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On the Optimal Volume of Labor Hoarding

by

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Abstract

Using a rational expectations model of profit maximizing firms facing demand uncertainty, this paper derives a closed-form relationship between the optimal volume of labor hoarding and other important economic variables such as profit, expected demand, interest rate, inventory level, output price and wage costs. An important insight gained from the analysis is that profit-seeking firms have incentives to enhance supply flexibility by holding not only goods inventories but also excess supplies of labor in reserve, so as to fully guard against demand uncertainty. The optimal target level of labor hoarding is shown to be a function of the variance of demand, the price level as well as the costs of production. The analysis confirms Blinder's (1982) conjecture regarding firms' strategic behavior under demand uncertainty. That is, inventories of labor are partial substitutes for inventories of goods as a means to cope with demand shocks.

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1 Introduction

Despite the fact that labor hoarding (or excess labor) is a widely observed behavior of firms (e.g., see Clark 1976, Medoff and Fay 1985, and Fair 1969, 1985) and that this concept has been constantly invoked in the literature as an explanation for the phenomenon of procyclical labor productivity (e.g., see Bernanke and Parkinson 1991, Dornbusch and Fischer 1981, Miller 1971, and Rotemberg and Summers 1990, among many others), yet we know surprisingly little about firms' optimal decision rules of labor hoarding. For example, how does the optimal volume of labor hoarding depend on profits, expected demand, inventory level, interest rate, output price and wage costs, etc.?[1]

Using a rational expectations model of profit maximizing firms facing demand uncertainty, this paper derives a closed-form relationship between the optimal volume of labor hoarding and other important economic variables such as profit, expected demand, interest rate, inventory level, output price and wage costs. An important insight gained from the analysis is that profit-seeking firms have incentives to enhance supply flexibility by holding not only goods inventories but also excess supplies of labor in reserve, so as to fully guard against demand uncertainty. The optimal target level of labor hoarding is shown to be a function of the variance of demand, the price level as well as the costs of production. The analysis confirms Blinder's (1982) conjecture regarding firms' strategic behavior under demand uncertainty. That is, inventories of labor are partial substitutes for inventories of goods as a means to cope with demand shocks.

The theoretical model provides not only a convenient framework for econometric analysis in related empirical issues, but also a genuine micro-foundation for understanding aggregate employment and labor productivity movements. It is shown that as a consequence of labor hoarding, measured labor productivity is procyclical despite non-increasing returns to scale. Interestingly, the model does not require the assumption of unobservable labor effort as an additional production factor in order to explain procyclical labor productivity. Unobservable

[1] Using a simple optimization model, Clark (1976) derives an optimal maximum length of time that is profitable for a firm to keep a worker on the payroll without having him/her work. Clark's model, however, does not deal with uncertainty and it cannot be used for understanding the optimal volume of labor hoarding.

effort is a necessary condition for the other types of labor hoarding theory to explain the procyclical output-labor ratio (e.g., see Rotemberg and Summers 1988, Burnside, Eichenbaum and Rebelo 1993, and Wen, 2002).

2 The Model

Assume that the firm's demand, denoted θ, is a stationary $AR(1)$ process:

$$\theta_t = \eta + \rho\theta_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is an $i.i.d.$ random variable with normal distribution, $N(0, \sigma^2)$. The maximum amount the firm can sell in period t is its inventory as of the end of the previous period, denoted by s_{t-1}, plus whatever it produces during period t (y_t). Assuming that the goods price p is sufficiently high, we have

$$\tau_t = \min\{\theta_t, s_{t-1} + y_t\}, \tag{2}$$

where τ_t denotes actual sales in period t. Also assuming that production takes one period, hence the decision for y_t needs to be made one period in advance based on information available in period $t-1$. Further, assuming that labor is the only factor of production and the production technology has constant returns to scale,

$$y_t = m_t. \tag{3}$$

It is useful to distinguish between workers on line (m) and workers on reserve (h). Workers on line are those who are engaged in production. Workers on reserve are those who are on the payroll but do not produce output. The wage rate paid to workers on line each period is w per worker ($w < p$ is a known constant), and the wage rate paid to workers on reserve each period is δw, where $\delta \in (0, \beta)$ with $\beta < 1$. The total stock of workers on payroll is denoted $W_t \equiv m_t + h_t$. Since in each period the firm can either hire or fire workers to adjust the firm's labor stock, the law of motion for the stock of workers is given by $W_t = W_{t-1} + n_t$, or

$$m_t + h_t = m_{t-1} + h_{t-1} + n_t, \tag{4}$$

where n is a flow variable denoting new hiring (or firing) of workers. The right hand side, $(m_{t-1} + h_{t-1} + n_t)$, is hence the total stock of workers available for working in the beginning of a period. It is also assumed that decisions for n_t (hiring or firing) need to be made one period in advance prior to production (e.g., due to time involved in search and job training).

