Over-the-counter loans, adverse selection, and stigma in the interbank market*

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Abstract

It is often the case that banks in the US are willing to borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing at the Fed's discount window. This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. Most policymakers and empirical researchers consider the stigma hypothesis plausible. Yet, no formal treatment of the issue has ever been provided in the literature. In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; and (2) physical and informational frictions limit those trade opportunities. In our model, banks obtain loans in an over-the-counter market (involving search, bilateral matching, and negotiations over the terms of the loan) and hold assets of heterogeneous qualities that in turn determine their ability to repay those loans. When asset quality is not observable by outside investors, information about the actions taken by a bank in the credit market may influence the price at which it can sell its assets. In particular, under some conditions, discount window borrowing may be regarded as a negative signal about the quality of the borrower's assets. In such cases, some of the banks in our model, just as in the data, are willing to accept loans in the interbank market at higher rates than the ones they could obtain at the discount window. **Keywords:** Discount window lending, signaling, search, bargaining, private information, banking. **JEL Clasification Numbers:** G21, E58.

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1 Introduction

Occasionally, some banks in the U.S. are willing to borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing from the central bank, at the Fed's discount window (Peristiani, 1998, Furfine, 2001, Darrat el al. 2004). This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. The general argument is that market participants may eventually identify with some accuracy which banks have borrowed at the discount window and take such activity as a sign of weakness in the financial conditions of the borrowing institution.[1]

While most policymakers and empirical researchers consider the stigma hypothesis plausible, no formal treatment of the issue has ever been provided in the literature. In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; (2) physical and informational frictions limit those trade opportunities; and (3) under some specific conditions, which we clearly identify in the model, a stigma effect like the one commonly associated with the regularities in the data may arise. The objective in this paper is not to assess the extent to which stigma is prevalent in the US fed funds market. Rather, we take as a starting point that policymakers consider stigma a cause for concern. Governor Kohn (2010), for example, has recently said: "The problem of discount window stigma is real and serious." In this paper, we intend to provide a detailed formal analysis of an instance of stigma that is consistent with the more informal explanations commonly provided in policy circles.

Understanding the apparent reluctance of banks to use the discount window is essential to address many important policy issues. For example, the prevalence of stigma may limit the ability of the central bank to effectively implement a "hard ceiling" on the range of interest rates observable in the interbank market. Partly in an effort to address such issues, in 2003, the Federal Reserve completely changed the terms of operation of its credit facilities. In spite of such efforts, the evidence suggesting the presence of stigma could still be found in the data after the change (Furfine, 2005).

More recently, the reluctance of banks to borrow from the window bedeviled the central bank's attempts to inject liquidity in the market. As explained by Chairman Bernanke (2009), "In August 2007, ... banks were reluctant to rely on discount window credit to address their funding needs. The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness – the so-called stigma problem. The perceived stigma of borrowing at the discount window threatened to prevent the Federal Reserve from getting much-needed liquidity into the system." The creation of the Term Auction Facility (TAF), and some of its particular organizational features, can actually be regarded as trying to limit the possibility of

[1] Journalist Mathew Cowley expresses this popular view succinctly in his column at Dow Jones Newswires: "*There's traditionally been a stigma associated with borrowing [from the Fed's discount window], which is initiated by the financial institution and is therefore regarded as a sign of weakness.*" (1 August 2008).

stigma associated with accessing this source of central bank liquidity.[2] In this paper, we will discuss specific conditions under which stigma may arise in the context of our formal model. We believe that the resulting insights are useful for evaluating alternative arrangements and policy options directed to reduce the incidence of stigma in the interbank market.

Banks in our model obtain loans in an over-the-counter market, involving search, bilateral matching, and negotiations over the terms of the loans. To repay those loans, banks sell assets of heterogeneous quality to outside investors. When asset quality is (at least partially) observable by loan counterparties, information about the actions taken by banks in the credit market may influence the price at which they can sell their assets later. In particular, under some conditions, discount window borrowing may be regarded as a negative signal of the quality of the borrower's assets. In such cases, some of the banks in our model, just as in the data, are willing to accept loans in the interbank market at higher rates than the ones they could obtain at the discount window.

Aside from the possibility of stigma, our model generates some interesting outcomes in the interbank market, even when discount window borrowing is not possible.[3] For example, we find that under some parameter values, the asset market and the interbank market for loans may simultaneously shut down. This outcome occurs in our model due to the existence of an equilibrium interconnection between the two markets. If participants expect that the asset market will shut down (meaning that prices for assets of unobserved quality are expected to be equal to zero in that market), then they will not be willing to lend in the (prior) interbank market, and since there is no lending in the interbank market, no high (unobserved) quality assets are actually sold, which makes the expected zero price an equilibrium. In other words, in our model, when the asset market shuts down, the interbank market shuts down because of adverse selection (in the asset market) and the consequent repayment risk (in the interbank market).

We make some simplifying assumptions in our model. It is fairly easy to see that many of them could be readily generalized. However, our main objective here is to formalize in as simple a framework as possible an argument that is often used to explain certain apparently abnormal trading patterns in the U.S. interbank market for funds. Abstracting from some features of reality allows us to better capture the basic mechanism at play and to identify the main components of the logic involved. Some of these components may not have been fully appreciated before: for example, market frictions and bilateral negotiations play a critical role in our formal explanation of the phenomenon but not necessarily in the more heuristic one used in policy circles. We believe that highlighting these important components is one of the main contributions of our paper.

The model we develop combines several elements that are commonly regarded as important in

[2] "The TAF, apparently because of its competitive auction format and the certainty that a large amount of credit would be made available, appears to have overcome the stigma problem to a significant degree." (Bernanke, 2008).

[3] There is now a large literature providing formal treatment of various issues related to the functioning of the interbank market. Some prominent examples are Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas and Holthausen (2004). More recent contributions include Freixas and Jorge (2007), Allen et al. (2009), and the working papers by Acharya et al. (2008), Heider et al. (2009), Freixas et al. (2009) and Bolton et al. (2009). For a good discussion of this literature see the introduction of Allen et al. (2009).

explaining the nature of financial (and, in particular, interbank) market outcomes. First, as in Freeman's (1996) article, and the large literature that followed, spatial separation plays a key role in limiting the ability of some agents (banks) to trade with other agents (outside investors) at a certain point in time. Second, search and bilateral negotiations determine the terms of trade in the market, as in Duffie et al. (2005) and Lagos and Rocheteau (2009).[4] Third, informational asymmetries and asset-quality heterogeneity play a crucial role in determining equilibrium interest rates and prices (as, for example, in Eisfeldt, 2004). Furthermore, the theory in this paper is in line with the long tradition, launched by Leland and Pyle (1977), of studying the role of signaling in financial markets.

The paper is organized as follows. In the rest of this section we discuss some evidence that has often been regarded as indicating the presence of stigma attached to lending from the Fed's discount window. Then, in the next section, we introduce our formal model of the interbank market, based on intertemporal trade with physical and informational frictions. In Section 3 we study equilibrium in the basic framework when the discount window is not available to banks. This section is intended to provide a description of the basic economics involved in the model. In Section 4 we introduce discount window lending and derive the equilibrium conditions for this more complicated case. In Section 5, then, we study an equilibrium in which discount window lending becomes a negative signal and, hence, results in stigma. In that context, we discuss the particular conditions that can give rise to such phenomenon. In Section 6 we briefly discuss other possible equilibrium configurations and, in Section 7, we provide a summary discussion and conclusions.

1.1 Interest rates in the fed funds market and the discount window

During the 1980s and 1990s, the Fed provided discount window loans to banks at a rate below the fed funds target rate (i.e., the rate announced as the target for monetary policy). Supervisory scrutiny was used to control the amount borrowed by banks. On January 9, 2003, the U.S. Federal Reserve dramatically changed its discount window lending policy and started to operate a standing facility, offering loans to eligible depository institutions at an interest rate higher than the fed funds target rate (at the time, the spread was set at 100 basis points). No other restriction or special supervision was associated with borrowing from the discount window. In principle, under this new regime (a so-called Lombard-type facility), the rate at the discount window (plus the implicit cost of collateral) should act as a ceiling for the fed funds market rate. However, there exists extensive evidence showing that it is common for banks to choose to borrow from another bank at a higher rate than the one they could get at the Fed's discount window. More generally, the evidence seems to indicate a persistent reluctance of U.S. banks to borrow from the discount window.

For example, Furfine (2003) compares the amount of borrowing at the discount window after January 2003 with the volume that one would have predicted in advance, given the historical (pre-standing facility) empirical distribution of across-banks interest rates paid in the market. He finds

[4] Ashcraft and Duffie (2007) argue that these are realistic features of the US interbank market for funds.

that borrowing from the discount window was significantly lower than what one might have expected based on that past data. Also, he finds that, at rates equal or higher than that offered by the central bank, for each day during the period from January 9, 2003, to March 31, 2003, on average, there was more than 57 times more borrowing activity in the fed funds market than at the discount window.

Interestingly, evidence of similar behavior can be found even before the system's overhaul in 2003.[5] A case in point is the study by Furfine (2001) of the borrowing activity at the fed funds market during the operation of the Special Lending Facility (SLF). The SLF was a temporary standing (Lombard-type) facility put in place from October 1, 1999, to April 7, 2000, to respond to possible spikes in demand for liquidity associated with the Y2K problem. Furfine finds that whenever market interest rates rose noticeably, borrowing in the overnight federal funds market at 150 or more basis points above the Fed's target rate dwarfed lending at the SLF. Furfine, then, concludes that commercial banks were extremely reluctant to borrow from the Fed during that time.[6]

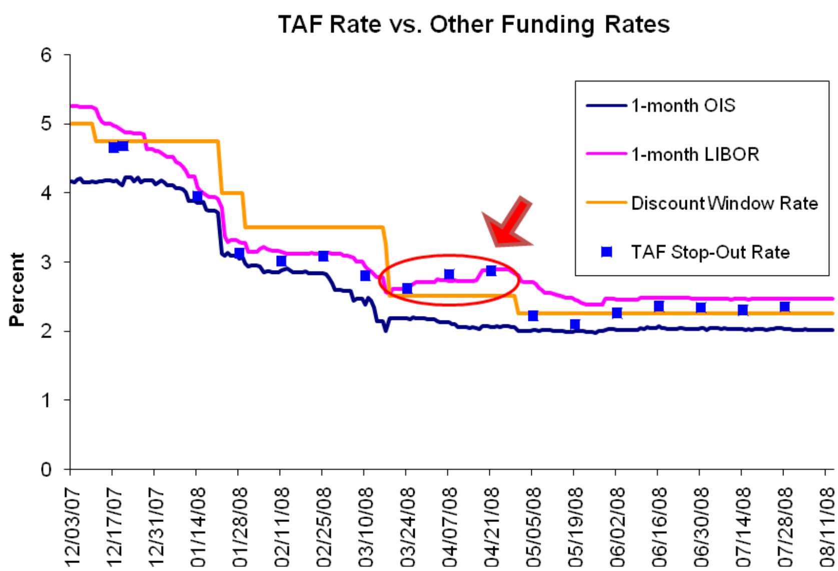

Figure 1: TAF stop-out rate behavior

In August 2007, as a response to the incipient financial crisis, the Fed lowered the spread in the discount window rate and started to allow eligible institutions to borrow funds at longer terms

[5] The issue has a long history in the US. For example, when Friedman and Schwartz (1963) discuss the creation in January 1932 of the Reconstruction Finance Corporation (RFC), which had the authority "to make loans to banks and other financial institutions, as well as to railroads," they say that: "... a provision of an act passed in July 1932 was interpreted as requiring publication of the names of banks to which the RFC had made loans in the previous month, and such publication began in August. The inclusion of a bank's name on the list was correctly interpreted as a sign of weakness, and hence frequently led to runs on the bank. Hence banks were fearful of borrowing from the RFC."

