optima are described for two examples of a monetary model with random meetings and some perfectly monitored people and some nonmonitored people. One example describes optimal inflation; the other describes optimal seasonal policy. Although the numerical examples are in most respects arbitrary, the results are consistent with three general conclusions: if the model is known, then intervention is desirable; even the qualitative aspects of the optimal intervention are not obvious; optimal intervention depends on all the details of the model. The results, therefore, are reminiscent of the conclusions of 'second-best' theory.

Key words: money, monitoring, optima.

JEL classification: E52, E58

1 Introduction

The work of Ostroy [22], Townsend [24], and Kocherlakota [14] initiated the mechanism-design approach to monetary theory. The goal of that approach is to find settings in which money helps to achieve good outcomes—or, in Hahn's teminology, in which money is *essential*—and to use those settings to study the consequences of monetary

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and fiscal policy. Here, I present two examples that do that. The words *attractive* and *surprising* in the title are meant to challenge readers: they should decide whether the model is *attractive* and whether the results are *surprising*.

There is one general conclusion that has emerged from the mechanism-design approach to monetary theory: *imperfect monitoring*, some privacy of the history of individual actions, is *necessary* for essentiality of money (see Wallace [26]). However, there are no general necessary and sufficient conditions for essentiality of money. Therefore, many monetary models rule out monitoring completely—either implicitly or explicitly—and make other extreme asumptions.[1] Roughly speaking, the following conditions are sufficient for essentiality of money: no monitoring, discounting (that is not taken to the limit of no discounting), a large number of agents, some background absence-of-double-coincidence, and no durable objects other than money. While useful for some purposes, such economies have no credit and, somewhat less obviously, no taxation. Indeed, they are best viewed as extreme versions of underground economies. In those respects, they are very special. The absence of credit seems particularly troublesome because the role of central banks is widely viewed to be intervention involving credit.

To get money and credit, we need to have some monitoring—but not so much as to eliminate a role for money. About a dozen years ago, Ricardo Cavalcanti and I (see [5]) formulated such an intermediate situation by having some exogenous fraction of the population be perfectly monitored, labeled m-people, and having the rest, labeled n-people, be not monitored at all. The model was designed to compare inside (private money) and outside money as alternative monetary systems. Here, I review some work by Alexei Deviatov and me that uses an outside-money version of that model to study optimal policy.

The results below are optima for two arbitrary numerical examples: one geared to finding optimal inflation; the other geared to finding optimal seasonal policy. Despite the arbitrariness of those examples, they strongly hint at three general and related conclusions—general in the sense that they should hold for almost all models that give rise to a role for money. First, if the model is known, then intervention is optimal. Second, it is not easy to guess at even the qualitative nature of optimal intervention. Third, the optimum depends on all the details of the model.

Those conclusions are reminiscent of second-best theory; if the first-best is unattainable—for example, because some sector of the economy produces unavoidable externalities—then optimal policy can call for interventions throughout the economy, interventions that depend on a detailed description of the entire economy. Somewhat similar con-

[1]Whenever borrowing and lending is ruled out—as, for example, in Lucas [20], Bewley [2], and many other papers—the implicit assumption is no monitoring.

clusions apply to optimal policy in any reasonably robust model that gives rise to a role for money. In such a model, the features of a model that give rise to a role for money make the first-best unattainable. As a consequence, some kind of intervention is desirable. However, the nature of the beneficial intervention depends on the details of the economy and may not be consistent with simple general principles like the *Friedman rule* or *lean-against-the wind*.