The profits in period t are simply revenue minus costs, $p\tau_t - w(m_t + \delta h_t + n_t)$. Denoting $\beta \in (0,1)$ as the firm's time discounting factor (the inverse of interest rate), the problem for the firm is to choose sequences of new hiring (or firing), $\{n_j\}_{j=t}^{\infty}$, the number of workers on line $\{m_j\}_{j=t}^{\infty}$, the number of workers on reserve $\{h_j\}_{j=t}^{\infty}$, and goods inventories $\{s_j\}_{j=t}^{\infty}$ to solve

$$\max_{\{n_{t+j}\}} E_{t-2}\left\{\max_{\{m_{t+j},h_{t+j}\}} E_{t-1}\left\{\max_{\{s_{t+j}\}} E_t\left\{\sum_{j=0}^{\infty}\beta^j\left[p\tau_{t+j} - w\left(m_{t+j} + \delta h_{t+j} + n_{t+j}\right)\right]\right\}\right\}\right\}$$

subject to the law of motion for inventories in the goods market (without loss of generality, assuming that inventories depreciate at zero rate):

$$\tau_t + s_t = s_{t-1} + y_t, \tag{5}$$

the production function (3), the law of motion for employment stock (4), and two non-negativity constraints on inventory stocks for goods and labor respectively:

$$s_t \geq 0, \tag{6}$$

$$h_t \geq 0; \tag{7}$$

where (6) is implied by (2). The expectation operators, $\{E_{t-2}, E_{t-1}, E_t\}$, in the objective function reflect the relevant information sets in the sequence of decision making: the firm decides first how many workers to hire (lay off) based on information available in period $t-2$, then how much output to produce one period later by choosing the number of workers on line and the number of workers on reserve based on updated information on expected demand in period $t-1$, and then on the level of sales or inventory holdings after observing the actual demand in period t.

Denoting $\lambda^s\,(>0)$ and $\lambda^h\,(>0)$ as the Lagrangian multipliers associated with the goods market resource constraint (5) and the labor market resource constraint (4) respectively, and $\pi^s\,(\geq 0)$ and $\pi^h\,(\geq 0)$ as the Lagrangian multipliers associated with the non-negativity constraints (6) and (7) respectively, the first-order conditions with respect to $\{n_t, m_t, h_t, s_t\}$ are given respectively by:

$$w = E_{t-2}\lambda_t^h \tag{8}$$

$$\lambda_t^h + w = \beta E_{t-1}\lambda_{t+1}^h + E_{t-1}\lambda_t^s \tag{9}$$

$$\lambda_t^h + \delta w = \beta E_{t-1}\lambda_{t+1}^h + \pi_t^h \tag{10}$$

$$\lambda_t^s = \beta E_t \lambda_{t+1}^s + \pi_t^s \tag{11}$$

where the complementary slackness constraints on goods inventories and labor inventories are given respectively by

$$\pi_t^s s_t = 0,$$

$$\pi_t^h h_t = 0.$$

These six equations together with equations (4) and (5) determine eight unknowns in the economy in each period t, $\left\{n, m, h, s, \lambda^s, \lambda^h, \pi^s, \pi^h\right\}$.

3 Analysis

Given output price (λ_t^s) and the availability of finished goods ($y_t + s_{t-1}$) as well as the realized demand shocks (θ) at the beginning of period t, the firm chooses inventory holdings (s_t) to maximize profit. Consider two possible cases:

Case A_1. The demand shock is below "normal", hence the existing supply ($y_t + s_{t-1}$) is sufficient to meet demand. In this case, we have $s_t \geq 0$ and $\pi_t^s = 0$. Equation (11) implies that the Lagrangian multiplier of goods is a constant, $\lambda_t^s = \bar{\lambda}$. Since demand can be satisfied, we have $\tau_t = \theta_t$. The level of inventory holdings is given by

$$s_t = y_t + s_{t-1} - \theta_t.$$

Since $s_t \geq 0$, the threshold level of preference shock determining that the firm is in case A_1 is then given by $y_t + s_{t-1} - \theta_t \geq 0$ or equivalently, by

$$\theta_t \leq y_t + s_{t-1}.$$

Case B_1. The demand shock is above "normal", and the existing goods supply $(y_t + s_{t-1})$ falls short in meeting the demand. In this case we have $s_t = 0$ and $\pi_t^s > 0$. Since sales are constrained by supply, we have

$$\tau_t = y_t + s_{t-1}.$$

Notice that the probability distribution of case A_1 and case B_1 (i.e., the probability of stocking out in period t) depends not only on demand but also crucially on the supply (production) level of goods (y_t), which is determined in period $t-1$ by the optimal amount of workers put on line (m_t).

Equation (9) shows that m_t is be chosen such that the cost of putting one more worker on line (= wage cost plus the shadow value of labor resource, $w + \lambda_t^h$) equals the benefit of having an extra worker on line (= the value of the marginal product of a worker, which is the expected next period value of goods, $E_{t-1}\lambda_t^s$, plus the discounted wage cost saved next period by having the worker on stock, βw). Denoting

$$z_t \equiv E_{t-1}s_t = y_t + s_{t-1} - E_{t-1}\theta_t$$

as the level of inventory holdings such that the firm will stock out in period t if and only if $\varepsilon_t \geq z_t$, then the expected goods price can be expanded into two terms (denoting the *p.d.f* of ε as $f(\varepsilon)$),

$$E_{t-1}\lambda_t^s = \int_{-\infty}^{z} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{z}^{\infty} p f(\varepsilon)d\varepsilon,$$

where $\lambda^s = \bar{\lambda}(< w)$ with probability $\int_{-\infty}^{z} f(\varepsilon)d\varepsilon$ if there is no stockout (i.e., case A_1),[2] and $\lambda^s = p$ with probability $\int_{z}^{\infty} f(\varepsilon)d\varepsilon$ if there is a stockout (i.e., the value of inventory equals market price p when the goods are sold). Given the law of motion for preference shocks,

[2]For a proof of $\bar{\lambda} < w$, see proposition 3.3.

