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Understanding the Inventory Cycle
I. Partial Equilibrium Analysis

by

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Abstract

Careful examination of aggregate data from the U.S. and other OECD countries reveals that production and inventory behavior exhibit paradoxical features: 1) Inventory investment is strongly **countercyclical** at very high frequencies (e.g., 2-3 quarters per cycle); it is procyclical only at relatively low cyclical frequencies such as the business-cycle frequencies (e.g., 8-40 quarters per cycle). 2) Production is **less volatile** than sales around the high frequencies; it is more volatile than sales only around business-cycle or lower frequencies. 3) Unlike capital investment or GDP, the bulk of the variance of inventory investment is concentrated around high frequencies rather than around business-cycle frequencies. These features of production and inventory behavior at the low and high frequencies provide a litmus test for inventory theories. This paper shows that the stockout-avoidance theory (Kahn, AER 1987) has a much better potential than any other competing theories for explaining the seemingly paradoxical features of inventory fluctuations observed at different cyclical frequencies. My analysis suggests that demand shocks are the source of the business cycle.

Keywords: Business cycles; Inventories; Stockout; Production smoothing; Demand shocks.

JEL classification: E13; E22; E32.

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1 Introduction

Understanding inventory fluctuations is a key step towards understanding the business cycle. Blinder and Maccini (1991) show that the drop in inventory investment accounts for 87 percent of the drop in total output during the average postwar recession in the U.S. There are two basic theories in the literature to explain the role of inventories: the production-smoothing theory and the stockout-avoidance theory. According to the production-smoothing theory, firms hold inventories to smooth the time path of production so as to reduce average costs of production under demand uncertainty when the cost function is convex. This theory predicts that production is less variable than sales and inventory investment is countercyclical with respect to sales (e.g., see Blinder 1986). According to the stockout-avoidance theory, firms hold inventories in order to avoid losses of opportunity for prospective sales when production takes time and is hence incapable of responding to demand shocks instantaneously. Consequently, firms may have the incentive to "overreact" to changes in expected demand by producing excess inventories, resulting in excess volatility in production relative to sales and procyclical inventory investment (e.g., see Kahn 1987).

A consensus in the empirical literature is that real-world production is more variable than sales and inventory investment is procyclical (e.g., see Blanchard, 1983, Blinder 1986, Ramey and West, 1999). Since the production-smoothing theory predicts countercyclical inventory investment and smooth production relative to sales, it has not been fared well empirically and historically (see Blinder 1986 and 1991 for critical reviews on this theory).[1] The real challenge, therefore, has been to provide models that can genuinely explain the apparent lack of production smoothing observed in most industry-level and aggregate data. Important work includes introducing stockout-avoidance motives (e.g., Able 1985, Blanchard 1983, Kahn 1987, West 1986), supply-side shocks (e.g., Blinder 1986, Eichenbaum 1989), nonconvex costs of production (e.g., Ramey 1991). Among them, Kahn's work is perhaps most provocative. In a theoretical paper (Kahn 1987), Kahn claims that a stockout-avoidance motive due to a nonnegativity constraint on inventories together with serially correlated demand shocks are sufficient for explaining why variance of production may exceed variance of sales. Kahn (1992) further claims that most of the interesting features of inventory and production behaviors can be attributed to firm's responses to demand uncertainty with a stockout-avoidance motive under a nonnegativity constraint on inventories.

A careful re-examination of quarterly aggregate data from the U.S. and other OECD economies, however, reveals that production and inventory behavior exhibit paradoxical features: 1) Inventory

[1] But see Fair (1989) and Krane and Braun (1991) for some empirical evidence of production smoothing found in physical-product data of certain industries.

investment is strongly countercyclical at very high frequencies (e.g., 2-3 quarters per cycle); it is procyclical only at relatively low cyclical frequencies such as the business-cycle frequencies (e.g., 8-40 quarters per cycle). 2) Production is less volatile than sales around the high frequencies; it is more volatile than sales only around business-cycle or lower frequencies. 3) Unlike capital investment growth or GDP growth, the bulk of the variance of inventory investment growth is concentrated around high frequencies rather than around business-cycle frequencies.

The stylized fact that production and inventories exhibit drastically different behaviors at the high- and low-cyclical frequencies offers a litmus test for theories regarding the motives for holding inventories and the sources of shocks that cause inventory fluctuations. For example, the features of production and inventory behavior at the high frequencies seem fully consistent with the production-smoothing theory while those at the business-cycle frequencies seem fully consistent with the stockout-avoidance theory.

This paper shows that the stockout-avoidance theory of Kahn (1987), nevertheless, holds more truth than any other competing theories in explaining the seemingly paradoxical features of production and inventory behaviors. The production-smoothing theory as well as the associated cost-shock theory fail to predict that inventories exhibit opposite behaviors at high- and low-cyclical frequencies. The intuition is quite simple. Kahn (1987) has shown that if demand shocks are serially correlated and if production takes time, it is then rational for firms to engage in speculative production by accumulating excess inventories in the short run so as to avoid possible stockouts, giving rise to more variable production than sales and procyclical inventory investment. However, these effects of the stockout-avoidance mechanism can be observed only at relatively low frequencies such as the business-cycle frequencies; because at the high frequencies, due to sluggish adjustment in production in the short run, inventories must act as a buffer stock to absorb demand shocks, resulting in countercyclical and volatile changes in inventories. Thus, the drastically different production and inventory behaviors observed at the high and low frequencies are consistent with the stockout-avoidance theory. In contrast, under demand uncertainty the production-smoothing theory predicts less variable production than sales and negative correlations between inventory investment and sales at both high and low frequencies, hence it is not consistent with the data at the business cycle frequencies. Although allowing for technology or cost shocks in the production-smoothing model can help inducing more variable production than sales, such supply-side shocks cannot explain why the correlation between inventory investment and sales is negative at the high frequencies but positive at the low frequencies. The reason is that supply shocks either generate procyclical inventory investment at both high and low frequencies or do not have any effect on the correlation between inventory investment and sales when a nonnegativity constraint on inventories is relevant.

Among all the prominent features of inventory behavior documented in this paper, there is one the stockout-avoidance model cannot explain, however. That is the relatively less volatile production than sales at the high frequencies. For most of the OECD countries examined in this paper except the U.S. and Finland, production is about 50% to 20% less volatile than sales at the high frequencies (2-3 quarters per cycle). The stockout-avoidance model of Kahn (1987) predicts that production is more volatile than sales at both high and low frequencies. Clearly, allowing for supply-side shocks does not help resolving this puzzle. It thus remains a challenge to the stockout-avoidance inventory theory to explain why production appears to be smooth relative to sales at the high frequencies.