2 The environment

The background setting is borrowed from Shi [23] and Trejos-Wright [25], a *pure-currency* economy with pairwise meetings at random. Time is discrete and there is a nonatomic measure of people, each of whom maximizes expected discounted utility with discount factor $\beta \in (0,1)$. Production and consumption occur in pairwise meetings that occur at random in the following way. Just prior to such meetings, each a person looks forward to being a consumer (a buyer) who meets a random producer (seller) with probability $1/K$, looks forward to being a producer who meets a random consumer with probability $1/K$, and looks forward to no pairwise meeting with probability $1-(2/K)$, where integer $K \geq 2$. The period utility of someone who becomes a consumer and consumes $y \in \mathbb{R}_+$ is $u(y)$, where u is strictly increasing, strictly concave, differentiable, and satisfies $u(0)=0$. The period utility of someone who becomes a producer and produces $y \in \mathbb{R}_+$ is $-c(y)$, where c is strictly increasing, convex, and differentiable, and satisfies $c(0)=0$. In addition, $y^* = \arg\max_{y \geq 0}[u(y) - c(y)]$ exists and is positive. Production is perishable; it is either consumed or lost.[2] In addition, either u is bounded above or c is such that y is bounded above, an assumption that allows us to invoke the principle of one-shot deviations.

People in the model are ex ante identical but the fraction α become permanently monitored (m-people), while the rest are permanently nonmonitored (n-people).[3] For m-people, histories and money holdings are common knowledge; for n-people, they are private. However, the monitored status and consumer-producer status of people in a pairwise meeting are common knowledge. And, no one except the planner can commit to future actions.

At each date, there are two stages. The first stage has the pairwise meetings just described. There is a second stage at which transfers are made. There is neither

[2] If K exceeds two, then, as is well-known, it can be interpreted as the number of goods and specialization types in Trejos and Wright [25] and Shi [23].

[3] The interpretation is that the fraction α realize a zero cost of attaining m-status and that the rest realize a prohibitively high cost of attaining m-status. We will see that m-people want to become monitored.

production nor consumption at the second stage. Money is uniform and indivisible, and each person's holding of money is limited to be in the set $\{0, 1\}$ at any time.

The only feasible punishment is permanent banishment of an individual m-person to the set of n-people. Underlying this assumption about punishment is free exit at any time from the set of m-people into the set of n-people and the ruling out of global punishments—like the shutting down of all trade in response to individual defections.

3 Implementable allocations and the optimum problem

The search for an optimum is limited to allocations that are steady states and are symmetric, where symmetry means that all people in the same situation take the same action, an action that can be a lottery. (In general, lotteries here have the form of a deterministic amount of output exchanged for a probability of getting money.) The state of the economy entering a date is (θ_m, θ_n), where $\theta_m \in [0, \alpha]$ is the fraction who are m-people with money and $\theta_n \in [0, 1-\alpha]$ is the fraction who are n-people with money. With $S = \{m, n\} \times \{0, 1\}$, the state of a meeting is an element in $S \times S$, where the first component is the state of the producer and the second is that of the consumer. The planner chooses (θ_m, θ_n), trades in meetings (as a function of the states of the producer and the consumer in the meeting), and second-stage transfers.

The planner is constrained by the steady-state restriction and by self-selection constraints that follow from the specification of private information and of punishments. The trades that the planner chooses for pairwise meetings are restricted to be individually rational (IR), immune to cooperative defection by the pair in any meeting, and incentive compatible (IC) for n people. At the transfer stage, transfers are subject to being *IR* and *IC.*

Several comments are in order about this notion of implementability. First, defection by an n-person has no further consequences for the person. Second, any defection by an m-person means permanent loss of m-status beginning at the next stage or date. The cooperative defection in meetings is static and assures only that the trade is in the pairwise core for the meeting taking as given the relevant continuation values. Moreover, as noted above, I am ruling out aggregative punishments in response to individual defections.