[6] Of course, one could argue that, at the time, banks still believed that tapping the discount window was likely to trigger some extra scrutiny from supervisors. It may take time to change the culture and the perceptions of participants. The question of whether this is a factor today still remains (see Duke, 2010). However, Artuç and Demiralp (2010) present evidence in support of the argument that the Federal Reserve was effective in reducing the fear of regulatory scrutiny after the change of policy in 2003.

(instead of just overnight, as was usually the case). The change generated little to no additional borrowing. In December 2007, the Fed created the TAF, a biweekly auction of a fixed amount of 28-day funding for depository institutions eligible to obtain (unrestricted) credit at the discount window. From its creation, borrowing at the TAF was in high demand.

Several features of the TAF may have made it less likely to generate stigma. The auction of a fixed, large amount of funds with a cap on individual bids guaranteed participation by multiple bidders and made them more likely to remain anonymous. Furthermore, a period of three days was set between the auction day and the settlement day (when the funds were transferred to the winners of the auction). This delay might have helped to decrease the perception that participants were in desperate need for funding (Bernanke, 2009). Interestingly, during March and April 2008 the stop-out rate at the TAF (i.e., the rate at which funds were allotted) was higher than the discount window rate (see Figure 1). That is, banks seemed to prefer to borrow at the TAF at a higher rate than the one they could obtain at the discount window.[7]

Date	Daily effective fed funds rate	Range		Standard Deviation
		Low	High	
09/08/08	1.92	1.00	2.50	0.22
09/09/08	1.96	1.62	2.19	0.07
10/09/08	2.12	1.50	2.62	0.18
09/11/08	2.00	1.00	2.50	0.19
09/12/08	2.10	1.75	2.94	0.10
09/15/08	**2.64**	**0.01**	**7.00**	**1.72**
09/16/08	**1.98**	**0.01**	**6.00**	**1.10**
09/17/08	**2.80**	**0.25**	**6.00**	**0.78**
09/18/08	**2.16**	**0.00**	**6.00**	**1.13**
09/19/08	**1.48**	**0.00**	**3.50**	**1.01**
09/22/08	1.51	0.00	3.50	0.92
09/23/08	1.46	0.06	3.00	0.69
09/24/08	1.19	0.01	3.25	0.66
09/25/08	1.23	0.00	3.00	0.66
09/26/08	1.08	0.00	3.00	0.69

Table 1: Fed funds rates in September 2008

As recently as September 2008, for many days, the (average) interest rate in the fed funds market was above the discount window rate, which was 2.25 percent during that period. As is clear from Table 2, during the week of September 15 the high effective fed funds rate (i.e., the average) and the high standard deviation of the observed distribution of rates imply that a relatively high proportion of the trade volume was executed at rates significantly above the discount window rate.

[7] This tendency to borrow at the TAF and not at the discount window is even more evident when the effective cost of discount window funding is computed as the 30-day OIS rate plus the primary credit spread. See Armantier, Krieger, and McAndrews (2008).

2 The model

The economy lasts for three periods, $t = 1, 2, 3$. There are a large number of banks and investors that can interact with each other over time, subject to some specific limitations described below. Each bank owns an asset that pays a return v in period 3 if held to maturity (and zero in periods 1 and 2). The return v can take one of two values, R or 0. If the return of the asset is R we say that the asset is high quality. If it is 0, then we say that the asset is low quality. The probability that the asset is high quality is $p \in (0, 1)$. Then, with probability $1 - p$ the asset is low quality. Asset quality is realized in period 1 even though the return only becomes available in period 3.[8]

The asset has some degree of specificity that determines that, if sold to another bank, its return v becomes much lower. Investors, instead, have the ability to manage the asset appropriately after buying it from a bank, and hence to maintain the potential return unaffected. While it could also be interesting to consider the case where some banks are good managers of purchased assets, we do not consider this case here and maintain the stark distinction between banks and investors throughout, just for simplicity. Also for simplicity, assume that investors have deep pockets and can always store their funds from one period to the next in a technology that gives a return $i > 0$ per unit stored.

At the beginning of period 1, banks get an observable liquidity shock: Some banks need to make a payment of size 1 and, hence, become illiquid; and some banks receive a payment of size 1 that makes them liquid.[9] Assume that half the banks become liquid and the other half, illiquid. Banks have access to the same storage technology as investors and hence can always obtain the return i on any positive fund holdings. For many banks participating in the U.S. fed funds market, this 'reservation' return is given by the interest on reserves paid by the central bank. The return i will play such a role in our model.

Illiquid banks, on the other hand, when unable to make their required payment by the end of period 1, suffer a penalty ρ, with $i < \rho < R$. We broadly interpret the penalty ρ as the costs (explicit and implicit) for the bank of not being able to fund a preestablished commitment through the 'normal' funding channels. The premium from incurring an overnight overdraft in the bank's account at the central bank is one (pecuniary) component of this cost, but there are many other (non-pecuniary) components that are just as important (see Clouse and Dow, 2002, p. 1792). For simplicity, we assume that the penalty ρ is all non-pecuniary in our model.[10]

In period 1, after the quality of the assets and the liquidity shocks are realized, banks can interact in an over-the-counter market for funds. Illiquid banks search for liquid banks to obtain immediate

[8] Our simple three-period model is (in principle) suitable to be incorporated into an infinite-horizon general equilibrium framework like the one studied by Lagos and Rocheteau (2007) following the technical innovations in Lagos and Wright (2005). The research in this paper could be regarded as a first step in that direction.

[9] It is possible to provide further microfoundation for the liquidity shock experienced by banks by introducing a set of depositors with random withdrawal demands as, for example, in Bhattacharya and Gale (1987) and Allen et al. (2009). For a sophisticated model of banks' demand for funds that includes many specific features of the U.S. system, see Clouse and Dow (2002).

[10] The assumption $\rho < R$ simplifies dealing with payments feasibility, but it is not essential.

funding. Investors cannot participate in this market. An illiquid bank finds a liquid bank with probability $\sigma \in (0,1)$. When two banks match, the liquid bank can costlessly verify the quality of the asset held by its illiquid counterparty.[11] The two banks in the match then decide whether or not to enter a lending agreement with each other and, finally, bargain over the terms of the loan. For simplicity, we assume that the outcome of the negotiations is determined according to Nash bargaining with θ being the bargaining power of the lender.[12] Loan maturity is one period and, at the time of repayment (i.e., in period 2), if a bank is not able to pay back the loan in full, it is still obligated to sell the asset and transfer the proceeds to the lender.

In period 2, banks and investors participate in a centralized market in which participants can trade funds and assets with each other, and make payments to each other. Each bank has a probability α that the quality of its asset becomes publicly observable at the beginning of period 2. With probability $1-\alpha$, the quality of the asset remains unknown to investors and other banks. At the end of period 2 all banks and investors part ways and, consequently, there are no possible (business) interactions in the economy during period 3.

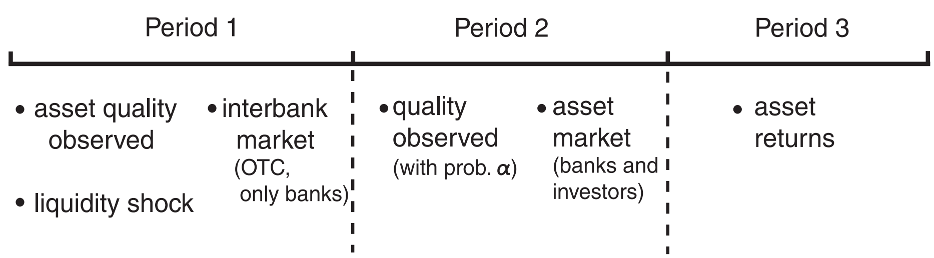

Figure 2: Timing

We assume that investors in period 2 cannot observe whether an illiquid bank has borrowed from another bank in period 1. This assumption is intended to limit the amount of information that investors can obtain from observing banks' prior activities. If, for example, the terms of the loan obtained by an illiquid bank were available to investors, this information could become fully revealing about the quality of the asset. In general, it would be interesting to study alternative specifications of the model in terms of the information available to investors. Here, to maintain the focus of our analysis, we make this simplifying assumption since it allows us to more clearly isolate the possible signaling role of discount window lending.

Basically, the set of frictions that characterize our environment are designed to capture a situation where some banks own illiquid assets but have an immediate need for funding. While, in principle, there are enough funds in the economy (on investors' hands) to cover all immediate needs, banks cannot access such liquidity directly. Instead, in the short run, illiquid banks can only trade with

[11] In our model, then, lenders have perfect information about the financial conditions of their counterparties in the interbank market. Including information frictions at the lending stage would be an interesting extension to consider. For an early contribution in this direction see Flannery (1996).

[12] See Bartolini et al. (2005) for evidence that suggests that the relative bargaining power of borrowers and lenders plays a significant role in the determination of interest rates in the fed funds market. See also Bech and Klee (2009).

other banks in a market with frictions. Trade in this market is based on the premise that banks will have access to investors' funds in the medium term. In summary, illiquid banks have resources in period 3 that they need in period 1. They effectively transfer (at least part of) those resources to period 2 by trading with investors and, to period 1 by taking loans from liquid banks (see Figure 2). We are interested in studying the implications of private information in this process of intertemporal reallocation of funds via borrowing and asset trading.[13]

3 Equilibrium

We will solve for a Perfect Bayesian Equilibrium of this economy. To do so, it is helpful to proceed going backward in time, dealing with the last period first. To start, then, we compute asset prices in period 2 given investors' beliefs about the trading strategies of banks. Then, in period 1, illiquid banks look for liquid banks and when matched, negotiate over the terms of a loan taking into account their equilibrium prediction about asset prices in period 2. In equilibrium, the interactions in period 1 confirm the beliefs of investors in period 2.

Two important properties of the equilibrium loan contracts result directly from the assumed isolation of agents in period 3. First, all contracts among banks involve one-period loans (from period 1 to period 2); and second, upon default in period 2, the lending bank will take possession of the borrower assets and sells them immediately to investors in the market.