The rest of the paper is organized as follows. Section 2 documents the dramatically different nature of inventory fluctuations at different cyclical frequencies in the U.S. and other OECD countries. Sections 3 and 4 compares these stylized facts with the predictions of various inventory theories, including the production-smoothing theory and the stockout-avoidance theory. Section 5 concludes the paper with remarks on further research.

2 Reality: Inventory Cycles at High and Low Frequencies

This section documents the stark differences in inventory fluctuations at high and low cyclical frequencies for post-war U.S. and some OECD economies.[2] I use the band-pass filter (Baxter and King, 1995) to isolate movements of aggregate inventory investment, production, and sales in the frequency range of 2-3 quarters per cycle (called the "high frequency" interval) and the frequency range of 8-40 quarters per cycle (called the "business-cycle frequency" interval).[3] Before applying the filter, all data series except inventory investment are logged and then detrended using a linear time trend. Since inventory investment series contain negative values and do not have noticeable time trend, they are normalized (divided) by the median of output series in the respective country

[2] All data are seasonally adjusted, quarterly real data. Total sales is defined as total output minus inventory investment. The US data are taken from Citibase (1960:1 - 1994:4). The rest are taken from OECD data bank (1960:1 - 1994:4). Countries with missing data or very short data series (less than 30 observations) are excluded from the sample.

[3] The results do not change significantly if the high-frequency interval is extended from 2-3 quarters per cycle to 2-4 quarters per cycle and the business-cycle frequency interval is extended from 8-40 quarters per cycle to 6-100 quarters per cycle. The band-pass filter uses a maximum lag length of $k = 8$ as the truncation window parameter, implying totally 16 observations are lost from each end of the data series. This choice is based on the length of samples. See Baxter and King (1995) for discussions on this issue.

(region).[4]

Table 1 reports the relative volatilities of production with respect to sales and correlations between inventory investment and sales. Several striking patterns of inventory and production behavior emerge from the table:

1) At the high frequencies (2nd column) production is less volatile than total sales. The ratio of standard deviations between output and sales, for example, is 0.91 for OECD and 0.83 for European countries as a whole. The only exceptions at the individual country level are the United States and Finland.

2) At the high frequencies (3rd column) inventory investment is strongly countercyclical with respect to sales. The correlation between inventory investment and sales, for example, is -0.43 for OECD countries and -0.51 for European economies. This is true for all individual countries in the sample.

3) In stark contrast, at the business cycle frequencies (4th column) production is more volatile than sales. The ratio of standard deviations between GDP and sales, for example, is 1.39 for OECD and 1.55 for European countries as a whole. There is no exception for any individual countries in the sample.

4) At the business cycle frequencies (5th column), inventory investment is positively correlated with sales. Except for Austria and Switzerland, the correlations between inventory investment and sales are all positive. It is, for example, 0.58 for OECD and 0.47 for European countries as a whole.

[4] The statistics reported in table 1 are robust to the data transformation method used. For example, they do not change significantly if the band-pass filter is applied directly to the raw data series without the transformations.

Table 1. Stylized Facts

	High Freq. (2-3 Quarter)		B-C Freq. (8-40 Quarter)	
Countries	σ_y/σ_z	$cor(z,i)$	σ_y/σ_z	$cor(z,i)$
Australia	0.78	-0.62	1.29	0.04
Austria	0.49	-0.85	1.37	-0.02
Canada	0.80	-0.69	1.29	0.52
Denmark	0.79	-0.69	1.27	0.24
France	0.80	-0.68	1.57	0.14
Finland	1.22	-0.27	1.25	0.21
Great Britain	0.82	-0.57	1.33	0.35
Japan	0.81	-0.60	1.14	0.44
Netherlands	0.64	-0.79	1.37	0.37
Switzerland	0.65	-0.75	1.41	-0.06
United States	1.17	-0.24	1.20	0.60
OECD	0.91	-0.43	1.39	0.58
Europe (15)	0.81	-0.53	1.56	0.52
Europe	0.83	-0.51	1.55	0.47

*Data source: The U.S. data are taken from the Citibase. The rest is taken from OECD data bank. All data are seasonally adjusted. The statistics reported do not change dramatically if the high-frequency interval is extended to 2-4 quarters per cycle and the business-cycle frequency interval is extended to 6-100 quarters per cycle.

It has been a consensus in the existing literature that inventory investment is procyclical and production is more volatile than sales (e.g., see Blinder 1986 and 1991, and Ramey and West 1997). Consequently, theories have been designed and judged based almost exclusively on these two characteristics of inventory behavior. But table 1 reveals that these two prominent characteristics of inventory behavior hold true only at the lower cyclical frequencies (such as the business cycle frequencies). The opposite, however, is true at the very high frequencies (i.e., frequencies higher than 4 quarters per cycle).[5] These new empirical facts should prove useful in trying to understand inventory behavior and testing different hypotheses.

Clearly, the production and inventory behaviors identified at the high frequencies are fully consistent with the conventional production smoothing theory, while those identified at the business-cycle frequencies seem fully consistent with the stockout-avoidance theory. The intriguing question

[5]Some of these features have also been noticed by Hornstein (1998) in U.S. industrial data. It needs to be emphasized that these high-frequency movements are not seasonal movements, as the data used in producing table 1 are all seasonally adjusted.

is, does there exist a simple model that can simultaneously explain these inventory cycles across different frequencies? This issue is addressed in the rest of the paper.

3 Theories

Assume that the firm's demand, denoted θ_t, is a stationary $AR(1)$ process:

$$\theta_t = \gamma + \rho\theta_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is an *i.i.d.* random variable with normal distribution $N(0, \sigma^2)$. The maximum amount the firm can sell in period t is its inventory as of the end of the previous period, denoted by s_{t-1}, plus whatever it produces during period t, y_t. Assuming that the price p is sufficiently high, we have

$$z_t = \min \{\theta_t, s_{t-1} + y_t\}, \tag{2}$$

where z_t denotes actual sales in period t. Also assuming that production takes one period, hence the decision for y_t needs to be made one period in advance based on information available in period $t-1$. The cost of production is $a_t c(y_t)$, where $c' > 0, c'' \leq 0$ and a is a stationary $AR(1)$ process representing cost shocks:

$$a_t = \phi + \rho_a a_{t-1} + v_t.$$

Profits in period t are simply revenue minus costs $pz_t - a_t c(y_t)$. The problem for the firm is to choose production and inventory holding each period to solve

$$\max_{\{y_s\}} E_{t-1} \left\{ \max_{\{s_s\}} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[pz_s - a_s c(y_s) \right] \right\} \right\}$$

subject to the resource constraint

$$z_t + s_t = s_{t-1} + y_t \tag{3}$$

and the nonnegativity constraint (2). Notice that (2) can also be expressed as

$$s_t \geq 0. \tag{4}$$

Denoting λ and π as the Lagrangian multipliers associated with constraints (3) and (4) respectively, the first order conditions with respect to output and inventory holdings are:

$$c'(y_t) E_{t-1} a_t = E_{t-1} \lambda_t \tag{5}$$

$$\lambda_t = \beta E_t \lambda_{t+1} + \pi_t \tag{6}$$

The interpretations of the first-order conditions are straightforward. In equation (5) the expected marginal cost of production equals the expected shadow goods price (λ). In equation (6) the cost of increasing inventories by one unit is λ_t due to the lost opportunity for sale in period t, and the benefit for having one extra unit of inventories available for sale in the next period is the discounted next period price. However, the marginal cost for keeping one extra unit of inventories in period t must be adjusted by a complementarity slackness multiplier $-\pi_t \leq 0$. Namely, π is positive when stockout occurs and zero when stockout does not occur. In equilibrium, the marginal costs equal the marginal benefits.