The planner's objective is ex ante expected utility, where α is the probability of becoming an m-person and where the probabilities of starting with money are determined by (θ_m, θ_n).[4] This notion of welfare is easily shown to be proportional to a weighted average of the surpluses in meetings; namely,

$$\sum_{s \in S} \sum_{s' \in S} \pi_s \pi_{s'} [u(y_{ss'}) - c(y_{ss'})],$$

where $y_{ss'}$ is production and consumption when the producer is in state s and the consumer is in state s' and where

$$(\pi_{m1}, \pi_{m0}, \pi_{n1}, \pi_{n0}) = (\theta_m, \alpha - \theta_m, \theta_n, 1 - \alpha - \theta_n).$$

It follows that the first-best is $u(y^*) - c(y^*)$—namely, output equal to y^* in every (single-coincidence) meeting. Below, we express welfare as a fraction of that first-best welfare.

4 Optimal inflation

In models with divisible money, a standard normalization holds the stock of money fixed and represents inflation by a proportional tax on money holdings. The approach taken here is the same, except that the discreteness of money in the model—each person's money holding is constrained to be in the set $\{0, 1\}$—dictates that we use a probabilistic version of such a tax: a person who ends up after trade with a unit of money loses it with some probability. This way of modeling inflation, which was first used by Victor Li (see [17] and [18]) and has been used by others, has the same effects on incentives to acquire money as inflation in a model with divisible money. A literal interpretation is that money is made of stuff such that each unit disintegrates at each date with a probability that the planner chooses.

The following example is taken from Deviatov and Wallace [8]: $u(y) = 1 - e^{-10y}$, $c(y) = y$, $K = 3$, $\beta = .59$, $\alpha = 1/4$. All the choices are arbitrary, except that for β. It is chosen to satisfy two conditions on optima for $\alpha \in \{0, 1\}$, the extreme situations with regard to monitoring. First, given the other aspects of the specification, β is such that if everyone is an m-person ($\alpha = 1$), then the first best is implementable. In other words, only the presence of n-people prevents implementability of the first best. Second, if everyone is an n-person ($\alpha = 0$), then it would be desirable to pay interest on money if doing so were feasible. Here are the details.

[4] One could also study Pareto allocations by varying the weight in the welfare criterion attached to different people—for example, m-people and n-people or even types distinguished by both monitored status and money holdings.

First consider $\alpha = 1$. This is an economy with no role for money and one in which punishment is permanent autarky for a defector. In it, output y in every single-coincidence meeting is implementable if and only if

$$c(y) \leq \beta[u(y) - c(y)]/K(1 - \beta), \tag{1}$$

or, equivalently,

$$c(y) \leq \beta u(y)/[\beta + K(1 - \beta)]. \tag{2}$$

Let $\bar{y}$ be the largest y for which (2) holds at equality. Then any $y \in [0, \bar{y}]$ is implementable. The best implementable y is $\min\{y^*, \bar{y}\}$. For our choice of u, c, and K, the smallest β such that $\bar{y} \geq y^*$ is $\beta \approx .51$. Therefore, for our choice, $\beta = .59$, the optimum is the first best if $\alpha = 1$.

Now consider $\alpha = 0$. Then trade occurs only if the producer has no money and the consumer has money and optimal inflation is zero. The relevant participation constraint is easily shown to be

$$c(y) \leq \beta u(y)/[\beta + K(1 - \beta)/(1 - \theta_n)]. \tag{3}$$

Also, if $y = y^*$ and $\theta_n = 1/2$ satisfy (3), then the optimum has $y = y^*$, $\theta_n = 1/2$, and ex ante welfare equal to 1/4 of the first best—1/4 because that is the fraction of single-coincidence meetings in which the producer has no money and the consumer has money when $\theta_n = 1/2$. (Obviously, $\theta_n = 1/2$ maximizes the fraction of single-coincidence meetings in which the producer has no money and the consumer has money.) If not, then the optimum has $y < y^*$, $\theta_n < 1/2$, ex ante welfare less than 1/4 of the first best, and payment of interest on money would be desirable (if it were feasible). For our choice of u, c, and K, the smallest β for which (3) holds with $y = y^*$ and $\theta_n = 1/2$ is $\beta \approx .67$. Thus, with $\beta = .59$ (the mid-point between $\beta = .51$ and $\beta = .67$), it would be desirable to pay interest on money because doing so would loosen constraint (3).[5]

With $\alpha = 1/4$, the optimum has ex ante welfare equal to 34% of the first best, has $\theta_m = 1/4$ (all the m-people have money), has $\theta_n = .18$ (only about one-quarter of the n-people have money), has a 16% inflation (disintegration) rate, and has no transfers to n-people at the second stage. There are five meetings in which trade can occur (see Table 1).