Liquid banks have no incentives to sell their assets in the market. When quality is observable, they can get the same payoff from holding the asset than they would get from selling it off. When quality is not observable, the extreme degree of adverse selection implies that the only price consistent with equilibrium is exactly equal to zero. If the price is less than R then no liquid bank holding a high quality asset will try to sell it when its quality is unobserved. But, at a positive price all liquid banks holding low (unobserved) quality assets would try to sell it off. Then, if a liquid bank is attempting to sell its asset it must be that the asset is low quality and hence its price would be equal to zero. From now on, we assume that liquid banks do not sell their asset and we concentrate exclusively on the problem of the illiquid banks.

3.1 Observable asset quality

Suppose that $\alpha = 1$; that is, investors in period 2 can perfectly observe the quality of the assets being sold in the market. Then, since low quality assets give zero return in period 3, investors are not willing to pay any positive amount for low quality assets in period 2. On the other hand, due

[13] See Acharya et al. (2008) for another model where the interaction between the interbank market and the asset market plays a critical role. In the terminology of Bolton et al. (2009), we assume that there are two distinct sources of outside liquidity, bank loans and investors funds, and no inside liquidity. While bank loans are available on short notice, access to investors funds take time and involve the sale of assets of (possibly) uncertain quality. We study the interaction between these two markets for outside liquidity.

to competition among investors, a high quality asset can be sold at price R in period 2.

In period 1, if an illiquid bank holding a low quality asset finds a liquid bank, the former will not be able to borrow from the latter. The lender in this case will anticipate that the borrower will have no funds to pay back the loan in period 2. Furthermore, by taking possession of the asset in period 2, the lender cannot sell the asset for any positive amount. In other words, an illiquid bank holding a low quality asset has no borrowing capacity in period 1 and, hence, will get no loan.

The situation is different if the illiquid bank is holding a high quality asset. In this case, if the illiquid bank finds a liquid bank then it will be able to take a loan from the liquid bank. After agreeing on a loan, the two banks will bargain over the interest rate, denoted by r_H. In particular, the interest rate will solve the following problem:

$$\max_{r_H \leq R} (r_H - i)^{\theta} (\rho - r_H)^{1-\theta}.$$

Since $\rho < R$ the solution to this problem is given by $r_H = i + \theta(\rho - i)$. Note that the interest rate is increasing on the bargaining power of the liquid bank and is always below the penalty rate ρ.

In equilibrium, only illiquid banks that find a liquid bank and hold a high quality asset are able to take loans in the interbank market in period 1. Hence, the interbank market interest rate is given by r_H. Note that whenever θ is positive r_H is greater than the risk-free opportunity cost of funds in period 1, which is given by i. This premium over the risk-free rate is the result of bargaining power by lenders and not default risk. In this equilibrium, banks that could be expected to default do not get loans in period 1 and, hence, do not influence the *observed* interest rates in the interbank market.

3.2 Unobservable asset quality

Suppose now that the quality of the asset held by a bank becomes observable in period 2 only with a probability less than one; that is, assume that $0 \leq \alpha < 1$. As in the previous section, when the quality of the asset becomes observable the price is equal to R if the asset is high quality and zero if the asset is low quality.

The more interesting case is when the quality of the asset is not observed. In this case, pricing in period 2 will depend on the beliefs of investors about the relative prevalence of high and low quality assets in the market (as in Eisfeldt, 2004). Let q be the (equilibrium) belief of investors that a given asset being sold in the market in period 2 is high quality. Then, the price of an asset of unobserved quality, P_U, will be equal to qR.

We need to determine now the possible equilibrium values of q. The first thing to note is that whenever $q \geq 0$ all illiquid banks holding assets of low unobserved quality will want to sell their assets in the market. What makes equilibrium determination nontrivial is the action of illiquid banks holding assets with high unobserved quality. These banks may or may not take a loan in period 1 depending on the value of q. In turn, whether these banks take a loan or not determines the relative prevalence of high quality assets in the market and, hence, the values of q consistent with equilibrium.

Proposition 1. When $\rho - i - (1-\alpha)R < 0$ there is an equilibrium with $q = 0$.

Proof: Recall that we are assuming that $R > \rho > i$. We will show that when the condition in the proposition holds and $q = 0$ no loans are made in period 1. The reason for this is as follows. If an illiquid bank with a high quality asset does not take a loan in period 1, his payoff is $R - \rho$. Hence, this bank should get at least as much in expected terms from entering a loan contract. Since $q = 0$ the maximum expected payoff obtainable from the asset in $t = 2$ is αR. Then, a borrower can get a maximum expected repayment equal to $\alpha R - (R - \rho)$, but he can get i from not making the loan. We can rewrite the condition of the proposition as $\alpha R - (R - \rho) < i$. So, under this condition, if banks expect that investors will not be willing to pay for an asset of unobserved quality (i.e., if $q = 0$), it is not possible to have the liquid and illiquid banks agreeing on a feasible loan contract. But then, since illiquid banks with assets of high unobserved quality do not have a loan to repay, they have no reason to sell their assets (they get zero from doing so, instead of R if they do not). Therefore, only low quality assets will be put for sale in period 2, which is consistent with the belief expectation $q = 0$. #

The proposition gives us a condition under which the asset market in period 2 could shut down and, in anticipation of that fact, illiquid banks get screened out of the loan market in period 1 even when they are holding a high quality asset. It is interesting to note that the condition is more likely to hold when the probability α that the quality of an asset will become observable in period 2 is low; that is, when the information frictions in the asset market are expected to be large.[14]

This "no credit" equilibrium does not exist if $\rho - i - (1-\alpha)R > 0$. Furthermore, even if the condition of the proposition is satisfied, another equilibrium with credit in the interbank market may be possible. We study such equilibrium with credit next.[15]

Suppose now that $q > 0$ in equilibrium. In such case, we know that in period 2 there will be $\frac{1}{2}(1-\alpha)(1-p)$ low unobserved quality assets in the market. Furthermore, for $q > 0$ to be (part of) an equilibrium, it must be true that the high quality assets of illiquid banks that manage to obtain a loan in the interbank market are put for sale in period 2 (otherwise q would be equal to zero). We provide parameter conditions below for which this is the case.

[14] Changes in the quality of the asset, as reflected by changes in the return R, have two opposing effects. On the one hand, an increase in R increases the availability of funds for repayment; but, on the other hand, it increases the outside option for the potential borrower, reducing his incentives to take the loan. In our setup, the second effect dominates and, as a consequence, increases in R make the possibility of a shut-down of the interbank credit market compatible with a larger set of values for the other relevant parameters.

[15] Heider, Hoerova, and Holthausen (2009) study a model of the interbank market with private information about the riskiness of banks' asset holdings and, hence, about banks' repayment risk. They discuss the possibility of equilibria with no credit, similar to the one studied here. In their model, however, adverse selection in the interbank market is the result of (exogenously) assuming that safer assets have lower liquidation costs. For a model where the market for bank assets freezes because of reasons unrelated to asymmetric information and adverse selection, see Diamond and Rajan (2009).

Since in this equilibrium the total amount of high quality assets in the market will then be equal to $(1-\alpha)\frac{1}{2}p\sigma$, consistent beliefs are given by:

$$q^* \equiv \frac{p\sigma}{(1-p)+p\sigma}.$$

Let $TS(q) = \rho - i - (1-\alpha)(1-q)R$ be the expected total surplus from a loan relationship between a liquid bank and an illiquid bank holding a high quality asset. As we will see, if $TS(q^*) > 0$ then an illiquid bank holding high quality assets will be able to obtain a loan from a liquid bank whenever the two of them match in period 1. The negotiated value of r_H will, in turn, determine how this surplus gets divided between the two parties.

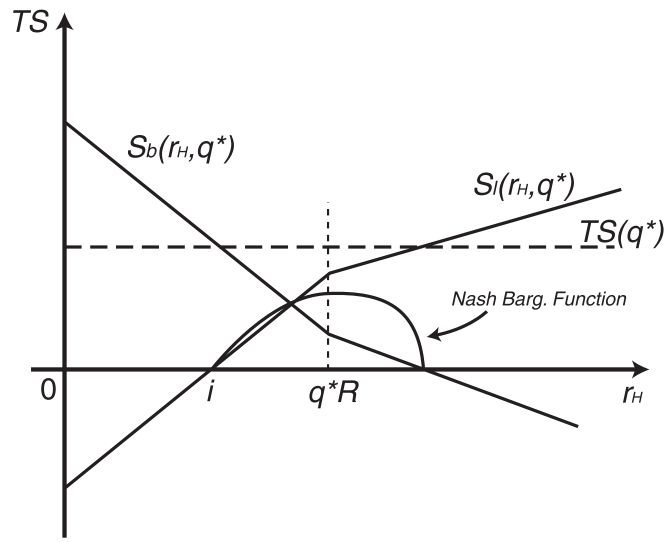

Figure 3: The Nash bargaining solution

Given a value of r_H the surplus for the borrower is given by:

$$S_b(r_H, q^*) = \alpha(R - r_H) + (1-\alpha)\max\{q^*R - r_H, 0\} - (R-\rho)$$

and the surplus for the lender is given by:

$$S_l(r_H, q^*) = \alpha r_H + (1-\alpha)\min\{q^*R, r_H\} - i.$$

Note, of course, that $S_b(r_H, q^*) + S_l(r_H, q^*) = TS(q^*)$. The equilibrium interest rate, then, solves the following Nash bargaining problem:

$$\max_{r_H} S_l(r_H, q^*)^{\theta} S_b(r_H, q^*)^{1-\theta}$$

subject to $S_l(r_H, q^*) \geq 0$ and $S_b(r_H, q^*) \geq 0$. Define the functions $\widetilde{r}_H(\theta, q^*) \equiv i + \theta TS(q^*)$ and $\widehat{r}_H(\theta, q^*) \equiv \frac{1}{\alpha}\left[i - (1-\alpha)q^*R\right] + \frac{\theta}{\alpha}TS(q^*)$. Then, we have that the solution to the Nash bargaining problem is given by:

$$r_H(\theta, q^*) = \begin{cases} \widetilde{r}_H(\theta, q^*) & \text{if} \quad \theta < \theta^T(q^*) \\ \\ \widehat{r}_H(\theta, q^*) & \text{if} \quad \theta \geq \theta^T(q^*) \end{cases}$$

where $\theta^T(q^*) = \max\{0, \min\{(q^*R - i)/TS(q^*), 1\}\}$. Note that when $\theta^T(q^*) \in (0,1)$ we have that $\widetilde{r}_H(\theta^T(q^*), q^*) = \widehat{r}_H(\theta^T(q^*), q^*) = q^*R$ (see Figure 3).[16]

After some algebra, it can be shown that $S_b(r_H(\theta, q^*), q^*) = (1-\theta)TS(q^*)$ and $S_l(r_H(\theta, q^*), q^*) = \theta TS(q^*)$. Hence, as long as $TS(q^*)$ is positive both the liquid and illiquid banks in a match will agree to participate in a loan agreement. Define the threshold value q^T as:

$$q^T \equiv \max\left\{0, \frac{(1-\alpha)R - (\rho - i)}{(1-\alpha)R}\right\},$$

such that $TS(q) \geq 0$ for all $q \geq q^T$. For concreteness, let us assume that whenever indifferent, banks enter a loan relationship. Then, we have the following proposition that provides conditions on parameter values such that an equilibrium with interbank credit exists (see Figure 4).