$\theta_t = E_{t-1}\theta_t + \varepsilon_t$, equation (9) then becomes[3]

$$\begin{aligned} (1-\beta)w + \lambda_t^h &= \bar{\lambda}\int_{-\infty}^{z} f(\varepsilon)d\varepsilon + p\int_{z}^{\infty} f(\varepsilon)d\varepsilon \\ &\equiv \Gamma(z_t), \end{aligned} \tag{12}$$

where $\Gamma()$ is a monotonically decreasing function of z. Since z is proportional to $y(= m)$ by definition, the more workers being put on line in period $t-1$ (m), the less likely there is a stockout in period t given expected demand $E_{t-1}\theta_t$. Thus equation (12) determines the optimal cut-off value of z and hence the optimal amount of workers on line for production in period $t-1$.

Turning to the left-hand side of equation (12), the shadow value of labor (λ^h) depends on the tightness of the firm's labor resource (i.e., whether the nonnegativity constraint on workers in reserve binds). There are two cases to consider for the possible values of λ^h:

Case A_2. The expected demand for goods is below "normal" and hence the demand for workers on line (m_t) is below "normal". In this case we have $h_t \geq 0$ and $\pi_t^h = 0$. Hence equation (10) implies that the value of labor is constant,[4]

$$\lambda_t^h = (\beta - \delta)w.$$

The interpretation for $(\beta - \delta)w$ is straightforward. In case there is no stockout in workers, by having one more worker on reserve the firm gets to save on the wage cost of new hiring next period (with a discounted value of βw). Subtracting from this the cost of keeping the worker on reserve in the current period (δw) gives the net benefit of having a worker on reserve, $(\beta - \delta)w$, which must equal the value of labor (λ^h).

In this case, equation (12) implies that the optimal target inventory level z_t based on period $t-1$ information is a constant: $z_t = k$, where k solves

$$\Gamma(k) = (1-\delta)w.$$

This implies that the optimal production level (y_t) is determined by the equation,

$$k = y_t + s_{t-1} - E_{t-1}\theta_t,$$

[3]Equation 8 implies $E_{t-1}\lambda_{t+1}^h = w$.
[4]Equation 8 implies $E_{t-1}\lambda_{t+1}^h = w$.

or equivalently, by

$$y_t = E_{t-1}\theta_t + (k - s_{t-1}). \tag{13}$$

Hence, optimal production is characterized by a policy that specifies a constant target level for inventory holdings (k) or a target level of inventory investment ($k - s_{t-1}$), such that production moves one-for-one with expected demand ($E_{t-1}\theta_t$) given the target inventory investment level, provided that the firm is in case A_2 (i.e., provided there is no stockout in workers: $h_t \geq 0$). This inventory target policy is similar to that derived by Kahn (1987) in a model without labor hoarding.

Using equation (4), the requirement $h_t \geq 0$ implies that the threshold level of expected demand that determines the probability of "stockout" in workers on reserve is given by,

$$m_{t-1} + h_{t-1} + n_t - y_t \geq 0,$$

or equivalently, by

$$E_{t-1}\theta_t \leq m_{t-1} + h_{t-1} + n_t - (k - s_{t-1}), \tag{14}$$

where we have substituted out y_t using the optimal policy (13). That is, if expected total demand (which equals expected sales, $E_{t-1}\theta_t$, plus inventory investment demand, $k - s_{t-1}$) is less than the potential supply (which equals the total supply of the stock of workers according to the one-to-one transformation technology, $y = m$), then some workers should be put on reserve (i.e., $h_t \geq 0$) and the optimal number of workers on line ($m = y$) is characterized by the policy (13).

Case B_2. The expected demand for goods is above "normal", and hence the demand for workers on line is above "normal". In this case, we have $h_t = 0$ and $\pi_t^h > 0$, implying that there is a stockout in workers on reserve. Hence, the optimal number of workers on line is simply

$$m_t = m_{t-1} + h_{t-1} + n_t.$$

In this case, the tightness in labor resource pushes shadow price of labor upwards, so that $\lambda_t^h = (\beta - \delta)w + \pi_t^h > (\beta - \delta)w$.

Clearly, whether the firm is in position A_2 or position B_2 depends not only on expected demand in period $t-1$ ($E_{t-1}\theta_t$), but also on the availability of labor stock ($m_{t-1} + h_{t-1} + n_t$)

in period $t-1$, which depends on hiring/firing decisions (n_t) made in period $t-2$. Thus, the optimal decision rules of the entire model depend on the decision rule for n.