Note that in a certainty-equivalent steady state, optimal inventory holding (s^*) equals zero since

$$\pi^* = \lambda^* (1 - \beta) > 0.$$

This implies that inventories are valued by the firm only in the "short run" (i.e., off the steady state) as long as uncertainty exists to prevent planned supply from being equal to actual demand.[6] Inventories are valued off the steady state because they can be used to avoid stockouts or to smooth production against uncertainty. With perfect foresight (such as in the steady state), however, optimal inventory stock is zero since planned production can always be made to match actual demand in the absence of uncertainty.

A. The Production Smoothing Theory

To analyze the production-smoothing theory, consider the case where production is instantaneous and costs shocks are absent ($a_t = 1$). (5) can then be written as

$$\lambda_t = c'(y_t),$$

and (6) can be written as

$$c'(y_t) = \beta E_t c'(y_{t+1}) + \pi_t, \tag{7}$$

which shows the incentive for cost (production) smoothing when the firm does not stockout (i.e., $\pi_t = 0$). Hence there are two cases to consider: case A $\pi_t > 0$ and case B $\pi_t = 0$.

Case A: $\pi_t > 0$ and $s_t = 0$. This is the case where the nonnegativity constraint (4) binds due to a high demand shock. The resource constraint (3) then implies[7]

$$y_t = z_t - s_{t-1},$$

[6] This does not mean that the average level of inventories is zero in the model. Due to the non-negativity constraint on inventories, the posterior mean of inventory stock is strictly positive under uncertainty: $ES_t > 0$.

[7] Without loss of generality, we can assume that the distribution of demand shocks has a finite support, $[a, b]$, such that $pb > c(b)$.

suggesting that production keeps track of sales when demand is above "normal". In such a case the variance of production is exactly equal to the variance of sales since it responds one-for-one to changes in sales.

Case B: $\pi_t = 0$ and $s_t \geq 0$. This is the case where the nonnegativity constraint (4) does not bind due to a low demand shock. The intertemporal Euler equation (7) suggests that production be smoothed. In such a case, as is shown by Samuelson (1971) and Deaton and Laroque (1996) in a similar environment, the solution for (7) is

$$c'(y_t) = \eta,$$

or

$$y_t = y^*(\eta)$$

where η is a constant.[8] The resource constraint then implies

$$s_t = y^* + s_{t-1} - \theta_t.$$

The condition $s_t \geq 0$ gives the threshold level of demand shock as

$$\theta^* = y^* + s_{t-1},$$

such that production is constant if $\theta_t \leq \theta^*$ and production is increasing in sales if $\theta_t > \theta^*$. The optimal solution of the model can thus be summarized by the following decision rules:

$$s_t = \begin{cases} 0 & \text{if } \theta_t > \theta^* \\ y^* + s_{t-1} - \theta_t & \text{if } \theta_t \leq \theta^* \end{cases}$$

$$y_t = \begin{cases} \theta_t - s_{t-1} & \text{if } \theta_t > \theta^* \\ y^* & \text{if } \theta_t \leq \theta^* \end{cases} .$$

Note that sales are given by

$$\begin{aligned} z_t &= \begin{cases} y_t + s_{t-1} & \text{if } \theta_t > \theta^* \\ \theta_t & \text{if } \theta_t \leq \theta^* \end{cases} \\ &= \theta_t. \end{aligned}$$

Proposition 1 *Under the production-smoothing motive, the variance of production is less than the variance of sales at all cyclical frequencies $\omega \in [0, \pi]$.*

[8] This implies $y^* = c'^{-1}(\eta)$ and $c'(y_{t+1}) = \frac{\eta}{\beta(1-\delta)} + \mu_{t+1}$ where $E_t\mu_{t+1} = 0$.

Proof. Denote $P = \Pr[\theta_t > \theta^*]$. Hence the spectrum of output is given by $f_y(e^{-i\omega}) = P f_\theta(e^{-i\omega})$ (note that $s = 0$ whenever $\theta > \theta^*$) and the spectrum of sales is given by $f_z(e^{-i\omega}) = f_\theta(e^{-i\omega})$. Clearly, $f_y(e^{-i\omega}) < f_z(e^{-i\omega})$ for all $\omega \in [0, \pi]$ as long as $P < 1$.■

Proposition 1 reflects the fact that inventories help smooth the time path of production. However, due to the nonnegativity constraint on inventories, production cannot be smoothed completely. If inventory could be negative, then optimal production would always be constant regardless sales. On the other hand, if inventory were not allowed, then production would be fully determined by sales, hence would be exactly as volatile as sales. The nonnegativity constraint on inventory implies that production is smoothed only in the direction where inventory is positive (i.e., only when demand is below "normal").

Proposition 2 *Under the production smoothing motive, the covariance between inventory investment and sales is negative at all cyclical frequencies $\omega \in [0, \pi]$.*

Proof. Denote $P = \Pr[\theta_t > \theta^*]$ and denote inventory investment as $i_t = s_t - s_{t-1}$. The relationship between inventory investment and sales is given by (note that $s = 0$ whenever $\theta > \theta^*$, hence $i = 0$ whenever $\theta > \theta^*$):

$$i = \begin{cases} 0 & \text{if } \theta > \theta^* \\ y^* - z & \text{if } \theta \leq \theta^* \end{cases}.$$

The cross spectrum between i and z is then given by

$$f_{iz}(e^{-i\omega}) = -(1 - P) f_z(e^{-i\omega}),$$

which is negative for all ω.■

Inventory investment is negatively correlated with sales in the production smoothing model because inventory serves purely as a buffer stock for demand shocks in the model. Hence changes in inventories reflects the opposite changes in sales (at least up to the point of stocking out).