[5] The desirability of paying interest on money should be interpreted as follows. Consider two side-by-side economies that are identical except in one respect. In one the money is barren; in the other the money throws off a real dividend at each date, is a so-called Lucas tree. Welfare in the tree economy is higher by more than the additional consumption implied by the dividend.

Table 1. Optimal trades	
(producer)(consumer)	output/(money transferred)
(n0)(n1)*	0.573/(1.0)
(n0)(m1)*	0.573/(1.0)
(m1)(n0)	0.113/(0)
(m1)(n1)*	0.381/(1.0)
(m1)(m1)*	0.381/(na)

In the table, output is reported as a fraction of y^* and (*) means a binding producer *IR* constraint. (Positive output in rows 3-5 implies that the pre-meeting welfare of an m-person is higher than that of an n-person with money, which, in turn, is higher than that of an n-person without money.) When the producer *IR* constraint is binding, the trade is that implied by a take-it-or-leave-it offer by the consumer. Given those binding *IR* constraints, it easy to see why output is lower in the last two rows than in the first two rows. In the first two rows, the binding defection payoff for the producer is the discounted value of entering the next date in state $(n, 0)$; in the last two rows, it is the higher discounted value of entering the next date in state $(n, 1)$.[6] And, even though the third-row trade does not have the m-person on the verge of defecting, a higher output in that meeting would decrease the discounted value of being in state $(m, 1)$ and increase that of being in state $(n, 0)$, and, thereby, lead to a violation of the *IR* constraints in all the other meetings.

Because only about one-quarter of the n-people have money, the inflow of money into holdings by n-people (the second-row meeting) is roughly three times the outflow (the fourth-row meeting). The inflation reconciles those flows with a constant θ_n, while transfers at the second stage reconcile those flows with a constant θ_m. Here is one way to describe the transfers. The m-people could have a risk-sharing arrangement among themselves that has those who end stage-one with two units money surrender one unit with the proceeds distributed to those who end stage-1 without money. Given the above first-stage trades, transfers by the planner at each date are needed to reconcile those insurance payments with a constant θ_m.

[6] That the defection payoff for an m-person with money is that of an $(n, 1)$ person depends on the assumption that money is uniform and that global punishments have been excluded. If the m-person had a person-specific money, which is one interpretation of inside money, and that person-specific money is worthless after a defection by the holder of that money, then better allocations are implementable. See Deviatov and Wallace [9] for the comparable inside-money result.

5 Optimal seasonal policy

In order to discuss seasonal policy, the setting is modified so that it contains a deterministic seasonal (see Deviatov and Wallace [7]). That is done by having a two-date periodic c function: at odd dates (winter), the disutility of production is higher than at even dates (summer). In other respects, the model is the same except that we now rule out inflation (there is no possible disintegration of money) and we look for the best two-date periodic implementable allocation taking the first date to be winter.[7]

Our example has $\alpha = 1/4$, $K = 3$, $u(y) = 2y^{1/2}$, $\beta = .95$, and

$$c_t(y) = \begin{cases} y/(.8) \text{ if } t \text{ is odd (winter)} \\ y/(1.25) \text{ if } t \text{ is even (summer)} \end{cases}. \tag{4}$$

For this example, maximum surplus is attained at $y = .8$ in a winter meeting and at $y = 1.25$ in a summer meeting.[8] Also, first-best welfare is proportional to $[.8 + \beta(1.25)]$. If everyone is monitored, then the first-best is implementable. And if no one is monitored, then the optimum is $\theta_n = 1/2$ at every date, output such that the maximum surplus is attained when the consumer has money and the producer does not, and welfare equal to 1/4 of first-best welfare. With $\alpha = 1/4$, the optimum is described in Tables 2 and 3.