Equation (8) shows that n_t should be chosen such that the marginal cost of hiring, w, equals the expected next period value of labor resource (the shadow value of labor, $E_{t-2}\lambda_t^h$). Since the value of λ^h depends on the tightness in the reserved labor resource, there are two possibilities to consider: in case A_2, $\lambda^h = (\beta - \delta)w$; and in case B_2, $\lambda^h = E_{t-1}\lambda^s - (1-\beta)w$ (see equation 9). Denote

$$\zeta_t \equiv \frac{1}{(1+\rho)}\left[m_{t-1} + h_{t-1} + n_t - E_{t-2}\theta_t\right] \tag{14'}$$

as the level of labor hoarding (workers on reserve), which equals total labor stock available for working (or potential supply of goods, $m+h+n$) minus expected workers on line (measured by expected goods demand, $E_{t-2}\theta$), such that the firm stockout of workers in period $t-1$ if and only if demand in that period is too high ($\varepsilon_{t-1} \geq \zeta$).[5] Then the expected value of labor based on information available in period $t-2$ (equation 8) can be expanded into two terms,

$$\begin{aligned} w &= E_{t-2}\lambda_t^h \qquad (15) \\ &= \int_{-\infty}^{\zeta_t} (\beta - \delta) w f(\varepsilon) d\varepsilon + \int_{\zeta_t}^{\infty} \left[E_{t-1}\lambda_t^s - (1-\beta)w\right] f(\varepsilon) d\varepsilon, \end{aligned}$$

[5] To see how ζ is derived, note that s_{t-1} in the expression (14) can take only two possible values

$$s_{t-1} = \begin{cases} k - \varepsilon_{t-1}, & \text{if } \varepsilon_{t-1} \leq k \\ 0, & \text{if } \varepsilon_{t-1} > k \end{cases}$$

Hence (14) can be rewritten as

$$E_{t-1}\theta_t \leq \begin{cases} m_{t-1} + h_{t-1} + n_t - \varepsilon_{t-1} & \text{if } \varepsilon_{t-1} \leq k \\ m_{t-1} + h_{t-1} + n_t - k & \text{if } \varepsilon_{t-1} > k \end{cases}$$

Utilizing the identity, $E_{t-1}\theta_t = E_{t-2}\theta_t + \rho\varepsilon_{t-1}$, the above two inequalities can be rearranged as

$$(1+\rho)\varepsilon_{t-1} \leq \begin{cases} m_{t-1} + h_{t-1} + n_t - E_{t-2}\theta_t & \text{if } \varepsilon_{t-1} \leq k \\ m_{t-1} + h_{t-1} + n_t - E_{t-2}\theta_t - (k - \varepsilon_{t-1)} & \text{if } \varepsilon_{t-1} > k \end{cases}$$

Clearly, the first inequality implies the second inequality given that $(k - \varepsilon_{t-1}) < 0$ in the second inequality. Hence, (14) is identical to

$$\varepsilon_{t-1} \leq \frac{1}{(1+\rho)}\left[m_{t-1} + h_{t-1} + n_t - E_{t-2}\theta_t\right] \equiv \zeta_t.$$

where the first term represents the probability of case A_2 and the second represents the probability of case B_2. Equation (15) implicitly determines the cut-off value for ζ_t as the optimal volume of labor hoarding. Given ζ_t, the optimal hiring/firing policy for n is consequently determined. The following proposition shows that the optimal solution for ζ_t is a constant.

Proposition 1 *The optimal volume of labor hoarding is a constant, $\zeta_t = \gamma$, that solves the following implicit equation,*

$$\begin{aligned} w &= (\beta-\delta)w\Phi(\frac{\gamma}{\sigma}) - (1-\beta)w\left(1-\Phi(\frac{\gamma}{\sigma})\right) \\ &\quad + \int_{\gamma}^{\infty}\left[\bar{\lambda}\Phi(\frac{(1+\rho)\gamma-\rho\varepsilon_{t-1}}{\sigma}) + p\left(1-\Phi(\frac{(1+\rho)\gamma-\rho\varepsilon_{t-1}}{\sigma})\right)\right] f(\varepsilon)d\varepsilon, \end{aligned} \tag{16}$$

where Φ is the cumulative standard normal distribution function of ε.

Proof. See Appendix 001.■

This optimal policy for labor hoarding, $\zeta_t = \gamma$, reflects a precautionary motive of the firm to avoid stockouts in workers when demand is uncertain and when labor replenishment takes time. This labor-hoarding target (γ) determines the optimal decision rule for n_t by equation (14′),

$$n_t = E_{t-2}\theta_t + (1+\rho)\gamma - (m_{t-1} + h_{t-1}).$$

This equation says that in equilibrium the optimal hiring/firing plan (n) is such that the total labor stock ($n_t + m_{t-1} + h_{t-1}$, which equals potential output supply) equals the expected labor usage (which equals the expected demand, $E_{t-2}\theta_t$) plus a target volume of labor hoarding (γ).

Proposition 2 *The optimal inventory target for labor is higher than that for goods,*

$$\gamma \geq k.$$

Proof. See Appendix 002.■

The intuition behind proposition 2 can be understood as follows. Based on the one-for-one transformation production technology, one worker's labor is equivalent to one unit of output. Hence labor inventories can be viewed essentially as intermediate goods inventories.

The probability of a stockout in finished goods in period-t is affected by the probability of a stockout in intermediate goods (labor) in period $t-1$, as the potential supply of finished goods is determined by the potential supply of intermediate goods (labor), which is determined by employment (hiring/firing) decisions made in period $t-2$. Under a stockout-avoidance motive, the optimal size of inventory targets, $\{k, \gamma\}$, are positively influenced by the degree of uncertainty in final demand. The earlier the decision has to be made, the harder it is to forecast final demand due to the increased uncertainty, hence the larger precautionary inventory stock it is needed in order to be better positioned to take advantage of periods in which demand is higher than normal.