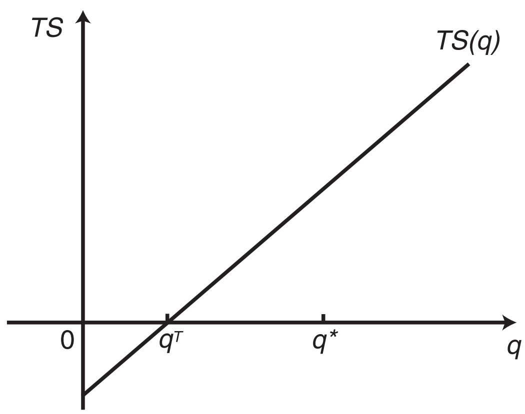

Figure 4: The surplus function

Proposition 2. When $q^* \geq q^T$ there is an equilibrium with interbank credit.

Proof: Suppose that investors conjecture that, of the unobserved quality assets for sale in period 2, a proportion q^* are high quality. Then, the expected price of unobserved quality assets in period 2 is q^*R. Since $q^* > 0$ all illiquid banks holding a low unobserved quality asset will sell it in period 2. Similarly, if a bank holding a high unobserved quality asset in period 2 has taken a loan in period 1, then its asset will be sold in the market in period 2. Also, since $q^* < 1$, banks who do not manage to find a counterparty in the interbank market in period 1 and are holding a high unobserved quality asset will not sell their asset in period 2 (since they have no loan to repay). Finally, since $q^* \geq q^T$

[16] Since $R > \rho$ we have that $r_H(\theta, q^*) < R$ for all (θ, q^*) and, in consequence, $r_H(\theta, q^*)$ is always reasonable in the sense that there are at least some borrowers that are able to pay as much for a loan.

we have that $TS(q^*) \geq 0$ and, hence, illiquid banks holding a high quality asset take a loan from liquid banks whenever they find a counterparty in period 1. We have, then, that the assets of all illiquid banks with low unobserved quality, and of all banks with high unobserved quality and a loan to repay, will be sold in the market in period 2. This implies that the expected value of assets of unobserved quality being sold in period 2 is q^*R, which is consistent with the investors' initial conjecture. #

Note that, in this equilibrium, illiquid banks holding low quality assets may or may not receive credit depending on whether $(1-\alpha)q^*R$ is greater than or less than i, respectively. To see this, denote by r_L the interest rate arranged by these banks in a loan agreement. It only makes sense to consider values of r_L less than or equal to q^*R, since this is the maximum amount that a lender could obtain in period 2 from a borrower holding a low quality asset. It is clear, then, that the lender, in expected terms, can get no more than $(1-\alpha)q^*R$ from the borrower, and if this quantity is less than i, the lender would not agree to participate in the loan. Note, also, that this implies that a loan may not take place even if the surplus from the loan agreement $\rho - i$ is positive (as we assume it is).[17]

Corollary 1. When $q^* \geq q^T > 0$ there are two equilibria, one with interbank credit and one where interbank credit shuts down.

Proof: Note that the parameters determining q^* are different from those determining q^T. Then, we can easily find parameters such that the conditions in the corollary hold (see Figure 4). In such a case, since $q^T > 0$ implies that the condition in Proposition 1 holds, we have that a "no credit" equilibrium exists. Furthermore, since $q^* \geq q^T$, by Proposition 2, an equilibrium with interbank credit also exists. #

This corollary tells us that, for a set of the parameter values, the model is consistent with multiple equilibria. Furthermore, these equilibria have significantly different implications for the outcomes on the interbank credit market. If banks expect that pessimistic investors will price assets of uncertain quality, then they may not be willing to enter into loan relationships, and this shutdown of the credit market, in turn, will result in a selective reduction of bank participation in the asset market (i.e., high-quality-asset holders will be out of the market), which would justify investors' initial pessimism.

Corollary 2. When $(1-\alpha)R \leq \rho - i$ we have that $q^T = 0$ and there is a unique equilibrium with interbank credit. The equilibrium price of the unobserved quality asset is $P_U = q^*R$.

Proof: Since $(1-\alpha)R \leq \rho - i$ we have that $TS(q) \geq 0$ for all $q \in [0,1]$. Then, high quality illiquid banks that find a match in the interbank credit market always enter a loan relationship. This implies

[17] Recall that ρ is a cost incurred by the bank if it cannot fund its liquidity shock in period 1. When the surplus from the loan agreement is positive and the loan does not happen, the illiquid bank would like to be able to use some of the resources dedicated to cover ρ to make a payment to the potential lender. We assume that ρ includes non-pecuniary costs (such as increased scrutiny by regulators) so these resources are not available for such type of payment. Alternatively, we could think that the emergency funding used to pay ρ is restricted and cannot be used in period 2 for the purpose of debt repayment.

that the assets of those banks will be put for sale in the market in period 2 and, hence, that the only consistent equilibrium value of q is q^*. #

Lemma 1. When the threshold value q^T is greater than zero, it is increasing in R and i and decreasing in ρ and α. The equilibrium value q^* is increasing in p and σ.

Proof: The results can be obtained by simple differentiation of the expression for q^T and q^*. #

The results in this lemma can be given interesting interpretations. For example, according to the lemma, higher values of α make the credit equilibrium consistent with a larger set of parameter values. More broadly, then, we can say that less informational frictions in the asset market make interbank credit more likely in our model.[18] Similarly, since q^* is increasing in σ, we can say that the potential for a more liquid credit market makes the credit equilibrium more likely to arise. Perhaps somewhat less intuitive is the case of different values for R, the return of high quality assets. Higher values of R make the credit equilibrium less likely. The reason for this fact is that when a bank holding a high quality asset enters a credit relationship, it anticipates that with some probability it will have to sell the asset when its quality is not observed. This sale entails a loss proportional to R, which discourages credit relationships ex ante.

The last proposition of this section provides a characterization of the equilibrium interest rate in the interbank credit market when such a market is open. An important aspect of this characterization is the determination of the conditions under which a borrower is able to pay the arranged interest rate when the quality of its asset is not observable by investors. Recall that if the borrower cannot pay the interest rate, the asset gets sold and the lender gets the proceeds. Whether this happens or not depends on how $r_H(\theta, q^*)$ compares with q^*R. If $r_H(\theta, q^*) > q^*R$, then the borrower cannot pay the arranged interest rate when the quality of the asset is not observed by investors.

Proposition 3. Assume $q^* \geq q^T$. If $\alpha i \geq \rho - (1-\alpha)R$ then there is an equilibrium with interbank credit in which the interest rate is given by $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*) \leq q^*R$ for all θ. If $\alpha i < \rho - (1-\alpha)R$ then there is an equilibrium with interbank credit in which the interest rate is given by the following expressions:

1) If $q^*R \leq i$ then $r_H(\theta, q^*) = \widehat{r}_H(\theta, q^*) \geq q^*R$ for all θ.

2) If $i < q^*R < \frac{1}{\alpha}[\rho - (1-\alpha)R]$ then $\theta^T(q^*) \in (0, 1)$ and

$$r_H(\theta, q^*) \begin{cases} \widetilde{r}_H(\theta, q^*) < q^*R & \text{if} \quad \theta < \theta^T(q^*) \\ \\ \widehat{r}_H(\theta, q^*) > q^*R & \text{if} \quad \theta \geq \theta^T(q^*) \end{cases}$$

(3) If $\frac{1}{\alpha}[\rho - (1-\alpha)R] \leq q^*R$ then $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*) \leq q^*R$ for all θ.

[18] Note, however, that higher values of α imply higher repayment risk for holders of low-quality assets. Hence, higher values of α may be associated with less volume of trade in the interbank market, as only banks with high quality assets get credit.

Proof: See the Appendix. #

In this section, we have studied the functioning of an interbank market for funds in the presence of frictions that limit the ability of banks to trade with each other. As a result of these frictions, some banks are not able to borrow during period 1 even when they hold high quality assets that have a present value larger than the face value of the loans that the banks seek to obtain. A natural question to ask is how would equilibrium outcomes change if a central bank lending facility (i.e., a discount window) is available to all banks in this environment. In the next section, we extend the model to allow for discount window lending and describe the equilibrium conditions corresponding to such case.

4 Discount window lending

Assume now that all banks have access in period 1 to the central bank's discount window, where they can obtain loans at the interest rate $r_W > i$. We will assume that discount window lending is uncollateralized and observable by the agents in the economy. These are extreme, simplifying assumptions, but not essential.[19]

Discount window loans in the U.S. are fully collateralized. We could easily amend our notation to include the opportunity cost of holding collateral as part of the cost of borrowing from the discount window. A more delicate issue would be to deal with a more realistic treatment of asset holdings by banks in our model. The Fed carefully assesses the value of the assets that are pledged as collateral for discount window borrowing purposes. Presumably, banks holding what we are calling low quality assets would not be able to access the Fed's primary credit program.[20] However, it should be kept in mind that asset characteristics are very stark in our simple framework. Introducing a more complete taxonomy of assets, with various degrees of riskiness and liquidity, would permit us to discriminate between banks in sound financial condition and those that, while able to pledge collateral with the central bank, are regarded as representing a significant repayment risk in the interbank market for overnight loans. If discount window borrowing is to become (possibly) a negative signal of the financial condition of a bank, as is required in the usual explanation of stigma, then less than perfect screening at the window must be possible in equilibrium. Our extreme assumption here makes imperfect screening simple and allows us to, then, more clearly identify the economic mechanism

[19] In Acharya et al. (2008) discount window loans play a related funding role by reducing illiquid banks' exposure to the risk of having to sell their assets in the market at a very significant loss. Here, the market for assets is actually closed at the time the illiquid bank needs funding in period 1. In a sense, our assumption is an extreme version of that considered in Acharya et al. (2008).

[20] Depository institutions in the US have access to three types of discount window credit: primary credit, secondary credit, and seasonal credit. Primary credit is available to depository institutions that are in sound financial condition. Its provision is associated with minimal administrative requirements and its usage is essentially unrestricted. Secondary credit is available to depository institutions that are not eligible for primary credit. It is provided only in particular situations and the institutions borrowing from the secondary credit program are closely monitored by the Fed. Seasonal credit is provided to assist small depository institutions to manage seasonal swings in loans and deposits.

that results from it.