Table 2. Optimal quantity of money and welfare		
	beginning of winter	beginning of summer
θ_m	1/4	1/4
θ_n	0.312	0.309
welfare/first-best welfare	.4558	

Again, all the m-people have money at the start of each date and there are no transfers to n-people at the second stage of either date. Notice that the stock of money is larger at the start of winter than at the start of summer. The trades in meetings appear in Table 3.

[7] See Cavalcanti and Nosal [4] for a similar background setting, but with only n-people. They permit random confiscation of money held by n-people, which is not allowed according to the notion of implementability used here. Whether their class of policies could instead be described as positive transfers and inflation is not immediately apparent.

[8] Although this example does not satisfy the boundedness requirement, it is easily amended to satisfy it in a way that does not affect the optimum. One possibility is to assume that (4) holds only for $y \leq \bar{y}$, where $\bar{y} = 100y^*$and that $c(y) = \infty$ for $y \geq \bar{y}$.

Table 3. Optimal trades		
meeting	output/(money transferred)	
(prod)(con)	winter	summer
$(n0)(n1)$	0.951/(1.0)	0.947/(1.0)
$(n0)(m1)$	$0.850^*/(.505)$	$0.777^*/(.776)$
$(m1)(n0)$	0.161/(0)	0.171/(0)
$(m1)(n1)$	$1.177^\dagger/(.813)$	$0.836^{*\dagger}/(1.0)$
$(m1)(m1)$	$1.000/(n/a)$	$0.836^*/(n/a)$

In the table, output at each date is expressed as a fraction of the respective first-best output, (*) denotes a binding producer IR constraint, and (†) denotes a binding truth-telling constraint (the n-person with money is indifferent between the fourth-row trade and the third-row trade). When money transferred is in $(0, 1)$, as in the second row, it is the probability that the consumer transfers 1 unit to the producer.

In order to interpret optimal intervention, we again focus on inflows into and outflows from money holdings of n people. In winter, the difference is proportional to $(.750 - .312)(.505) - .312(.813)$, which is negative. And, because there are no transfers to n-people at the second stage, those flows imply that fewer n-people have money at the start of summer than at the start of winter (see Table 2). In summer, an exactly offsetting net flow occurs—a consequence of the restriction to two-date periodic allocations without inflation. To reconcile those flows with every m-person starting with money at every date, m-people in the aggregate surrender money at the beginning of summer and receive an exactly offsetting amount at the beginning of winter. Those transfers can be interpreted as zero-interest loans: the loans are extended at the beginning of winter and are repaid at the beginning of summer.[9]

[9] One might have guessed that the loans would be extended at the start of summer (when goods are plentiful) and would be repaid at the start of winter (when goods are scarce), as hinted at in the following statement:

> '*[For the Bank of England in 1805] knowing the direction of the wind was [important] ... If ... from the east, ships would soon be sailing up the Thames to unload goods in London. The Bank would need to supply lots of money..... If a westerly was blowing, the Bank would mop up any excess money..., thereby avoiding inflation.' ...Mervyn King, the current governor, told the FT in an interview .... (Winds of change by Chris Giles, Financial Times, May 14, 2007).*

6 Concluding remarks

There are several attractive aspects of the model and examples: the model is built up from fundamental ideas about the role of money; it has endogenous taxation and endogenous division of the gains from trade in meetings; and the examples were not selected to produce particular outcomes. However, the model is very special in all sorts of ways.