Proposition 3 *The equilibrium decision rules for goods inventories (s), sales (τ), production (y) - which also equals equilibrium utilization of labor stock (m), labor hoarding (h), and new hiring/firing (n) are given respectively by:*

$$s_t = \begin{cases} k - \varepsilon_t & ;\ \text{if } \varepsilon_t \leq k \ \&\ \varepsilon_{t-1} \leq \gamma \\ (1+\rho)\gamma - \varepsilon_t - \rho\varepsilon_{t-1} & ;\ \text{if } \varepsilon_t \leq k \ \&\ \varepsilon_{t-1} > \gamma \\ 0 & ;\ \text{if } \varepsilon_t > k \end{cases}$$

$$\tau_t = \begin{cases} E_{t-1}\theta_t + \varepsilon_t & ;\ \text{if } \varepsilon_t \leq k \\ E_{t-1}\theta_t + k & ;\ \text{if } \varepsilon_t > k \ \&\ \varepsilon_{t-1} \leq \gamma \\ E_{t-2}\theta_t + (1+\rho)\gamma & ;\ \text{if } \varepsilon_t > k \ \&\ \varepsilon_{t-1} > \gamma \end{cases}$$

$$y_t = m_t = \begin{cases} E_{t-1}\theta_t + \varepsilon_{t-1} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} \leq \gamma \\ E_{t-1}\theta_t + k - (1+\rho)\gamma + \varepsilon_{t-1} + \rho\varepsilon_{t-2} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} > \gamma \\ E_{t-1}\theta_t + k & ;\ \text{if } k < \varepsilon_{t-1} \leq \gamma \\ E_{t-2}\theta_t + (1+\rho)\gamma & ;\ \text{if } \varepsilon_{t-1} > \gamma \end{cases}$$

$$h_t = \begin{cases} (1+\rho)\gamma - (1+\rho)\varepsilon_{t-1} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} \leq \gamma \\ 2(1+\rho)\gamma - k - (1+\rho)\varepsilon_{t-1} - \rho\varepsilon_{t-2} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} > \gamma \\ (1+\rho)\gamma - k - \rho\varepsilon_{t-1} & ;\ \text{if } k < \varepsilon_{t-1} \leq \gamma \\ 0 & ;\ \text{if } \varepsilon_{t-1} > \gamma \end{cases}$$

$$n_t = E_{t-2}\theta_t - E_{t-3}\theta_{t-1}.$$

where $\bar{\lambda} = \beta(1-\delta)w$.

Proof. See Appendix 003.■

The intuition behind the decision rule for hiring/firing is the simplest to understand. Since labor is a perfectly durable input in the model, hence whether to hire or fire more workers depends solely on changes in expected demand. If expected demand remains constant, then there is no need to adjust the firm's stock of workers. If demand is expected to increase, then the firm opts to hire, otherwise it opts to layoff.

Proposition 4 *The optimal volume of labor hoarding depends positively on the variance of demand (σ), the price level (p), and negatively on interest rate ($\frac{1}{\beta}$), labor hoarding cost (δ) and wage cost (w):*

$$\frac{\partial\gamma}{\partial\sigma} > 0, \frac{\partial\gamma}{\partial p} > 0, \frac{\partial\gamma}{\partial\beta} > 0, \frac{\partial\gamma}{\partial\delta} < 0, \frac{\partial\gamma}{\partial w} < 0.$$

Proof. See Appendix 004.■

4 Procyclical Labor Productivity

It is well known that measured labor productivity (or output per worker) is strongly procyclical. This phenomenon is puzzling because given the assumption of non-increasing returns to labor, one percent increase in labor can lead to no more than one percent increase in output, hence output per worker is expected to be negatively correlated with output level – in sharp contrast to what the data suggest. Labor hoarding has been one of the most popular explanations offered in the literature for resolving this long-standing puzzle. The intuition is that if firms hoard labor during a downturn, then the drop in output will appear to be larger than the drop in measured employment based on firms' payroll statistics. On the other hand during an upturn, firms can increase output by utilizing hoarded labor without new hiring. Thus measured productivity appears to be procyclical.

In the current model, since production technology is linear, the output/labor ratio is expected to be constant (acyclical). However, since the firm opts to hold excess labor on reserve so as to be better positioned to take advantage of periods in which demand is higher than normal, the resulting measured output/labor ratio is procyclical.

To see this, notice that based on the decision rules given above, output-to-workers ratio is given by (note, $E_{t-1}\theta_t = E_{t-2}\theta_t + \rho\varepsilon_{t-1}$)

$$\frac{y_t}{m_t + h_t} = \begin{cases} \frac{E_{t-2}\theta_t + (1+\rho)\varepsilon_{t-1}}{E_{t-2}\theta_t + (1+\rho)\gamma} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} \leq \gamma \\ \frac{E_{t-2}\theta_t + k - (1+\rho)\gamma + (1+\rho)\varepsilon_{t-1} + \rho\varepsilon_{t-2}}{E_{t-2}\theta_t + (1+\rho)\gamma} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \&\ \varepsilon_{t-2} > \gamma \\ \frac{E_{t-2}\theta_t + k + \rho\varepsilon_{t-1}}{E_{t-2}\theta_t + (1+\rho)\gamma} & ;\ \text{if } k < \varepsilon_{t-1} \leq \gamma \\ 1 & ;\ \text{if } \varepsilon_{t-1} > \gamma \end{cases}.$$