In terms of information assumptions, we conduct the main analysis assuming that discount window activity is perfectly observable. We later demonstrate that our main result is still valid when discount window activity is only observed with some positive probability. In the U.S., discount window borrowing is not perfectly observable. The Federal Reserve periodically announces the total amount of discount window lending granted the previous two weeks (but not the size or recipients of the individual loans). This is, potentially, a very noisy signal of the participation of particular banks in the reported discount window activity. However, under certain circumstances, market participants may be able to put together various pieces of information (like a prior funding request by a particular institution, for example) which, in combination with the Fed's reported number, may partially reveal the identity of the borrowing banks (Furfine, 2001, Duke, 2010). Our results are consistent with this fact, in the sense that all we really need is that with some positive probability agents obtain accurate information about banks' activities at the discount window.

In the U.S., in principle, banks could borrow funds at the discount window to lend them later to other banks in the fed funds market. We do not consider this possibility in our model. Liquid banks meet at most one illiquid bank in period 1. Since they have access to funds at the opportunity cost i, they have no incentives to take loans from the discount window at rates higher than i. In our analysis below, we assume that r_W is greater than i in all cases, so the lend-to-borrow strategy is not profitable for liquid banks. Some illiquid banks could also try to borrow extra at the window to later lend to other illiquid banks. For simplicity, we assume that each bank can take only one side in the market. This assumption is a feature of the matching technology and rules out the lend-to-borrow strategy for illiquid banks. More generally, all one needs to assume is that search frictions in the market for interbank loans limit the ability of banks to arbitrage interest-rate differentials by following the lend-to-borrow strategy at the discount window. This general premise, and not the particular details used to capture it in the model, is what is crucial for the theory in our paper.

We solve, again, for a Perfect Bayesian equilibrium and start by identifying possible outcomes in the market for assets during period 2. As before, when the quality of an asset becomes observable, the price of the asset is either R or zero depending on whether the asset is of high or low quality, respectively.

When the quality of the asset is not observable, things become more complicated. Whether or not a bank borrowed at the discount window could be an informative signal about the quality of the asset that the bank is trying to sell. This possibility is the result of two important assumptions in our model. On the one side, banks in the interbank market are able to obtain some accurate information about the quality of the asset held by counterparties, which in turn influences their lending behavior. On the other side, sometimes investors are not able to observe directly the quality of the asset being traded, nor the seller's private dealings in the prior interbank market, but do get to observe the seller's past transactions with the central bank.

In period 2, then, investors form beliefs about the quality of a given asset of unobserved quality that depend on whether or not the seller of the asset has borrowed at the discount window. Let q_W be the belief probability that the asset is high quality if the holder has borrowed at the window; and let q_N be the corresponding probability (of high quality) if the seller has not borrowed at the window. These are equilibrium beliefs that will depend also on the decisions taken by all banks, given those beliefs. We study bank decisions next.

Illiquid banks that do not find a liquid counterparty in the interbank market have to decide whether or not to borrow from the discount window. For this decision, the bank compares the payoff of taking each possible action. To calculate this payoff, we assume that all banks that have borrowed at the window sell their asset in period 2 to pay back the loan (in full or partially).[21] Define $P(q_W, r_W) \equiv \max\{q_W R - r_W, 0\}$. Then, an illiquid bank that has not found a counterparty and is holding a high quality asset will borrow at the window if:

$$\alpha(R - r_W) + (1-\alpha)P(q_W, r_W) \geq -\rho + R;$$

and a bank holding a low quality asset will borrow at the window if:

$$(1-\alpha)P(q_W, r_W) \geq -\rho + (1-\alpha)q_N R.$$

Illiquid banks that do find a counterparty in period 1 must decide among three possible alternatives: they could either borrow from the liquid bank, from the window, or not borrow at all. If an illiquid bank borrows in period 1, it will have to sell its asset in period 2 to repay (all or some of) the loan. Define $P(q_N, r_j) \equiv \max\{q_N R - r_j, 0\}$ with $j = H, L$, where r_j is the interest rate on a loan from a liquid bank to an illiquid bank holding a j asset. Then, an illiquid bank that finds a liquid counterparty and is holding a high quality asset will agree to take the loan if:

$$S_b(r_H, q_N, q_W) = \alpha(R - r_H) + (1-\alpha)P(q_N, r_H) - \max\{\alpha(R - r_W) + (1-\alpha)P(q_W, r_W), -\rho + R\} \geq 0.$$

An illiquid bank that finds a liquid bank in period 1 and is holding a low quality asset will enter a lending relationship with the liquid bank if:

$$S_b(r_L, q_N, q_W) = (1-\alpha)P(q_N, r_L) - \max\{(1-\alpha)P(q_W, r_W), -\rho + (1-\alpha)q_N R\} \geq 0.$$

Finally, we need to consider the decision of liquid banks upon entering a match with an illiquid bank. If the illiquid bank is holding a high quality (H) asset, then the liquid bank will agree to

[21] It could be interesting to consider alternative treatments of those borrowers that cannot repay discount window loans in full. In the simple case we study here, loans from the discount window differ from loans granted by private counterparties only in the way the interest rate is determined. At the window, the rate is exogenously set and is not contingent on asset quality.

make a loan if and only if:

$$S_l(r_H, q_N) = \alpha r_H + (1-\alpha)\min\{q_N R, r_H\} - i \geq 0.$$

Similarly, when the illiquid bank is holding a low quality asset, the liquid bank will agree to make a loan if and only if:

$$S_l(r_L, q_N) = (1-\alpha)\min\{q_N R, r_L\} - i \geq 0.$$

Define the total surplus in a match as $TS_j(q_N, q_W) = S_b(r_j, q_N, q_W) + S_l(r_j, q_N)$ for $j = H, L$. Whenever the total surplus in a match is positive, banks will agree to enter a lending relationship and will negotiate over the interest rate. The outcome of such negotiation is the solution to the following Nash problem for $j = L, H$:

$$\max_{r_j} S_l(r_j, q_N)^{\theta} S_b(r_j, q_N, q_W)^{1-\theta}$$

subject to $S_l(r_j, q_N) \geq 0$ and $S_b(r_j, q_N, q_W) \geq 0$. Call the solution to this problem $r_j(\theta, q_N, q_W)$ for $j = L, H$.

In period 2, those banks that have taken a (private or discount window) loan in period 1 will sell their asset in the market. If $q_N > 0$ then all illiquid banks holding a low quality asset will sell their asset even if they do not have a loan to repay. If $q_N < 1$ then banks holding a high quality asset that do not have a loan to repay will not sell their asset. These cases exhaust all the possibilities.

A Perfect Bayesian equilibrium, then, can be characterized by a set of beliefs (q_N, q_W), loan agreements with the corresponding interest rates, and asset sales and prices such that: (1) all agents make optimal lending and asset sale decisions given those beliefs (as described above); (2) asset prices reflect those beliefs; and (3) the agents' decisions validate those equilibrium beliefs in the sense that they are the result of applying Bayes Rule on equilibrium outcomes (i.e., a fixed point in beliefs).

We close this section by providing some general lemmas that can be used to simplify certain equilibrium expressions and to facilitate the construction of an equilibrium in our model.

Lemma 2. The equilibrium terms of credit for illiquid banks holding low quality assets satisfy the following condition:

$$\frac{i}{1-\alpha} \leq r_L(\theta, q_N, q_W) \leq q_N R.$$

Proof: The maximum amount that an illiquid bank can repay is $q_N R$. Hence, only values of r_L lower that $q_N R$ are relevant. Furthermore, if $(1-\alpha) r_L < i$ the liquid bank would not accept to lend. #

Note that this lemma can be used to simplify the equilibrium expressions for $S_b(r_L, q_N, q_W)$ and

$S_l(r_L, q_N)$. The next lemma shows that, in equilibrium, if an illiquid bank holding a high quality asset borrows at the discount window when it cannot find a counterparty, then so does an illiquid bank that is holding a low quality asset and also cannot find a counterparty.

Lemma 3. If the condition $\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) \geq -\rho + R$ holds, then condition $(1 - \alpha)P(q_W, r_W) \geq -\rho + (1 - \alpha)q_N R$ also holds.

Proof: The proof follows from the fact that $q_N \leq 1$ and $r_W > 0$. #

In principle, there are many possible configurations of equilibrium outcomes in this model, depending on parameter values. Furthermore, as in the previous section, for some parameter values multiple equilibria may exist. Studying the different cases can provide interesting insights about the influence of discount window policy on interbank lending activity. In the next section, we take a partial step in this direction. To keep the analysis focused on the issue of stigma, first we study equilibrium for a particular range of parameter values for which the possibility of stigma is present. Later, we discuss other possible equilibrium configurations.

5 Stigma

The objective in this section is to study, in the context of the model, the empirical and theoretical arguments discussed in the introduction of the paper. In particular, we want to construct an equilibrium in which stigma is associated with lending from the discount window and, for this reason, some banks take loans in the interbank market at rates higher than the discount window rate r_W. In the process, we identify specific conditions under which such a situation is theoretically possible and draw some conclusions about its empirical plausibility.

Let $r_W = i + \tau$ for $\tau > 0$; that is, borrowing at the discount window involves paying a premium τ over the opportunity cost of funds i.

Proposition 4. Define $\xi(p) = \frac{p - \sigma p}{1 - \sigma p}$ and assume that $\tau \in (0, A)$ where $A = \min\left\{\rho - i, \frac{\alpha}{1-\alpha} i\right\}$. Then, there exists a threshold value $\overline{p}_\tau < 1$ such that if $p \in (\overline{p}_\tau, 1)$ then there is an equilibrium with both interbank credit and discount window lending in which $q_W = \xi(p)$, $q_N = 1$, and $r_H = i + \theta T S_H(q_N, q_W)$.

Proof: See the Appendix. #

Since $\xi(p) < 1$ banks that borrow at the window and try to sell an asset of unobserved quality can only do so at a discount (i.e., $q_W < 1$). In other words, having borrowed at the window is regarded as a negative signal in the asset market. This effect is how our model captures the idea of stigma formally.

In the equilibrium of Proposition 4, only those illiquid banks that find a counterparty in the interbank market and are holding a high quality asset, borrow from another bank. Illiquid banks that do not find a counterparty, plus those banks that do but are holding a low quality asset, borrow from the discount window. Note that this configuration does not necessarily imply unrealistic levels of borrowing at the discount window. In fact, if both p and σ are close to unity, as they are likely to be in the empirically relevant case, then most banks actually borrow from the interbank market in this equilibrium.

Note that if τ is greater than A, that is, if the penalty rate from borrowing at the discount window is too large, then all illiquid banks that find a counterparty in the market will take a private loan. If this is the case and all banks borrowing at the window do so for exogenous reasons (i.e., because they did not find a counterparty) and not as the product of their choices, then there would be no adverse selection associated with banks' participation in central bank lending. As a consequence, having borrowed at the window could not be, in equilibrium, an informative signal about the quality of the assets held by the bank, and stigma would no longer be present.

The following corollary provides our main result dealing with the comparison between the equilibrium interest rate in the interbank market and the discount window rate.

Corollary 3. In the equilibrium described in Proposition 4, for θ close enough to unity, $r_H > r_W$.