People in the model meet in pairs to trade and meetings occur at random. As regards meetings in pairs, even if we set aside all the descriptions of absence-of-double-coincidence situations that presume such meetings, there are good reasons for using such models. Pairwise meetings are, of course, the standard model in labor economics. In addition, they have been used to study the following diverse topics in monetary economics, none of which are easily addressed in models in which trade occurs among a large group: float (see Wallace and Zhu [27]), the denomination structure of currency (see Lee *et. al.* [16]), coexistence of money and higher return assets (see Zhu and Wallace [28]), and counterfeiting (see, for example, Hu [12]). As regards randomness of meetings, such randomness is simple and could, at a small cost in terms of additional structure, be replaced by heterogeneous taste shocks.

Another special assumption is money holdings in $\{0, 1\}$. Such holdings do prejudice the result toward inflation in the first example. If n-people were to spend more than they earn in meetings with m-people, then money would have to be returned to n-people as transfers. With holdings in $\{0, 1\}$, the transfers would have go to those who would otherwise have no money. Such transfers have harmful incentive effects on n-people who are producers in meetings. If, instead, money holdings were richer, say $\{0, 1, 2, ..., B\}$ with B large, then such transfers could be paid in a way that approximates payment of interest on money held by n-people. Nevertheless, I am skeptical that an optimum in such a version would resemble the Friedman rule. Spending by m-people serves several purposes in the model. Therefore, taxing them by having them earn more than they spend in meetings is not costless in terms of welfare.[10]

The assumption that an exogenous fraction are perfectly monitored is a special case of a model with a smooth distribution of costs of getting monitored across the population. In such a model, the planner chooses a cut-off cost subject to people

[10] A similar effect on welfare appears in Antonolfi *et al* [1]. They have a two-sector model in which m-people interact soley with each other in a Kehoe-Levine [19] credit market, while n-people interact solely with each other in a market with spot trade in money. As in our model, m-people face the threat of being banished into the set of n-people. In other respects, the models are very different. In our model, as highlighted above, the in-equilibrium interactions between m-people and n-people are central to the results.

self-selecting in accord with that cut-off. Some examples with such specifications were explored in Deviatov and Wallace [8]. It was found that the extreme version used above is not misleading. More interesting and challenging is a departure from the extreme situation of some perfectly monitored people and others not monitored at all. Some preliminary work on a model of that sort is in Mills [21].

Of course, real business-cycle enthusiasts will notice that the seasonal example is a special case of a real business-cycle model. Nothing in principle prevents changing the model into one with a random process for the disutility of production. Such a model, but with only n-people, is studied in Cavalcanti and Erosa [3] and in Huang and Igarashi [13].

One concern is that optima are difficult to describe, or, in other words, that the model is *intractable*. That difficulty is due to a general feature of the model. The above examples are such that the optima at both extremes—either all m people or all n-people—are easy to describe because neither extreme has an endogenous state variable. The version with only m-people is a repeated game. The version with only n-people is also a repeated game, because, with money holdings in $\{0, 1\}$, the fraction who end the first stage with money is unaffected by the first-stage trades. In contrast, a version with both types has an endogenous state variable: the distribution of money holdings between the two types. As a consequence, the trades at a date play two distinct roles: they affect the current payoff (current trades) and they help to determine the distribution at the next date which matters for future payoffs (future trades). That multiple role for trades, which is not special to models with trade in pairs, and the impossibility of attaining the first-best account for why the optimum is difficult to describe.

There are well-known devices for eliminating that multiple role of trades. One is the so-called *large family* (for a recent example, see Gertler and Kiyotaki [10]). Another is periodic centralized trade with quasi-linear preferences (see Lagos and Wright [15] and its many offshoots). But is it desirable to make such assumptions? If we conclude, as I believe we should, that the above multiple role of trades and the impossibility of attaining the first-best are features of almost all models of monetary economies, then we should live with those features and with the difficulty of describing their implications. Indeed, that difficulty is the message in the following sense. Optimal intervention, even its direction, depends on all the details of the model even in the very simple settings studied above.

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