Notice that as long as there exists excess labor on reserve (i.e., as long as $\varepsilon_{t-1} \leq \gamma$), all the ratios of output to labor stock given above (i.e., the top 3 rows) are procyclical for two reasons. First, a positive innovation in demand in period $t-1$ (i.e., $\varepsilon_{t-1} > 0$) raises production (the numerator) instantaneously while leaving the total labor stock (the denominator) intact, since labor on reserve serves as a buffer to replenish labor on line.[6] Second, following this positive innovation in demand the increase in productivity is persistent, because an increase in ε_{t-1} translates into an increase in ε_{t-2} next period (which increases $E_{t-2}\theta_t$), this raises the numerator more than it raises the corresponding denominator since the constant terms in each numerator (e.g., $0, (k-(1+\rho)\gamma), k$) are strictly smaller respectively than the constant term in the corresponding denominator (i.e., $(1+\rho)\gamma$). Hence, due to labor hoarding, labor productivity should appear to be procyclical despite the fact that the production technology, $y_t = m_t$, shows a constant output-to-labor ratio.

5 Conclusion

This paper shows that under demand uncertainty it is rational for firms to hold not only excess supplies of goods in inventories but also excess supplies of labor on reserve as partial substitutes for good inventories. This confirms Blinder's (1981) conjecture about firms' strategic behavior under demand uncertainty: firms rationally create excess production capacity by all means in order to enhance supply flexibility. The optimal volume of labor

[6] Only when the innovation in demand is sufficiently large so that the firm stockout of workers on reserve ($h_t = 0$ if $\varepsilon_{t-1} > \gamma$), output moves one-for-one with labor stock (i.e., the 4th row), resulting in labor productivity being constant (acyclical).

hoarding is shown to be related positively to the variance of demand and price level, and negatively to interest rate and labor costs. The theoretical model provides not only a natural framework for further empirical work on related issues but also a genuine micro foundation for understanding the cyclical nature of employment and labor productivity.

The model can also be applied to studying optimal investment behavior and excess capacity, as the labor stock in the model can be interpreted as capital stock, labor hoarding as capacity hoarding, and hiring/firing of labor as purchase/sales of equipments (investment). Extra constraints may need be imposed on the model if investment is considered irreversible.

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Appendix 001 (Proof for proposition 1):

Given that

$$\begin{aligned} E_{t-1}\lambda_t^s &= \int_{-\infty}^{z_t} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{z_t}^{\infty} pf(\varepsilon)d\varepsilon \\ &= \int_{-\infty}^{y_t+s_{t-1}-E_{t-1}\theta_t} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{y_t+s_{t-1}-E_{t-1}\theta_t}^{\infty} pf(\varepsilon)d\varepsilon, \end{aligned}$$

equation (10) can be further expressed as

$$\begin{aligned} w &= \int_{-\infty}^{\zeta} (\beta-\delta)wf(\varepsilon)d\varepsilon - \int_{\zeta}^{\infty} (1-\beta)wf(\varepsilon)d\varepsilon \\ &\quad + \int_{\zeta}^{\infty} \left[\int_{-\infty}^{y_t+s_{t-1}-E_{t-1}\theta_t} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{y_t+s_{t-1}-E_{t-1}\theta_t}^{\infty} pf(\varepsilon)d\varepsilon \right] f(\varepsilon)d\varepsilon. \end{aligned}$$

Since $y_t(=m_t) = m_{t-1} + h_{t-1} + n_t$ if $h_t = 0$ (i.e., if $\varepsilon_{t-1} > \zeta$), and since $s_{t-1} = 0$ if $\varepsilon_{t-1} > k$, the third term in the above equation can be expressed as (conditional on $\varepsilon_{t-1} > \zeta$ and on the assumption $\zeta \geq k$)[7]

$$\begin{aligned} &\int_{\zeta}^{\infty} \left[\int_{-\infty}^{m_{t-1}+h_{t-1}+n_t-E_{t-1}\theta_t} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{m_{t-1}+h_{t-1}+n_t-E_{t-1}\theta_t}^{\infty} pf(\varepsilon)d\varepsilon \right] f(\varepsilon)d\varepsilon \\ &= \int_{\zeta_t}^{\infty} \left[\int_{-\infty}^{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}} \bar{\lambda} f(\varepsilon)d\varepsilon + \int_{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}}^{\infty} pf(\varepsilon)d\varepsilon \right] f(\varepsilon)d\varepsilon \\ &= \int_{\zeta_t}^{\infty} \left[\bar{\lambda}\Phi(\frac{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}}{\sigma}) + p\left(1-\Phi(\frac{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}}{\sigma})\right) \right] f(\varepsilon)d\varepsilon, \end{aligned}$$

where $\Phi()$ is the cumulative standard normal distribution function of ε. Hence we have

$$\begin{aligned} w &= (\beta-\delta)w\Phi(\frac{\zeta_t}{\sigma}) - (1-\beta)w\left(1-\Phi(\frac{\zeta_t}{\sigma})\right) \\ &\quad + \int_{\zeta_t}^{\infty} \left[\bar{\lambda}\Phi(\frac{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}}{\sigma}) + p\left(1-\Phi(\frac{(1+\rho)\zeta_t-\rho\varepsilon_{t-1}}{\sigma})\right) \right] f(\varepsilon)d\varepsilon. \end{aligned}$$

[7] The assumption, $\zeta \geq k$, will be confirmed in proposition 3.2.