Proof: Note that θ appears only in the condition that determines the surplus splitting rule between the liquid and the illiquid bank holding a high quality asset. Then, the existence of the equilibrium described in Proposition 4 is independent of the value of θ and the equilibrium exists for any value of θ, including those arbitrarily close to one. Since in such equilibrium we have that $r_H = i + \theta TS_H(q_N, q_W) = i + \theta\left[(1-\alpha)(1-q_W)R + r_W - i\right]$, which approaches $(1-\alpha)(1-q_W)R + r_W > r_W$ when θ approaches unity, the corollary holds. #

This corollary demonstrates that when the bargaining power of lenders is high, the equilibrium in our model may involve some banks that are willing to take a loan in the interbank market at a rate higher than the rate that they could obtain at the discount window. That is, under certain conditions, our model predicts the empirical pattern of interest rate that we discussed in the introduction.

The equilibrium premium of the observed private rate r_H over the discount window rate r_W is given by:

$$r_H - r_W = \theta(1-\alpha)(q_N - q_W)R - (1-\theta)\tau,$$

which is actually decreasing in τ. Hence, lowering the spread τ, a policy parameter, would tend to increase the extent to which some transactions in the market are executed at a rate higher than the discount window rate. This is an interesting finding. A common reaction by policymakers to the reluctance by banks to borrow from the window has been to lower the penalty spread.[22] Our model predicts that such a change could actually increase the extent by which observed market rates are higher than the discount window rate as a result of stigma.

[22] For a long time, since 2003, the Fed provided discount window credit at a rate 100 basis points over the target fed funds rate. Recently, with the advent of the 2007 financial crisis and, in part, to respond to the persistent reluctance of banks to borrow from the window, the Fed lowered the spread significantly, first to 50 basis points in August 2007, and later to 25 basis points in March 2008.

In the equilibrium of Proposition 4, the proportion of banks lending from the discount window is equal to $1-\sigma p$. During normal times, most banks in the U.S. do not borrow from the discount window. This empirical regularity suggests that the cases where the proportion of banks borrowing in the market, σp, is high are the relevant ones. Since q_W depend on σp, accommodating this fact has implications for the level of stigma that can be obtained in the model. However, it should be noted that, in principle, the model is consistent with relatively low values of q_W even when σp is high. Then, significant levels of stigma and equilibrium interest rate premia, as measured by $r_H - r_W \in (0, (1-\alpha)(1-q_W)R)$, can be obtained in "realistic" versions of the model.

The threshold value of A in Proposition 4 is positively related to the value of α, the probability that the quality of the asset held by a bank becomes observable by investors. In other words, when information about asset quality is more certain, the range of values of r_W for which the equilibrium in Proposition 4 can exists is larger. However, Corollary 3 also shows that the range of premia over the discount window rate that an illiquid bank would be willing to accept from a private counterparty is decreasing in the value of α. Then, while more information in asset markets can make stigma more likely for a given value of the discount window rate, the intensity with which equilibrium stigma is reflected in the observable variables actually decreases with such information.

In our model, the level of the interest rate that is observed in the market is given by r_H, which does not involve any repayment risk. Banks with low quality assets (which could be regarded as the risky ones in our setup) do not receive loans from private banks. They are just cut out of the interbank market. Furthermore, all banks borrowing at the market pay the same interest rate. In the data, however, any given day there is a distribution of rates observed in the market. We could generalize the model to capture these different rates by, for example, introducing some heterogeneity in the bargaining power of different banks. This modification may, in fact, be realistic (Ashcraft and Duffie, 2007). Banks in need of funds in a given day may find that their usual counterparty has no funds available that day. In that case, they need to search in the market for alternative counterparties and, depending on their network connections, they may find their bargaining power much reduced. In our model, illiquid banks with low bargaining power will pay higher interest rates. In fact, this kind of heterogeneity will be consistent with the fact that, most of the time, only some banks pay interest rates that are higher than the one they could obtain at the discount window.

An interesting feature of the equilibrium in Proposition 4 is that it requires p to be above a certain positive threshold. If we think that lower values of p are associated with a general deterioration of asset quality, our model tells us that the kind of equilibrium we are considering will not be possible when asset quality deteriorates beyond some point. A reason for why our equilibrium may break down as the value of p becomes lower is that some banks may refrain from borrowing at the window if the equilibrium value of q_W is too low (as it needs to be if the value of p is low). To be concrete, if the value of q_W is too low, it may be the case, for example, that it is no longer beneficial for banks holding low quality assets to borrow at the window (see the analysis in Section 6). But then, illiquid banks that find a match will borrow in the market and illiquid banks that do not find a match will borrow at the window. In consequence, the composition of banks borrowing at the window would be the same as that of banks borrowing in the market (a proportion p of banks holding high quality assets and a proportion $1-p$ holding low quality assets). Hence, borrowing at the discount window would not be regarded as a negative signal and the equilibrium of the model would no longer produce the outcomes associated with stigma.

Finally, note that higher values of the probability of finding a match in the interbank market, σ, (*ceteris paribus*) result in lower equilibrium values of q_W. In fact, very high values of σ will undermine the possibility of an equilibrium with stigma of the type we consider in Proposition 4. The reason for this fact is that, in our equilibrium, when a larger proportion of banks find a match in the interbank market, the composition of banks borrowing at the window shifts toward a relative abundance of banks holding low quality assets and, hence, q_W becomes smaller. For small enough values of q_W a bank that finds a match and is holding a low quality asset will actually prefer to take a loan from its private counterparty, an action that is inconsistent with the equilibrium proposed in Proposition 4.[23] If we think that the likelihood of finding a counterparty in the interbank market is an approximate measure of the liquidity in that market, then we can conclude that the possibility of equilibrium stigma is associated with low levels of liquidity in the interbank market. In other words, when the market for interbank loans is very liquid, the type of equilibrium with stigma studied in this section is likely to break down.

5.1 Imperfectly observable discount window activity

Suppose that after a bank borrows at the discount window, investors in the asset market become aware of such activity only with probability $\delta < 1$. Two main changes in equilibrium variables and decisions result from introducing this modification. First, prices in the asset market must reflect the fact that even when a bank shows no evidence of having been at the discount window, it is still possible that such a bank has indeed borrowed from the central bank. Second, when a bank is deciding whether to borrow at the window or borrow from another bank, it must take into account that activity at the window is not necessarily observed in the asset market.

In the equilibrium of Proposition 4 only illiquid banks that find a counterparty and are holding a high quality asset borrow from another bank. All the other banks borrow from the discount window. We will demonstrate here that an equilibrium with these characteristics still exists when $\delta < 1$ and the conditions in Proposition 4 are satisfied.

Define $P(q_W, q_N, r_W) = \delta \max\{q_W R - r_W, 0\} + (1-\delta)\max\{q_N R - r_W, 0\}$ then the expressions describing the decisions of banks are the same as in Section 4 but where the function $P(q_W, q_N, r_W)$ replaces the function $P(q_W, r_W)$. Let $\zeta(p) = \frac{p(1-\delta+\delta\sigma)}{1-\delta+p\delta\sigma}$ and A and $\overline{p}_\tau$ as defined in Proposition 4.

Proposition 5. Assume that $\tau \in (0, A)$. If $p \in (\overline{p}_\tau, 1)$ then there is an equilibrium with both

[23] If q_W is very low, taking a loan at the discount window entails a large (expected) discount at the time of selling the asset in period 2, on the eventuality that the quality of the asset is not observed by investors.

interbank credit and discount window lending in which $q_W = \xi(p)$, $q_N = \zeta(p)$.

Proof: See the Appendix. #

In the equilibrium of Proposition 5, just as in the equilibrium of Proposition 4, only illiquid banks with high quality assets borrow from another bank. Even though this is the case, the price of the assets sold by those banks showing no evidence of borrowing at the window still comprises a discount. The reason for this price discount, which is not present in Proposition 4, is that some of the banks in that group will actually have a low quality asset and will have borrowed at the window. Investors, of course, are aware of the uncertainty over the quality of the assets held by these banks and, hence, require a discounted price to agree to purchase the assets. Note, however, that since $\xi(p) < \zeta(p)$ the discount is higher for banks that can be identified as having borrowed from the window. In this sense, stigma is still present in equilibrium.

In line with the common intuition, the fact that discount window activity is not perfectly observable makes borrowing from the central bank more attractive for all banks in the model. Banks holding high quality assets, however, still prefer to borrow from another bank and not from the discount window. This is the case for two reasons: (i) there is still stigma attached to being identified as borrowing from the window, and (ii) the discount window rate is a penalty rate ($\tau > 0$). In fact, the "binding" condition for the existence of the type of equilibrium described in Propositions 4 and 5 involves verifying that banks holding *low quality* assets prefer to borrow from the window even when they find a counterparty in the interbank market. For this reason, changes in parameter that could break down the equilibrium configuration in Propositions 4 and 5 involve changes that make discount window lending *less* attractive. Imperfect observability makes discount window lending *more* attractive and hence does not disturb the pattern of behavior postulated in Propositions 4 and 5.

More generally, changes in the incidence of stigma will be associated with changes in the composition of banks borrowing at the window (in terms of the quality of their assets) relative to the composition of banks borrowing in the interbank market. The model we have presented allows only abrupt changes in this composition (either low quality banks that find a counterparty borrow at the window or they do not) and, for this reason, it is not especially suitable to conduct comparative statics on policy parameters which could, for example, change δ. Generalizing the patterns of heterogeneity in asset qualities would make such exercises more meaningful within the framework of this paper. If a policy measure tends to increase the prevalence of banks with relatively high quality assets borrowing at the window, then that policy will tend to decrease the level of stigma associated with such activity.

6 Other possible equilibria

There are other possible equilibrium configurations, depending on parameter values. One interesting alternative is the equilibrium in which illiquid banks that find a match in the interbank market borrow from their private counterparty, and only those illiquid banks that do not find a counterparty borrow at the discount window (with $\delta = 1$). In such an equilibrium, there is no stigma attached to borrowing from the window. The following proposition provides an explicit characterization.

Proposition 6. Assume that $\frac{\alpha}{1-\alpha} i < \tau < \rho - i$. There is a threshold value $\overline{p}_\tau < 1$ such that if $p \in (\overline{p}_\tau, 1)$ then there is an equilibrium with both interbank credit and discount window lending in which $q_W = q_N = p$ and $r_H = \theta r_W + (1-\theta)\, i < r_L = \theta r_W + (1-\theta)\, \frac{i}{1-\alpha} < r_W$.

Proof: See the Appendix. #

Note that this proposition requires τ to be greater than A, as defined in Proposition 4. Basically, the proposition in this section deals with equilibrium outcomes in the case in which the interest rate at the discount window, r_W, is relatively *high*. As it turns out, when r_W is relatively high, there is no equilibrium stigma at the discount window. Furthermore, note that $r_j < r_W$ for $j = H, L$, and hence observed market interest rates (and any weighted average of them) will be below the rate at the discount window. This result is in sharp contrast with the finding stated in Corollary 3.