B. The Cost Shock Theory

Clearly, if demand shocks are the major source of uncertainty, the production-smoothing model cannot explain data. However, as noticed by Blinder (1986) and Eichenbaum (1989) as well as many others, the fact that production is more variable than sales in the data may indicate that supply shocks are important. It is therefore worthwhile to reconsider the production-smoothing model under cost shocks. It can be shown, however, that if inventory cannot be negative and if demand shocks and cost shocks are uncorrelated, then allowing for cost shocks in the model does not help resolving the problem. The intuition is simple. If firms cannot hold negative inventories, then cost shocks have no effects on inventories when production cost is high or when demand is high (since $s = 0$ in these cases); and they have no effects on sales either unless demand is high. Hence allowing for cost shocks does not change the correlation between sales and inventory investment. Denote innovations in demand as ε and innovations in cost as v. Consider the reduced-form decision rules for inventory investment (i) and sales (z):

$$i_t = \begin{cases} 0 & \text{if high demand or high cost} \\ \sum_{j=0}^{\infty} a_{1j}\varepsilon_{t-j} + \sum_{k=0}^{\infty} a_{2k}v_{t-k} & \text{if low demand or low cost} \end{cases}$$

$$z_t = \begin{cases} \sum_{j=0}^{\infty} b_{1j}\varepsilon_{t-j} + \sum_{k=0}^{\infty} b_{2k}v_{t-k} & \text{if high demand or high cost} \\ \sum_{j=0}^{\infty} c_j\varepsilon_{t-j} & \text{if low demand or low cost} \end{cases}$$

where $\{a_{1j}, a_{2k}, b_{1j}, b_{2k}, c_j\}$ are coefficients determined by the structural parameters of the model. In the event of stockouts, either due to high demand shocks or high cost shocks, the firm does not accumulate inventories and hence the correlation between inventory investment and sales is zero. In the event of no stockout, on the other hand (either due to low demand shocks or low cost shocks), production can always meet demand and inventory investment will reflect both demand and cost innovations. However, since sales are not constraint by production in this case, they are determined solely by demand, implying that sales contain no information in costs but only information in demand. Hence in either case the correlation between inventory investment and sales is independent of cost shocks.[9]

To see this more precisely, consider the marginal cost function,

$$a_t c'(y_t) = a_t + y_t,$$

where the cost shock term (a_t) shifts the marginal cost curve. Equation (7) then becomes,

$$a_t + y_t = \beta E_t (a_{t+1} + y_{t+1}) + \pi_t. \tag{8}$$

There are two cases to consider:

Case A: Production cost is high. In this case, since output is low the firm opts not to hold inventories (the nonnegativity constraint binds and hence $\pi_t > 0$), and its production is either able to meet demand, $y_t = \theta_t - s_{t-1}$, or not able to meet demand because it reaches the point where the marginal cost equals the marginal revenue, $y_t = p - a_t$, whichever is smaller. Accordingly, sales are determined either by θ_t or by $p - a_t + s_{t-1}$, whichever is smaller.

[9]If, however, inventory can be negative, then it will contain information about cost shocks when production cost is high. In such a case inventories become negative due to a low production level. Since high costs can always translate into low sales due to low output, the correlation between inventory investment and sales will thus be affected by cost shocks.

Case B: Production cost is low. In this case, the firm opts to accumulate inventories, and production can always meet demand. This is the case where the nonnegativity constraint on inventory is not binding ($\pi_t = 0$), hence by equation (8) we have the case where the marginal cost equals a constant,

$$a_t + y_t = \eta,$$

where the constant term η is less than p. The inventory level is thus determined by

$$\begin{aligned} s_t &= y_t + s_{t-1} - \theta_t \\ &= \eta - a_t + s_{t-1} - \theta_t. \end{aligned}$$

The requirement, $s_t \geq 0$, gives the threshold value of the shocks,

$$a_t + \theta_t \leq \eta + s_{t-1},$$

such that we are in case B.

Hence, the decision rules of the model under both demand and cost shocks are given by:[10]

$$y_t = \begin{cases} p - a_t & \text{if } a_t + \theta_t > p + s_{t-1} \\ \theta_t - s_{t-1}, & \text{if } p + s_{t-1} \geq a_t + \theta_t > \eta + s_{t-1} \\ \eta - a_t, & \text{if } a_t + \theta_t \leq \eta + s_{t-1} \end{cases}$$

$$s_t = \begin{cases} 0 & \text{if } a_t + \theta_t > p + s_{t-1} \\ 0 & \text{if } p + s_{t-1} \geq a_t + \theta_t > \eta + s_{t-1} \\ \eta + s_{t-1} - a_t - \theta_t, & \text{if } a_t + \theta_t \leq \eta + s_{t-1} \end{cases}.$$

Note that sales are given by

$$z_t = \begin{cases} p - a_t + s_{t-1} & \text{if } a_t + \theta_t > p + s_{t-1} \\ \theta_t, & \text{if } p + s_{t-1} \geq a_t + \theta_t > \eta + s_{t-1} \\ \theta_t, & \text{if } a_t + \theta_t \leq \eta + s_{t-1} \end{cases}.$$

Also note that cost shocks enter the dynamics of sales only when production cost is high or when demand is high ($a_t + \theta_t > p + s_{t-1}$). Since in these cases optimal inventory investment is zero, cost shocks have no effects on the correlation between inventory investment and sales. Thus we have the following propositions.

Proposition 3 *The variance of production exceeds the variance of sales if and only if the variance of cost shocks exceeds the variance of demand shocks.*

[10] Assuming that the distribution of cost shocks has a finite support $[a, b]$ with $b < p$, so that optimal output is always positive.

Proof. The decision rules suggest that there are three cases to consider. Case (a): $s_t = 0, y_t = p - a_t$ and $z_t = p - a_t$. In this case production is exactly as variable as sales. Case (b): $s_t = 0, y_t = \theta_t$ and $z_t = \theta_t$. In this case, production is also exactly as variable as sales. Case (c): $s_t \geq 0, y_t = \eta - a_t$ and $z_t = \theta_t$. In this case production is more variable than sales if and only if $\sigma_a^2 > \sigma_\theta^2$. It can be shown trivially that this is true for all cyclical frequencies $\omega \in [0, \pi]$. ■

Proposition 4 *Inventory investment is negatively correlated with sales at all frequencies despite cost shocks.*

Proof. Consider the 3 cases discussed in the proof of proposition (3) above. In the cases of (a) and (b), inventory investment is zero. Hence it has no correlation with sales. In case (c), inventory investment is given by $i_t = \eta - a_t - \theta_t$ and sales are given by θ_t. Clearly, the correlation between i and z in case (c) is negative at all frequencies and is not affected by cost shocks (a_t) under the assumption that a_t is uncorrelated with θ_t.■