Clearly, a constant, $\zeta_t = \gamma$, solves the above equation; i.e., γ solves:

$$\begin{aligned} w &= (\beta - \delta) w \Phi(\frac{\gamma}{\sigma}) - (1-\beta) w \left(1 - \Phi(\frac{\gamma}{\sigma})\right) \\ &+ \int_{\gamma}^{\infty} \left[\bar{\lambda} \Phi(\frac{(1+\rho)\gamma - \rho\varepsilon_{t-1}}{\sigma}) + p \left(1 - \Phi(\frac{(1+\rho)\gamma - \rho\varepsilon_{t-1}}{\sigma})\right)\right] f(\varepsilon) d\varepsilon, \end{aligned}$$

where the integral is over ε_{t-1} and the expectation is based on period $t-2$ information set, hence the third term in the above expression is a constant after integration as the arguments inside the integral involve only innovations ($i.i.d.$ shocks) after $t-2$.[8]■

Appendix 002 (proof for proposition 2):

The goods inventory stock satisfies

$$s_t = \begin{cases} y_t + s_{t-1} - \theta_t & ; \text{ if } \varepsilon_t \leq k \\ 0 & ; \text{ if } \varepsilon_t > k \end{cases} .$$

Since the optimal production (y_t) satisfies

$$y_t = \begin{cases} E_{t-1}\theta_t + (k - s_{t-1}) & ; \text{ if } \varepsilon_{t-1} \leq \gamma \\ m_{t-1} + h_{t-1} + n_t & ; \text{ if } \varepsilon_{t-1} > \gamma \end{cases}$$

and since the optimal hiring/firing (n) satisfies

$$n_t = E_{t-2}\theta_t + (1+\rho)\gamma - (m_{t-1} + h_{t-1}),$$

we have

$$\begin{aligned} s_t &= \begin{cases} E_{t-1}\theta_t + k - \theta_t & ; \text{ if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} \leq \gamma \\ E_{t-2}\theta_t + (1+\rho)\gamma + s_{t-1} - \theta_t & ; \text{ if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} > \gamma \\ 0 & ; \text{ if } \varepsilon_t > k \end{cases} \\ &= \begin{cases} k - \varepsilon_t & ; \text{ if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} \leq \gamma \\ s_{t-1} + (1+\rho)\gamma - \varepsilon_t - \rho\varepsilon_{t-1} & ; \text{ if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} > \gamma \\ 0 & ; \text{ if } \varepsilon_t > k \end{cases} \end{aligned}$$

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[8] Namely, the conditional expectation, $E_{t-2}g(\varepsilon_{t-1})$, is always a constant, where $g()$ is any arbitrary function.

Notice that the lagged variable (s_{t-1}) can be further iterated backwards using the above dynamic equation to obtain:

$$s_{t-1} = \begin{cases} k - \varepsilon_{t-1} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \& \ \varepsilon_{t-2} \leq \gamma \\ s_{t-2} + (1+\rho)\gamma - \varepsilon_{t-1} - \rho\varepsilon_{t-2} & ;\ \text{if } \varepsilon_{t-1} \leq k \ \& \ \varepsilon_{t-2} > \gamma \\ 0 & ;\ \text{if } \varepsilon_{t-1} > k \end{cases},$$

which may lead to infinite regression, resulting in the series $\{s_t\}$ being nonstationary, which cannot be an equilibrium unless the condition

$$s_{t-1} = 0, \quad \text{if } \varepsilon_{t-1} > \gamma;$$

always holds, so that the decision rule for s_t can be written as

$$s_t = \begin{cases} k - \varepsilon_t & ;\ \text{if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} \leq \gamma \\ (1+\rho)\gamma - \varepsilon_t - \rho\varepsilon_{t-1} & ;\ \text{if } \varepsilon_t \leq k \ \& \ \varepsilon_{t-1} > \gamma \\ 0 & ;\ \text{if } \varepsilon_t > k \end{cases}.$$

Since we know that $s_{t-1} = 0$ if and only if $\varepsilon_{t-1} > k$, hence, if the condition, $\gamma \geq k$, holds, then $\varepsilon_{t-1} > \gamma$ automatically implies $\varepsilon_{t-1} > k$, which yields a stationary process for s_t and it is therefore an equilibrium. Hence, $\gamma \geq k$ must be true.■

Appendix 003 (proof for proposition 3):

The decision rule for s_t is proved in the proof for proposition 3.2. The rest can be obtained by following the discussions in section 3 above using straightforward substitutions. What is left to show is the equation, $\bar{\lambda} = \beta(1-\delta)w$. Note $\lambda_t^s = \bar{\lambda}$ if $\varepsilon_t \leq k$ (i.e., if $\pi_t^s = 0$). On the other hand, since the current period shadow price of intermediate goods is known based on last period information set, as all variables in the resource constraint (4) are known in period $t-1$, we then have $E_{t-1}\lambda_t^h = \lambda_t^h$ and $E_t\lambda_{t+1}^h = \lambda_{t+1}^h$. Since $\lambda_t^h = (\beta - \delta)w$ if $\varepsilon_{t-1} \leq \gamma$ according equation (10) and equation (8),[9] we have $\lambda_{t+1}^h = (\beta-\delta)w$ if $\varepsilon_t \leq \gamma$. According to proposition 3.2, $k < \gamma$, hence $\lambda_t^s = \bar{\lambda}$ implies $\lambda_{t+1}^h = (\beta - \delta)w$ (since $\varepsilon_t \leq k$ implies $\varepsilon_t \leq \gamma$). According to equation (9), $E_t\lambda_{t+1}^s = (1-\beta)w + \lambda_{t+1}^h$, substituting this into equation (11) gives

$$\lambda_t^s = \beta\left((1-\beta)w + \lambda_{t+1}^h\right) = \beta(1-\delta)w, \quad \text{if } \varepsilon_t \leq k.$$

[9] Equation (8) implies $E_{t-1}\lambda_{t+1}^h = w$.