Another interesting feature of the equilibrium in Proposition 6 is that, since $r_H < r_L$, some dispersion in interest rates is observed in the interbank market. In particular, banks with high quality assets obtain credit at lower interest rates than banks with low quality assets.[24] Finally, note that since $\frac{p-\sigma p}{1-\sigma p} < p < 1$, a bank borrowing at the window receives a higher price for an asset of unobserved quality than in the equilibrium in which borrowing at the window is regarded as a negative signal (i.e., the equilibrium of Proposition 4). However, for banks that do not borrow at the window, the price of the unobserved quality asset is actually lower in this equilibrium without stigma.

An implication of Propositions 4 and 6 is that stigma happens at moderate levels of the discount window (rate) spread τ. A natural question to ask is whether equilibria with no stigma could be constructed for low levels of r_W. It is easy to see that, for low values of r_W, one can construct equilibria where all banks borrow from the discount window. In such construction, off equilibrium beliefs become important, since investors need to form beliefs about the probability that a (deviating) bank not borrowing at the window holds a high quality asset. These beliefs are necessarily arbitrary in our model and can affect equilibrium outcomes and the interpretation of stigma. This kind of technical complication is common in signaling environments and a thorough discussion of the issues involved is beyond the scope of this paper.

[24] King (2008) presents evidence suggesting that banks with higher repayment risk tend to pay higher interest rates for loans in the US interbank market.

The issue of stigma is often discussed in relation to the possibility of multiple equilibria. Loosely speaking, the logic refers to an activity that "everybody does it" because there is no stigma, and there is no stigma because "everybody does it" (see, for example, the tax evasion example of Kim, 2003). This is not the idea behind the equilibria that we have constructed in this paper. In the model, stigma does not always decrease when participation at the window increases. In fact, when moving from the equilibrium in Proposition 6 to the equilibrium in Proposition 4, the proportion of banks borrowing at the discount window becomes higher and stigma actually increases.[25]

7 Conclusion

In this paper, we have provided a formal model of the interaction between the interbank market for funds and the asset market. Our model is capable of reproducing certain trading patterns by banks that are consistent with a situation in which outside investors attach some degree of stigma to the activity of borrowing from the central bank's discount window. The intention was to introduce the minimum number of elements in the model to allow us to capture such stigma effect. In this kind of signaling setting there is a delicate balance between information and frictions, which needs to be maintained so that, while some information flows to the market, the equilibrium does not become fully revealing. Achieving this balance in a parsimonious way is the main justification for most of our simplifying assumptions.

The main components of our model are the following. On the one hand, participants in the interbank market have (some) information about the quality of their counterparty's assets. On the other hand, private dealings in the interbank market are not observable by third parties. Actions in the interbank market, if observable, could reveal asset quality and, hence, pin down asset prices independent of any signal.

Repayment risk is endogenous in the model and depends on the equilibrium in the asset market. In turn, banks' activities in the asset market depend on their ability to borrow in the interbank market. In particular, some banks in the asset market may be selling their assets because they need to repay their interbank loans. Other banks, however, may be selling their assets just because they know that their assets are low quality and that, at the prevailing equilibrium prices, they are effectively overpriced.

When we introduce the possibility of borrowing from the central bank, bank participation at the discount window may be subject to adverse selection and, as a consequence, stigma may arise. We make information about borrowing activity at the window (partially) observable by outside investors. The idea of stigma clearly relies on some degree of observability; the underlying logic

[25] From the situation in Proposition 4, an increase in participation would indeed reduce stigma. In fact, if one assumes that $q_N = 0$ then constructing an equilibrium where that happens is actually possible. However, such equilibrium would involve off-equilibrium beliefs that are unlikely to be robust to the standard criteria used for equilibrium refinement.

is that information held by participants in the interbank market, which would otherwise remain private, can flow to asset market participants through reported activity at the discount window. Another important aspect associated with the possibility of stigma is that loan repayment risk, while partly endogenous, also depends on the true quality of the assets held by the banks. This fact is what implies that, in equilibrium, banks holding bad assets are more likely to not obtain loans in the interbank market and, hence, be borrowing at the discount window. Consequently, there is adverse selection in participation decisions at the discount window, justifying that borrowing from the central bank is regarded as a negative signal and, in this way, making stigma an equilibrium phenomenon.

This was a paper on positive economics. We made no attempt to address most of the relevant policy issues associated with our general subject of inquiry. The incidence of stigma in the activity of borrowing from the discount window has potentially important policy implications. For example, if some amount of discount window lending is optimal and stigma makes banks reluctant to access such liquidity (as it has often been argued), then the effectiveness of policy may be seriously impaired as a result. While addressing the policy questions is, of course, very important, we think that in the process of reaching reliable conclusions, an essential first step is to develop a better understanding of the fundamental nature of stigma in the interbank market. Taking this first step was the objective of this paper.

8 Appendix

Proof of Proposition 3: We divide the proof in two parts:

Part 1. When $\alpha i \geq \rho - (1-\alpha)R$ holds, we want to show that $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*)$ for all θ and all $q^* \geq q^T$. If we can show that $\widehat{r}_H(\theta, q^*) < \widetilde{r}_H(\theta, q^*) < q^* R$ then the result follows. Start by noting that if $\alpha i > \rho - (1-\alpha)R$ then $\widehat{r}_H(0, q^T) < \widetilde{r}_H(1, q^T)$. Since $\widehat{r}_H(\theta, q^*)$ is increasing in θ and decreasing in q^* for $q^* > q^T$ and $\widetilde{r}_H(\theta, q^*)$ is increasing in θ and q^* for $q^* > q^T$, we have that:

$$\widehat{r}_H(\theta, q^*) \leq \widehat{r}_H(0, q^T) < \widetilde{r}_H(1, q^T) = \widetilde{r}_H(\theta, q^T) \leq \widetilde{r}_H(\theta, q^*) \quad \text{for all } \theta \text{ and } q^* \geq q^T.$$

Furthermore, when $\alpha i > \rho - (1-\alpha)R$ we have (after some algebra) that $\widetilde{r}_H(\theta, q^*) < q^* R$ and since $\widehat{r}_H(\theta, q^*) < \widetilde{r}_H(\theta, q^*)$ we have that $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*)$ for all θ and all $q^* \geq q^T$. If $\alpha i = \rho - (1-\alpha)R$ then $\widehat{r}_H(0, q^T) = \widetilde{r}_H(1, q^T) = i$. We still have that $\widehat{r}_H(\theta, q^*) \leq i \leq \widetilde{r}_H(\theta, q^*) \leq q^* R$ for all θ and $q^* \geq q^T$ so, again, $r_H(\theta, q^*) = \widetilde{r}_H(\theta, q^*)$ for all θ and all $q^* \geq q^T$.

Part 2. When $\alpha i < \rho - (1-\alpha)R$ things are more complicated. In particular, $r_H(\theta, q^*)$ may be equal to $\widetilde{r}_H(\theta, q^*)$ or $\widehat{r}_H(\theta, q^*)$ depending on the values of q^* and θ. Start by noting that if $\alpha i < \rho - (1-\alpha)R$ then $\widehat{r}_H(0, q^T) > \widetilde{r}_H(1, q^T)$. Then, define two threshold values for q^* as follows:

$$q_1 \equiv i/R \quad \text{and} \quad q_2 \equiv [\rho - (1-\alpha)\, R]/\alpha R.$$

We have that $\widehat{r}_H(0,q_1) = i$ and $\widetilde{r}_H(1,q_2) = [\rho - (1-\alpha)R]/\alpha$. Also note that $q_2 > q_1 > q^T$. Given these thresholds, we need to consider three cases:

(1) If $q^* \in [q^T, q_1]$ then we have that $q^*R \leq q_1 R = i$. Since $\widehat{r}_H(\theta, q^*)$ is non-decreasing in θ we have that $\widehat{r}_H(0,q^*) \leq \widehat{r}_H(\theta,q^*)$ for $\theta \geq 0$ and $q^* \in [q^T, q_1]$. Also, since $\widehat{r}_H(\theta,q^*)$ is non-increasing in q^* we have that $\widehat{r}_H(\theta,q_1) \leq \widehat{r}_H(\theta,q^*)$ for $q^* \in [q^T, q_1]$. Then, we have that

$$\widehat{r}_H(\theta,q^*) \geq \widehat{r}_H(\theta,q_1) > \widehat{r}_H(0,q_1) = i = q_1 R \geq q^* R.$$

Since $\widetilde{r}_H(\theta,q^*)$ is non-decreasing in q^* we have that:

$$\widetilde{r}_H(\theta,q^*) \geq \widetilde{r}_H(\theta,q^T) = 1 = q_1 R > q^* R.$$

It then follows that for $q^* \in [q^T, q_1]$ we have that $r_H(\theta,q^*) = \widehat{r}_H(\theta,q^*)$.

(2) If $q^* \in (q_1, q_2)$ then $\widehat{r}_H(0,q^*) < i = \widetilde{r}_H(0,q^*)$ and $\widehat{r}_H(1,q^*) = [\rho-(1-\alpha)R]/\alpha = \widetilde{r}_H(1,q_2) > \widetilde{r}_H(1,q^*)$. Define the auxiliary function $\psi(\theta,q^*) \equiv \widehat{r}_H(\theta,q^*) - \widetilde{r}_H(\theta,q^*)$. It is easy to demonstrate that $\psi(\theta,q^*)$ is increasing in θ. From the inequalities above, we have that $\psi(0,q^*) < 0$ and $\psi(1,q^*) > 0$. By continuity, then, there exist $\theta^T \in (0,1)$ such that $\psi(\theta^T,q^*) = 0$, or similarly,

$$\widehat{r}_H(\theta^T,q^*) = \widetilde{r}_H(\theta^T,q^*),$$

where $\theta^T = (q^*R - i)/TS(q^*)$. Note that $\widetilde{r}_H(\theta^T,q^*) = i + \theta^T TS(q^*) = q^*R = \widehat{r}_H(\theta^T,q^*)$. For $\theta < \theta^T$ we have that $\widehat{r}_H(\theta,q^*) < \widetilde{r}_H(\theta,q^*) < q^*R$ and $r_H(\theta,q^*) = \widetilde{r}_H(\theta,q^*)$. For $\theta \geq \theta^T$ we have that $\widehat{r}_H(\theta,q^*) \geq \widetilde{r}_H(\theta,q^*) \geq q^*R$ and $r_H(\theta,q^*) = \widehat{r}_H(\theta,q^*)$.