C. The Stockout Avoidance Theory

Now consider the model with a production lag described by equations (3) - (6) above. To highlight the stockout-avoidance motive for holding inventories under demand shocks, assume $a_t = 1$ and $c'(y_t) = c$ as in Kahn (1987) so as to rule out any production-smoothing motive. Under these assumptions, (5) and (6) then become

$$E_{t-1}\lambda_t = c \tag{9}$$

$$\lambda_t = \beta c + \pi_t. \tag{10}$$

To solve for the production decision rule based on information available in period $t-1$, we can apply the expectation operator E_{t-1} on equation (10). Equations (9) and (10) then imply

$$E_{t-1}\pi_t = (1-\beta)\, c > 0. \tag{11}$$

Equation (11) indicates that based on information available in period $t-1$, the optimal expected inventory holding for period t must also be a nonnegative constant,

$$E_{t-1}s_t = k \geq 0,$$

where k is an optimal cut-off point that determines whether the firm stocks out or not. Thus the decision rule for production is given by

$$y_t = k + E_{t-1}\theta_t - s_{t-1}. \tag{12}$$

To determine the actual inventory holding in period t, consider the following two possible cases:

Case A. $\pi_t > 0$. In this case the realized demand is high and the firm stocks out, hence we have $z_t = y_t + s_{t-1}$ and $s_t = 0$.

Case B. $\pi_t = 0$. In this case the realized demand is low and the firm incurs inventories, hence we have $z_t = \theta_t$ and

$$\begin{aligned} s_t &= y_t + s_{t-1} - \theta_t \\ &= k - \varepsilon_t, \end{aligned}$$

where the second equality is based on the decision rule for output and the law of motion for demand shocks. The requirement $s_t \geq 0$ then implies $\varepsilon_t \leq k$.

The equilibrium decision rules of the model can thus be summarized as

$$s_t = \max\{0, k - \varepsilon_t\}$$

$$y_t = \gamma + \rho\theta_{t-1} + \min\{k, \varepsilon_{t-1}\}$$

$$z_t = \gamma + \rho\theta_{t-1} + \min\{k, \varepsilon_t\}$$

To determine the value of k, note that (9) implies

$$c = \int_{\varepsilon_t \leq k} \lambda_t \phi(\theta_{t-1}, d\theta_t) + \int_{\varepsilon_t > k} \lambda_t \phi(\theta_{t-1}, d\theta_t).$$

According to (10), $\lambda_t = \beta c$ if $\varepsilon_t \leq k$ (i.e., the value of goods equals discounted marginal cost if stockouts do not occur, hence the complementarity slackness constraint does not bind and $\pi_t = 0$). Furthermore, $\lambda_t = p$ if $\varepsilon_t > k$ (i.e., the value of goods from sale equals p in case demand is high). Hence the above equation can be written as

$$\begin{aligned} c &= \int_{\varepsilon_t \leq k} \beta c \phi(\theta_{t-1}, d\theta_t) + \int_{\varepsilon_t > k} p \phi(\theta_{t-1}, d\theta_t) \\ &= \beta c \Phi\left(\frac{k}{\sigma}\right) + p\left[1 - \Phi\left(\frac{k}{\sigma}\right)\right], \end{aligned}$$

where $\Phi()$ is the standard normal cumulative distribution function. To increase output production by one unit, the marginal cost is c (the left hand side), and the expected marginal revenue is p in case there is a stockout ($\varepsilon_t > k$ with probability $1 - \Phi$) plus the discounted cost saved for the next period's production in case the firm does not stock out ($\varepsilon_t \leq k$ with probability Φ). The optimal value of k is chosen such that the marginal cost equals the expected marginal revenue. Note that production is profitable only if $p > c$. The above equation can be solved as

$$\Phi\left(\frac{k}{\sigma}\right) = \frac{p - c}{p - \beta c},$$

which implies $k = \sigma\Phi^{-1}((p-c)/(p-\beta c))$. Hence, k increases with both uncertainty (σ) and the markup $(p-c)$. Since k is the maximum value of inventories where stockouts occur, it must be nonnegative; hence we have $k = 0$ if $c \leq p \leq 2c - \beta c$ and $k > 0$ if $p > 2c - \beta c$.

Proposition 5 *Under the stockout-avoidance motive, the variance of production exceeds the variance of sales if demand shocks are serially correlated. This is true at all cyclical frequencies except at frequency zero.*

Proof. Denote $P = \Pr[\varepsilon_t > k]$, the spectral density functions of sales and output are given respectively by:

$$\begin{aligned} f_z(e^{-i\omega}) &= P\left|\frac{\rho e^{-i\omega}}{1-\rho e^{-i\omega}}\right|^2 \sigma_\varepsilon^2 + (1-P)\left|\frac{1}{1-\rho e^{-i\omega}}\right|^2 \sigma_\varepsilon^2 \\ &= P\left\{\frac{\rho^2}{1+\rho^2-2\rho\cos\omega}\right\}\sigma_\varepsilon^2 + (1-P)\left\{\frac{1}{1+\rho^2-2\rho\cos\omega}\right\}\sigma_\varepsilon^2 \end{aligned}$$

$$\begin{aligned} f_y(e^{-i\omega}) &= P\left|\frac{\rho e^{-i\omega}}{1-\rho e^{-i\omega}}\right|^2 \sigma_\varepsilon^2 + (1-P)\left|\frac{1+\rho\left(1-e^{-i\omega}\right)}{1-\rho e^{-i\omega}}\right|^2 \sigma_\varepsilon^2 \\ &= P\left\{\frac{\rho^2}{1+\rho^2-2\rho\cos\omega}\right\}\sigma_\varepsilon^2 + (1-P)\left\{\frac{(1+\rho)^2+\rho^2-2\rho(1+\rho)\cos\omega}{1+\rho^2-2\rho\cos\omega}\right\}\sigma_\varepsilon^2. \end{aligned}$$

Clearly, term by term comparison indicates that $f_y\left(e^{-i\omega}\right) \geq f_z\left(e^{-i\omega}\right)$ if and only if the second term in f_y exceeds the second term in f_z. This is possible if only if (since $1+\rho^2-2\rho\cos\omega > 0$)

$$\rho^2 + (1+\rho)^2 - 2\rho(1+\rho)\cos\omega \geq 1.$$

Since $\cos\omega < 1$ for $\omega \neq 0$ and $\cos\omega = 1$ for $\omega = 0$, the left-hand side of this inequality is greater than or equal to $\rho^2 + (1+\rho)^2 - 2\rho(1+\rho) = 1$. Hence, we have $f_y(e^{-i\omega}) > f_z(e^{-i\omega})$ for $\omega \in (0, \pi]$ (provided $\rho > 0$) and $f_y(e^{-i\omega}) = f_c(e^{-i\omega})$ for $\omega = 0$.■

The intuition for proposition 5 is simple. Due to the lag in production, innovations in demand affect output in two ways: It increases production by reducing current inventories; and it raises expected demand in the next period. The first effect alone would cause a one-for-one change in output (at least up to the point of stocking out, see the decision rules above). Hence, due to serially correlated demand shocks, production responds to innovations in demand more than one-for-one. Since demand shocks are transitory, they have no permanent (long run) effects on expected future demand. Hence at frequency zero production is exactly as variable as sales.