■

Appendix 004 (proof for proposition 4):

Equation (16) can be rewritten as

$$(2-\beta)w = \int_{-\infty}^{\gamma}(1-\delta)wf(\varepsilon)\varepsilon + \int_{\gamma}^{\infty} F(\gamma,\sigma,\varepsilon_{t-1})f(\varepsilon)d\varepsilon, \tag{17}$$

where

$$F(\gamma,\sigma,\varepsilon_{t-1}) \equiv \left[\beta(1-\delta)w\Phi(\frac{(1+\rho)\gamma-\rho\varepsilon_{t-1}}{\sigma}) + p\left(1-\Phi(\frac{(1+\rho)\gamma-\rho\varepsilon_{t-1}}{\sigma})\right)\right].$$

The first term on the right hand side of (17) pertains to the case of no stockout in labor reserve, and the second term pertains to the case of a stockout in labor reserve. Hence the right hand side of equation (17) is a convex combination of two terms, $(1-\delta)\,w$ and $F(\gamma,\sigma,\varepsilon_{t-1})$. Since $(1-\delta)w < (2-\beta)w$, we must have $F > (2-\beta)w > (1-\delta)w$. Given $F > (1-\delta)w$, the right hand side of (17) clearly depends negatively on γ if F also depends negatively on γ. Furthermore, the right-hand-side of (17) moves in the same direction with F as the other parameters $\{\sigma,\beta,\delta,\rho\}$ change.

1). Show $\frac{\partial\gamma}{\partial\sigma} > 0$. First, we show that $F()$ is decreasing in γ and increasing in σ. Differentiating F with respect to γ gives:

$$\frac{\partial F}{\partial\gamma} = \left(\beta(1-\delta)w - p\right)\frac{\partial\Phi}{\partial\gamma} < 0.$$

Similarly we have

$$\frac{\partial F}{\partial\sigma} = \left(\beta(1-\delta)w - p\right)\frac{\partial\Phi}{\partial\sigma} > 0.$$

Since $\beta(1-\delta)w < p$, and since $\frac{\partial\Phi}{\partial\gamma} > 0$ and $\frac{\partial\Phi}{\partial\sigma} < 0$, hence we have $\frac{\partial F}{\partial\gamma} < 0$ and $\frac{\partial F}{\partial\sigma} > 0$. Therefore, the right-hand side of (17) increases with σ and decreases with γ. Given that the left hand side of (17) does not depend on $\{\gamma,\sigma\}$, an increase in σ thus must imply an increase in γ in order to keep the right hand side of (17) unchanged.

2). Show $\frac{\partial\gamma}{\partial p} > 0$. It is clear that F increases with p. Given that the left hand side of (17) does not depend on p, an increase in p must imply an increase in γ in order to keep the right hand side of equation (17) unchanged.

3). Show $\frac{\partial\gamma}{\partial\delta} < 0$. It is clear that the right side of (17) is decreasing in δ. Given that the right hand side of (17) is also decreasing in γ, an increase in δ must imply a decrease in γ in order to keep equation (17) unchanged.

4). Show $\frac{\partial\gamma}{\partial\beta} > 0$. Due to an increase in β, the marginal effect on the left hand side is $-w < 0$, and the marginal effect on the right hand side is

$$\int_{\gamma}^{\infty} \frac{\partial F}{\partial\beta} f(\varepsilon)d\varepsilon = \int_{\gamma}^{\infty} (1-\delta)w\Phi(\frac{(1+\rho)\gamma - \rho\varepsilon_{t-1}}{\sigma})f(\varepsilon)d\varepsilon > 0.$$

Given that the right hand side of (17) is decreasing in γ, an increase in β must imply an increase in γ so as to offset the positive effect of β on F.

5). Show $\frac{\partial\gamma}{\partial w} < 0$. For an increase in w, the marginal effect on the left hand side is $(2-\beta) > 0$, and the marginal effect on the right hand side is given by

$$\begin{aligned}
& \int_{-\infty}^{\gamma} (1-\delta)f(\varepsilon)\varepsilon + \int_{\gamma}^{\infty} \frac{\partial F}{\partial w} f(\varepsilon)d\varepsilon \\
= & \int_{-\infty}^{\gamma} (1-\delta)f(\varepsilon)\varepsilon + \int_{\gamma}^{\infty} \beta(1-\delta)\Phi(\frac{(1+\rho)\gamma - \rho\varepsilon_{t-1}}{\sigma})f(\varepsilon)d\varepsilon \\
< & \int_{-\infty}^{\gamma} (1-\delta)f(\varepsilon)\varepsilon + \int_{\gamma}^{\infty} \beta(1-\delta)f(\varepsilon)d\varepsilon \\
< & (1-\delta) \\
< & (2-\beta).
\end{aligned}$$

Hence the left hand side increases more than the right hand side does after w changes. Thus γ must decrease so as to balance out the relative fall on the right hand side of equation (17).$\blacksquare$