(3) If $q^* \geq q_2$ then $q^*R \geq q_2 R = [\rho-(1-\alpha)R]/\alpha$. Also, it is easy to see that $\widetilde{r}_H(\theta,q^*) \leq \widetilde{r}_H(1,q^*) \leq q^*R$. Since we have that:

$$\widetilde{r}_H(1,q^*) > \widetilde{r}_H(1,q_2) = \widehat{r}_H(1,q^*),$$

and since the slope of $\widehat{r}_H(\theta,q^*)$ with respect to θ is higher than the slope of $\widetilde{r}_H(\theta,q^*)$ with respect to θ, then we have that:

$$\widehat{r}_H(\theta,q^*) \leq \widetilde{r}_H(\theta,q^*) \leq q^*R,$$

which implies that $r_H(\theta,q^*) = \widetilde{r}_H(\theta,q^*)$ for all θ and all $q^* \geq q_2$.#

Proof of Proposition 4: Consider an arbitrary value of $\tau \in (0,A)$. We want to show that for a set of values of p there is an equilibrium with both interbank credit and discount window lending in which $q_W = \xi$, and $q_N = 1$. We organize the proof in five steps.

Step 1. Given τ, pick the value p_1 such that $q_1 \equiv \frac{p_1 - \sigma p_1}{1-\sigma p_1}$ satisfies $q_1 = \frac{i+\tau}{R}$.[26] Since $i+\tau < i+\rho-i = \rho < R$ we have that $q_1 < 1$ and hence $p_1 < 1$. Then, if $p > p_1$ and $q_W = \xi$ then we have that

[26] Note that the function $q(p) = \frac{p-\sigma p}{1-\sigma p}$ is increasing in p and that $\lim_{p\to 1} q(p) = 1$ and $\lim_{p \to 0} q(p) = 0$.

$q_W R > q_1 R = r_W$. Also note that:

$$\alpha (R - r_W) + (1-\alpha) P(q_W, r_W) = \alpha R + (1-\alpha) q_W R - r_W = R - r_W - [(1-\alpha)(1-q_W) R].$$

Now, pick the value p_2 such that $q_2 \equiv \frac{p_2 - \sigma p_2}{1-\sigma p_2}$ satisfies

$$q_2 = 1 - \frac{\rho - i - \tau}{(1-\alpha) R}.$$

Since $\rho - i - \tau > \rho - i - \rho + i = 0$ then $q_2 < 1$ and $p_2 < 1$ and if $p > \max\{p_1, p_2\}$ then

$$R - r_W - [(1-\alpha)(1-q_W) R] > R - r_W - (\rho - i - \tau) = R - \rho > 0.$$

Hence, whenever $p > \max\{p_1, p_2\}$ illiquid banks holding a high quality asset will borrow at the discount window when they do not find a match in the interbank market. By Lemma 3, illiquid banks holding a low quality asset will do the same. Furthermore, notice that the inequalities above demonstrate that the relevant alternative for those illiquid banks that do find a match is for them to borrow (rather than not to borrow at all).

Step 2. For any $\max\{p_1, p_2\} < p < 1$ we have that $q_W = \xi < 1$ and $TS_H = (1-\alpha)(1-q_W) R + r_W - i = (1-\alpha)(1-q_W) R + \tau > 0$. Then,

$$R - r_H = R - i - \theta TS_H > R - i - TS_H = R - (1-\alpha)(1-q_W) R - r_W,$$

which implies that $R - r_H > \alpha R + (1-\alpha) q_W R - r_W$ and hence illiquid banks that find a match in the interbank market and are holding a high quality asset borrow from the market.

Step 3. By Lemma 2, a liquid bank would agree to give a loan to an illiquid bank holding a low quality asset only if the agreed upon interest rate r_L is such that $r_L \geq i/(1-\alpha)$. Pick the value p_3 such that $q_3 \equiv \frac{p_3 - \sigma p_3}{1-\sigma p_3}$ satisfies

$$q_3 = 1 - \frac{\alpha i - (1-\alpha)\tau}{(1-\alpha) R}.$$

Since $\alpha i - (1-\alpha)\tau > \alpha i - (1-\alpha)[\alpha i/(1-\alpha)] = 0$ we have that $q_3 < 1$ and $p_3 < 1$. If $p > \max\{p_1, p_2, p_3\}$ then

$$(1-\alpha)(q_W R - r_W) > (1-\alpha)\left[\left(1 - \frac{\alpha i - (1-\alpha)\tau}{(1-\alpha) R}\right) R - r_W\right] = (1-\alpha) R - i \geq (1-\alpha)(R - r_L).$$

Hence, illiquid banks holding a low quality asset will borrow at the window even when they find a match in the interbank market. Note that the condition $(1-\alpha)(q_W R - r_W) > (1-\alpha) R - i$ is equivalent to the condition $TS_L < 0$.

Step 4. Let $\overline{p}_\tau = \max\{p_1, p_2, p_3\}$. For any $p \in (\overline{p}_\tau, 1)$, illiquid banks holding high quality assets borrow in the interbank market if they find a match and all other banks borrow at the window.

That is, $1-\sigma p$ banks borrow at the window, of which only a proportion $p(1-\sigma)$ are holding high quality assets. Hence, $q_N = 1$ and $q_W = p(1-\sigma)/(1-\sigma p) \equiv \xi$.

Step 5. In this equilibrium, we have that

$$S_b(r_H, q_N, q_W) = (1-\alpha)(1-q_W)R - r_H + r_W$$

and

$$S_l(r_H, q_N) = r_H - i.$$

Solving the Nash bargaining problem we get that $r_H = i + \theta[(1-\alpha)(1-q_W)R + r_W - i] = i + \theta TS_H$. #

Proof of Proposition 5: Since $p > \overline{p}_\tau$ we have that $q_W R - r_W > 0$ and that

$$\begin{aligned} \alpha(R-r_W) + (1-\alpha)[\delta(q_W R - r_W) + (1-\delta)(q_N R - r_W)] &> \alpha(R-r_W) + (1-\alpha)(q_W R - r_W) \\ &> -\rho + R, \end{aligned}$$

so illiquid banks that do not find a counterparty and are holding a high quality asset borrow from the window. This inequalities also imply that:

$$(1-\alpha)P(q_W, q_N, r_W) > -\rho + R - \alpha(R-r_W) > -\rho + (1-\alpha)q_N R,$$

so illiquid banks that do not find a counterparty and are holding a low quality asset borrow at the window. It is easy to verify that the Nash bargaining solution implies that $S_l(r_H, q_N) = \theta TS_H$ and $S_b(r_H, q_N, q_W, r_W) = (1-\theta)TS_H$ where $TS_H = (1-\alpha)\delta(q_N - q_W)R + \tau > 0$. Hence, illiquid banks that find a counterparty in the interbank market and are holding a high quality asset arrange a private loan. Finally, note that:

$$TS_L = (1-\alpha)[\delta(q_N - q_W)R + r_W] - i < (1-\alpha)[(1-q_W)R + r_W] - i.$$

The right-hand-side of this inequality was shown to be negative in the proof of Proposition 4. Then, illiquid banks that find a counterparty and are holding a low quality asset prefer to borrow from the discount window.

In the asset market, then, there is a proportion $p\sigma + (1-\delta)[p(1-\sigma) + (1-p)]$ of illiquid banks that cannot be identified as having borrowed from the discount window. Of these, only a proportion $p\sigma + (1-\delta)p(1-\sigma)$ hold high quality assets. Hence, the price of the assets held by banks showing no evidence of borrowing at the window is $P_N = q_N R$ where, by Bayes law, $q_N = \zeta(p)$ (the ratio of the two quantities discussed above). Similarly, there is a proportion $\delta[p(1-\sigma) + (1-p)]$ of illiquid banks in the asset market that are identified as having borrowed from the discount window. Of those, only $\delta(1-p\sigma)$ are holding high quality assets. By Bayes law, the price of assets for banks that can be verified as borrowing from the window is $P_W = q_W R = \xi(p)R$ where $\xi(p)$ is the ratio of $\delta(1-p\sigma)$ and $\delta[p(1-\sigma)+(1-p)]$ (and it is, in fact, equal to the value of q_W in Proposition 4). #

Proof of Proposition 6: Consider a value of τ such that $r_W = i+\tau \in \left(\frac{i}{1-\alpha}, \rho\right)$. We want to show that for a set of values of p there is an equilibrium where illiquid banks that find a counterparty borrow from the interbank market (regardless of the quality of the asset they hold) and illiquid banks that do not find a counterparty borrow from the discount window.

Given τ, pick the value $\overline{p}_\tau = \max\left\{\frac{i+\tau}{R}, 1-\frac{\rho-i-\tau}{(1-\alpha)R}\right\}$. Since $i+\tau = r_W < \rho < R$ we have that $\overline{p}_\tau < 1$ and $(\overline{p}_\tau, 1)$ is non-empty. Consider a value of $p \in (\overline{p}_\tau, 1)$. We want to show that:
(1) illiquid banks with a match borrow from the interbank market; that is,

$$\alpha(R-r_H)+(1-\alpha)P(q_N, r_H) > \alpha(R-r_W)+(1-\alpha)P(q_W, r_W),$$

and

$$(1-\alpha)P(q_N, r_L) > (1-\alpha)P(q_W, r_W);$$

(2) illiquid banks without a match borrow from the discount window; that is,

$$\alpha(R-r_W)+(1-\alpha)P(q_W, r_W) > -\rho+R,$$

and

$$(1-\alpha)P(q_W, r_W) > -\rho+(1-\alpha)q_N R;$$

(3) liquid banks are willing to lend to illiquid banks (regardless of asset quality); that is,

$$\alpha r_H + (1-\alpha)\min\{q_N R, r_H\} > i,$$

and

$$(1-\alpha)\min\{q_N R, r_L\} > i.$$

If these three sets of conditions hold, then it is easy to see that, of the banks borrowing in the market, a proportion p holds high quality assets. Furthermore, this proportion is also p for the banks borrowing at the discount window. Hence, $q_n = q_W = p$ in equilibrium. Since $p > \overline{p}_\tau \geq \frac{i+\tau}{R}$ we have that $pR > r_W$.

The total surplus from an interbank loan to a high-quality asset holder is $TS_H = r_W - i > 0$ and the total surplus from a loan to a low-quality asset holder is $TS_L = r_W - \frac{i}{(1-\alpha)} > 0$. Bargaining over the terms of the loan will then result on $r_H = i + \theta TS_H = \theta r_W + (1-\theta)i < r_W$ and $r_L = \frac{i}{1-\alpha} + \theta TS_L = \theta r_W + (1-\theta)\frac{i}{1-\alpha} < r_W$. In consequence, we have that $pR > r_L > r_H$.

Then, the two conditions in *(1)* hold since $r_H > r_W$ and $r_L > r_W$; and the two conditions in *(3)* hold since $r_H > i$ and $r_L > \frac{i}{1-\alpha}$. Since $p > \overline{p}_\tau > 1-\frac{\rho-i-\tau}{(1-\alpha)R}$ we have that the first condition in *(2)* holds as:

$$\alpha(R-r_W)+(1-\alpha)P(q_W,r_W)=\alpha R+(1-\alpha)pR-r_W>\alpha R+(1-\alpha)R-(\rho-i-\tau)-r_W=-\rho+R.$$

Finally, the second condition in *(2)* holds because $r_W<\rho<\frac{\rho}{1-\alpha}$. This completes the proof. #

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