Proposition 6 *Under the stockout-avoidance motive, the correlation between inventory investment and sales is negative at high frequencies (2-4 quarters per cycle) but positive at business-cycle frequencies (e.g., 8-40 quarters per cycle).*

Proof. To prove this proposition, it suffices to show that inventory investment is procyclical for $\omega < \frac{\pi}{2}$ and countercyclical for $\omega \geq \frac{\pi}{2}$. (Note: $\frac{\pi}{2}$ corresponds to 4 quarters per cycle). Denote L as the lag operator and $i = s_t - s_{t-1}$ as inventory investment. Consider two possible cases:

Case A: $\varepsilon_t > k$. The decision rules imply

$$\begin{aligned} z_t &= \rho\theta_{t-1} \\ i_t &= s_t - s_{t-1} = 0. \end{aligned}$$

Hence the cross spectrum of inventory investment (i_t) and consumption is zero at every frequency: $f_{iz}(e^{-i\omega}) = 0$ for all $\omega \in [0, 1]$.

Case B: $\varepsilon_t \leq k$. The decision rules imply

$$\begin{aligned} z_t &= \theta_t \\ i_t &= s_t - s_{t-1} = \varepsilon_{t-1} - \varepsilon_t. \end{aligned}$$

Note that inventory investment is related to sales by:

$$\begin{aligned} i_t &= (L-1)\varepsilon_t = (L-1)(1-\rho L)\theta_t \\ &= -(1-L)(1-\rho L)z_t. \end{aligned}$$

The cross spectrum of inventory investment and consumption is thus given by

$$\begin{aligned} f_{iz}(e^{-i\omega}) &= -\left(1 - e^{-i\omega}\right)\left(1 - \rho e^{-i\omega}\right) f_c(e^{-i\omega}) \\ &= -\left(1 + \rho e^{-2i\omega} - e^{-i\omega} - \rho e^{-i\omega}\right) \\ &= [(1+\rho)\cos\omega - \rho\cos 2\omega - 1] - [(1+\rho)i\sin\omega - \rho i \sin 2\omega] \\ &= co(\omega) + iqu(\omega), \end{aligned}$$

where the real part,

$$co(\omega) = (1+\rho)\cos\omega - \rho\cos 2\omega - 1,$$

is proportional to the covariance between i and z at frequency ω.

Given that case B happens with positive probability, the covariance between inventory investment and sales is determined entirely by $co(\omega)$. But $co(\omega) > 0$ if and only if

$$(1+\rho)\cos\omega - \rho\cos 2\omega > 1. \tag{A'}$$

Note first that If $\rho = 0$, then condition (A') implies $\cos\omega > 1$, which is impossible. Hence without serial correlation in demand shocks, the correlation between inventory investment and sales can never be positive for $\omega > 0$. To prove that inventory investment is negatively correlated with sales at the high frequencies regardless the value of ρ, note that condition (A') can never be satisfied within the high frequency interval $\pi \geq \omega \geq \frac{\pi}{2}$, which corresponds to cyclical frequencies between 2 quarters per cycle to 4 quarters per cycle. To see this, note that for $\omega \in \left[\frac{\pi}{2}, \pi\right]$, we have $\cos\omega \leq 0$ and $|\cos 2\omega| \leq 1$. Hence the maximum of the left-hand side of (A') is ρ, which cannot be greater than one by definition.

Now we prove that for $0 < \omega < \frac{\pi}{2}$, there always exists a sufficiently large value of $\rho \in (0, 1)$ such that (A') is satisfied. Note that (A') implies

$$\rho\left[\cos\omega - \cos 2\omega\right] > 1 - \cos\omega.$$

Since $\cos 2\omega < \cos\omega$ for $\omega < \frac{\pi}{2}$, the above inequality implies

$$\rho > \frac{1 - \cos\omega}{\cos\omega - \cos 2\omega}. \qquad (B')$$

In order for condition (B') to be satisfied with $\rho \in (0, 1)$, the right-hand side of (B') must be less than one, which is true if and only if $1 - \cos\omega < \cos\omega - \cos 2\omega$. This in turn implies $\cos\omega\,(1 - \cos\omega) > 0$, which further implies $0 < \cos\omega < 1$, which is clearly true as long as $0 < \omega < \frac{\pi}{2}$.■

Proposition 6 is very powerful. It provides a rigorous mathematical proof for why the stockout-avoidance mechanism is crucial for understanding the phenomenon that inventory investment in the U.S. and other OECD countries appears to be countercyclical at the high frequencies but procyclical at the business cycle frequencies. The intuition for proposition 6 is as follows. At the high frequencies, due to the sluggish adjustment in production in the short run, inventories serve primarily as a buffer stock to absorb demand shocks, hence resulting in countercyclical changes in inventories. In the longer run (e.g., at the business cycle frequencies), however, production is capable of responding to expected demand shocks if the shocks are serially correlated, the stockout-avoidance motive thus dominates, giving rise to procyclical changes in inventories.

This simple stockout-avoidance model of Kahn (1987), however, is not without failures. It predicts that production is more volatile than sales at both high and low frequencies, whereas in the data it is less volatile than sales at the high frequencies for most countries except for the U.S. and Finland (see table 1). Despite this shortcoming, the stockout-avoidance theory clearly out-performs the other theories in explaining the prominent features of production and inventory behaviors documented in table 1. In what follows, I will therefore scrutinize further on the stockout-avoidance theory by focusing on one more issue: Is this theory consistent with another stylized fact of inventories, namely, the fact that inventory investment appears to be excessively volatile relative to GDP at the high frequencies but not at the business-cycle frequencies?

4 Why is Inventory Investment so Volatile?

Another well know stylized fact puzzling economists is the extremely volatile changes in inventory investment relative to changes in GDP observed in the data. The literature reports that the drop in inventory investment accounted for 87 percent of the drop in total output during the average postwar recessions in the U.S. (Blinder, 1991). Based on such a surprisingly disproportionately large contribution to GDP fluctuations from inventory investment, given a much smaller average share of inventory investment in GDP (about 0.5%), Blinder concludes that business cycles are essentially inventory fluctuations. A careful re-examination of the data shows, however, that unlike GDP or capital investment, the bulk of the volatility in inventory investment is concentrated around the very high cyclical frequencies (2-3 quarters per cycle) rather than around the lower frequency interval such as the business-cycle frequencies (8-40 quarters per cycle).

To document this feature of inventory investment in our data set, define a relative volatility statistic (r) as the ratio between movement (relative to GDP) in inventory investment and movement (relative to trend) in GDP as

$$r = \frac{\frac{1}{T}\sum_{t=1}^{T}\left|\frac{\Delta S_t}{Y^*}\right|}{\frac{1}{T}\sum_{t=1}^{T}\left|\log Y_t - \gamma t\right|},$$

where ΔS denotes inventory investment or changes in the inventory stock, Y denotes GDP, $\frac{\Delta S_t}{Y^*}$ denotes inventory investment normalized by the median of GDP (Y^*), $(\log Y_t - \gamma t)$ denotes percentage deviations of GDP relative to a deterministic long-run trend, and $|\cdot|$ denotes the absolute-value operator. Thus, $\left|\frac{\Delta S_t}{Y^*}\right|$ measures the volatility of inventory investment relative to GDP and $|\log Y_t - \gamma t|$ measures the volatility of GDP relative to trend. The statistic r therefore measures the changes in inventory investment as a fraction of GDP with respect to changes in GDP around trend. For example, $r = 1$ implies roughly that for one percent change in inventory investment as a fraction of GDP there is associated one percent change in GDP relative to trend. Hence the larger the r, the more contributions inventory investment has to GDP fluctuations. To compare such contributions across different cyclical frequencies, the band-pass filter is applied to the time series $\{\frac{\Delta S_t}{Y^*}\}_{t=1}^{T}$ and $\{\log Y_t - \gamma t\}_{t=1}^{T}$ so that the r statistics can be estimated for data in different frequency bands.

Table 2. Volatility Ratio of Inventory Investment-to-GDP across Frequencies

	r_H ($\omega \in [2/3\pi, \pi]$)	r_L ($\omega \in [0.05\pi, 0.25\pi]$)	$\varkappa \left(\frac{r_H}{r_L}\right)$
Australia	0.85	0.64	1.3
Austria	1.51	0.67	2.3
Canada	1.12	0.33	3.4
Denmark	1.37	0.45	3.0
France	1.24	0.69	1.8
Finland	0.77	0.47	1.6
Great Britain	0.89	0.41	2.2
Japan	0.55	0.21	2.6
Netherlands	1.01	0.49	2.1
Switzerland	1.34	0.82	1.6
United States	0.70	0.25	2.8
Sample Mean	1.03	0.49	2.2

Table 2 reports the r statistics for the same OECD economics considered in table 1. It shows that at the high frequency interval $\omega \in [2/3\pi, \pi]$, which corresponds to movements between 2-quarter cycle to 3-quarter cycle, the r statistic is quite large, ranging from 0.55 to 1.51 (the sample mean of r for the 11 countries listed is 1.03), indicating very large contributions to GDP fluctuations from inventory investment. In contrast, at the business-cycle frequency interval $\omega \in [0.05\pi, 0.25\pi]$, which corresponds to movements from 8-quarter cycle to 40-quarter cycle, the estimated contributions of inventory investment to GDP fluctuations are much smaller, with the r statistic ranging from 0.21 to 0.82 (the sample mean is 0.49). Notice that although the r statistics change quite a lot across individual countries, for every country considered in the sample, r_H (the statistic at the high frequencies) is always larger than r_L (the statistic at the lower business-cycle frequencies).[11]

To better appreciate such a large difference in contribution across frequencies, define the ratio, $\varkappa = \frac{r_H}{r_L}$, as an index for the relative contribution change across the high and the low frequencies. The last column in table 2 shows that on average inventory investment contributes more than twice as much to GDP fluctuations at the high frequencies as it does at the business-cycle frequencies (the sample average of $\varkappa$ is 2.2), indicating that the bulk of variance in inventory investment is concentrated at the high frequencies. The smallest $\varkappa$ index comes from Australia (1.3), the largest

[11]This is also true for both aggregated OECD data and aggregated European country data. For example, for OECD countries as a whole, $r_H = 0.67$ and $r_L = 0.45$. For the European countries as a whole, $r_H = 0.74$ and $r_L = 0.36$.

$\varkappa$ index is from Canada (3.4). The $\varkappa$ index for the United States is 2.8.[12]

These stylized facts are qualitatively consistent with predictions of the stockout-avoidance model. Since production cannot respond to demand shocks in the short run, inventories act primarily as a buffer stock to absorb demand shocks, thus inventory investment appears more volatile relative to production at high frequencies. Since production can respond to demand shocks in the longer run, it reduces the impact of demand shocks on inventories, rendering inventory investment less responsive to shocks relative to production at the business cycle frequencies. In the very long run where production can be viewed as instantaneous (i.e., at frequency zero), inventories will appear to be extremely smooth relative to production. This pattern of inventory volatility at different cyclical frequencies should not be affected by the average ratio of inventories to output. Thus we have

Proposition 7 *Inventory investment is more volatile than output at the high frequencies but less volatile than output at the low frequencies in the stockout-avoidance model, regardless the average share of inventory investment in total output.*

Proof. Given the equilibrium decision rules for inventories and output:

$$s_t = \begin{cases} 0 & \text{if } \varepsilon_t > k \\ k - \varepsilon_t & \text{if } \varepsilon_t \leq k \end{cases}$$

$$y_t = \begin{cases} \gamma + \rho\theta_{t-1} + k & \text{if } \varepsilon_t > k \\ \gamma + \rho\theta_{t-1} + \varepsilon_{t-1} & \text{if } \varepsilon_t \leq k \end{cases}$$

and the decision rule for inventory investment:

$$i_t = s_t - s_{t-1} = \begin{cases} 0 & \text{if } \varepsilon_t > k \\ \varepsilon_{t-1} - \varepsilon_t & \text{if } \varepsilon_t \leq k \end{cases},$$

the power spectra of inventory investment and output are then given respectively by

$$\begin{aligned} f_i(e^{-i\omega}) &= P \times 0 + (1-P)(1-e^{-i\omega})(1-e^{i\omega})\sigma_\varepsilon^2 \\ &= 2(1-P)\,(1-\cos\omega)\,\sigma_\varepsilon^2 \end{aligned}$$

$$\begin{aligned} f_y(e^{-i\omega}) &= P\frac{\rho^2}{(1-\rho e^{-i\omega})(1-\rho e^{i\omega})}\sigma_\varepsilon^2 + (1-P)\left[\frac{(1+\rho-\rho e^{-i\omega})(1+\rho-\rho e^{i\omega})}{(1-\rho e^{-i\omega})(1-\rho e^{i\omega})}\right]\sigma_\varepsilon^2 \\ &= \left[P\frac{\rho^2}{1+\rho^2-2\rho\cos\omega} + (1-P)\frac{1+2\rho^2+2\rho-2(1+\rho)\rho\cos\omega}{1+\rho^2-2\rho\cos\omega}\right]\sigma_\varepsilon^2 \end{aligned}$$

[12] The $\varkappa$ index is 2.1 for all OECD countries and is 1.5 for all European countries.

where $P \equiv \Pr[\varepsilon_t > k] \leq 0.5$ (since $k \geq 0$). It can be shown easily that $f_i(e^{-i\omega})$ is monotonically increasing in $\omega \in [0, \pi]$ and $f_y(e^{-i\omega})$ is monotonically decreasing in ω, hence the ratio, $\frac{f_i(\omega)}{f_y(\omega)}$, is monotonically increasing in $\omega \in [0, \pi]$. Hence it suffices to consider the spectral densities of $\{i, y\}$ at two extreme points, $\omega = 0$ and $\omega = \pi$:

$$f_i(e^{-i\omega})/\sigma_\varepsilon^2 = \begin{cases} 4(1-P) & \text{if } \omega = \pi \\ 0 & \text{if } \omega = 0 \end{cases}$$

$$f_y(e^{-i\omega})/\sigma_\varepsilon^2 = \begin{cases} P\left(\frac{\rho}{1+\rho}\right)^2 + (1-P)\left(\frac{2+\rho}{1+\rho}\right)^2 & \text{if } \omega = \pi \\ P\left(\frac{\rho}{1-\rho}\right)^2 + (1-P)\left(\frac{1}{1-\rho}\right)^2 & \text{if } \omega = 0 \end{cases}$$

Clearly, $\left.\frac{f_i(\omega)}{f_y(\omega)}\right|_{\omega=0} = 0$. On the other hand, since $P \leq 0.5$, the magnitude of the power spectrum of y at the high frequency is decreasing in ρ. Hence the minimum value attainable for the variance ratio at $\omega = \pi$ is given by $\left.\frac{f_i(\omega)}{f_y(\omega)}\right|_{\omega=\pi,\rho=0} = \frac{4(1-P)}{2(1-P)} = 2$. Hence the variance ratio, $\frac{f_i(\omega)}{f_y(\omega)}$, is less than one at low frequencies and larger than one at high frequencies. Therefore, there exists a cut-off point $\omega^* \in (0, \pi)$ such that $\left.\frac{f_i(\omega)}{f_y(\omega)}\right| < 1$ for $\omega \leq \omega^*$ and $\left.\frac{f_i(\omega)}{f_y(\omega)}\right| > 1$ for $\omega > \omega^*$ for any value of $\rho \in [0, 1]$. It is clear intuitively that $\frac{d\omega^*}{d\rho} > 0$; namely, as the shocks become more persistent, the variance ratio decreases at each frequency point, hence the critical value ω^* shifts towards high frequencies. It is also clear that the results do not depend on the values of the structural parameters k and γ, indicating that these results can be observed in the real world even if inventory investment accounts only for a tiny fraction of GDP on average.■

5 Conclusion

This paper documents the stark differences in production and inventory behaviors at high and low cyclical frequencies. The distinction between high- and low-frequency movements proves very useful in understanding inventory fluctuations and testing different hypotheses. The analysis shows that models featuring production lags and serially correlated demand shocks have a much better potential than any other theories considered in this paper for explaining production and inventory behaviors observed in the data, supporting the view that inventories exists because they provide a means to avoid stockouts against demand uncertainty. This insight has implications for understanding the role of the retail industries which hold most of the finished goods inventories in the economy (see Blinder and Maccini, 1991). From the point of view of satisfying consumption demand, retail stores can provide better and faster services to consumers than can manufacturing firms since they are located closer to customers. Hence, if the single most important role of inventories is to meet consumption demand as the stockout-avoidance theory suggests, inventories should then be expected to be held mostly by retail stores rather than by manufacturing firms. My analysis also supports the conventional view that demand shocks hold the key for understanding the business cycle, since business cycles are, to a large extent, inventory fluctuations.[13]

The stockout-avoidance model, however, cannot explain why production is simultaneously smoother than sales at the high frequencies. This failure could be due to the fact that the simple model of Kahn (1987) is based on a partial equilibrium framework with exogenous demand as well as highly restrictive assumptions regarding structural parameters of the economy, such as constant marginal costs of production. Due to exogenous demand, price is incapable of responding fully to inventory changes in the goods market, resulting in possible distortions of production and sales.[14] With constant marginal costs of production, the motive for production smoothing is ruled out *a priori*. Highly restrictive assumptions like these may inhibit potential interactions between the production-smoothing mechanism and the stockout-avoidance mechanism, casting doubts on the robustness of the predictions of the simple model of Kahn (1987). Hence, to support the stockout-avoidance theory as a correct theory for understanding inventory fluctuations, extensions that relax these highly restrictive assumptions are clearly needed.[15]

Much more work along other lines also needs to be done, in order to complete our understanding on inventories. For example, the stockout-avoidance model cannot address the issue of input inventories. As documented by Blinder and Maccini (1991), input inventories account for 2/3 of total inventory stock and are at least twice as volatile as output inventories.[16] Second, this paper does not study the dynamics of the inventory-to-sales ratio, which is another important aspect of the inventory cycle. However, this issue is already starting to be addressed in the literature using the stockout-avoidance theory. For example, see Bils and Kahn (2000) and Daniele (2003). Finally,

[13] This, however, does not necessarily reject the RBC theory since RBC models can be shown to be capable of explaining business-cycle facts by demand shocks (e.g., see Wen, 2002a, 2002c).

[14] This point is clearly demonstrated by Kahn and Thomas (2002) using a general-equilibrium (S,s) inventory model. They show that the variance of sales relative to that of production is adversely affected by introducing inventories in general equilibrium. Also, Reagan (1982) shows that when price is allowed to vary in time and to fall below marginal cost of production, production may exhibit downward stickiness, which tends to reduce the variability of production relative to sales.

[15] Extensions of Kahn's (1987) model to allowing for storage costs and more general demand shock process are worked out by Maccini and Zabel (1996). These extensions, however, also rely on constant marginal cost of production and partial equilibrium analysis where demand is exogenous. Daniele (2003) considers increasing marginal cost in the model of Kahn (1987) with heterogeneous agents; but he also uses a partial equilibrium framework and he addresses entirely different questions from those considered here. See Wen (2002c) for a preliminary general equilibrium analysis of the stockout-avoidance theory. And see Kydland and Prescott (1982) and Christiano (1988) for general equilibrium models in which inventories are treated as factors of production.

[16] Also see Humphreys, Maccini and Schuh (2001).

it has long been known that durable goods inventories are much more volatile than nondurable goods inventories (e.g., see Feldstein and Auerbach, 1976). So far there have been few rigorous theoretical models offered to explain this puzzling phenomenon. It is not clear how the stockout-avoidance model can provide a natural environment in which this problem can be studied. But researches along this line could be fruitful.

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