Bargaining When Sunspots Matter*

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February 1999

Abstract

When agents know that the environment they live in is changing through time, the traditional Nash Bargaining Rule is no longer the limit of the sequential bargaining game. I derive the limit rule for bilateral bargaining when agents recognize that the aggregate economy follows a dynamic process that randomly switches back and forth between two possible states. Decentralized economies with bargaining are natural environments for the study of sunspot equilibria. The rule derive in this paper then becomes of special importance for those types of phenomena. Two simple applications are presented to illustrate this fact: one for the labor market and the other for a monetary random-matching economy. In the first example, a model of wage bargaining and trade externalities is introduced and it is shown that in those situations sophisticated bargaining can increase the volatility of the wage bill due to extrinsic uncertainty. In the second example, a Kiyotaki-Wright model of money, equilibrium prices are shown to fundamentally depend on our novel bargaining solution. JEL C78, E30, E24.

*I would like to thank Karl Shell for helpful conversations and encouragement. Guido Cozzi, Todd Keister, Salvador Ortigueira, Luis Rivas and the participants at the Spring 1999 Cornell University-Penn State University Joint Macroeconomics Workshop also provided useful comments on an earlier draft. All errors are my own. Research support from the Thorne Fund and the Center for Analytic Economics is gratefully acknowledge.

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1 Introduction

In a stationary equilibrium, there exists a close relationship between the solutions to the sequential bargaining game (Rubinstein, 1982) and the Nash Bargaining Rule. See, for example, Binmore, 1987. In fact, either when the period of time between each round of negotiations in the sequential bargaining game becomes very small, or when the discount factor goes to unity, the two theories deliver the same predictions. In general, however, this equivalence breaks down when the agents interact in a nonstationary environment (Coles and Wright, 1998). For the case in which the parameters of the bargaining game are changing over time *smoothly*, Coles and Wright develop a simple formula that can be applied to obtain the limiting solution (as the time interval between moves approaches zero) to the alternating-offers bargaining process. Some of those parameters for the game may be endogenous to the aggregate model and determined, for example, in the market (prices, wages, and other variables determining the payoffs from negotiations, etc.).

Several well-known examples of economies with decentralized trading show that equilibria are often sensitive to the influence of extraneous variables that coordinate expectations. After the seminal work of Cass and Shell (1983) (see also Shell, 1977), it has been shown in the literature that rational expectations economies in which the usual Arrow-Debreu set of assumptions do not obtain are generally subject to the existence of sunspot equilibria.[1] Following this tradition, trade frictions have shown to be an important factor to motivate this sunspot-type of phenomena (see for example, Diamond (1982), Howitt and McAfee (1988) for a labor market model, and Wright (1994), Shi (1995) and Ennis (1998) for random matching models of money). Usually, bilateral bargaining constitutes an essential part of these models. Clearly, if one applies the same principles that guided the Coles-Wright analysis, then one should expect that the outcomes from negotiations depend on the extrinsic uncertainty that drives the dynamics of the aggregate economy. In a sense, from the point of view of the agent involved in the bilateral relationship where bargaining is taking place, the parameters of the game are not stationary. However, they do not necessarily follow a smooth path over time. Instead, the focus here will be in those cases where the sunspot signal triggers jumps from one set of equilibrium values to another as it switches

[1] This is yet another case for the so-called Philadelphia Pholk 'Theorem'. See Shell (1987).

states. This calls for an adaptation of the Coles-Wright result to make it applicable in this specific situation. I proceed with this task in the present paper.

Using a similar method to the one used by Coles and Wright, we obtain a formula that can be used to derive the outcome of bargaining negotiations when the state of the economy, for whatever reason, randomly switches back and forth between a finite number of possible states. This theory is then applied to a couple of well known models of decentralized exchange for which the equilibrium may be subject to extrinsic uncertainty (sunspots). Specifically, an economy with bilateral production matchings and trade externalities (*à la* Diamond, 1982) and a monetary random matching economy (*à la* Kiyotaki and Wright, 1989) with bargaining are studied in detail to better understand the effects that the new derivations can have on aggregate outcomes.

The two examples provide with interesting insights on the fundamental factors that determine the sunspots' influence (through forward looking bargaining) upon equilibrium outcomes. The underlying driving force for most of the findings in that part of the paper is that the relative position of the agents at the two sides of the negotiation determines the final importance of sunspot volatility. Accordingly, if the agents in the bilateral bargaining are identical in all respects, sunspots do not alter the surplus-splitting rules. Otherwise, sunspots do have important consequences over the bargaining power of the negotiators and require special consideration.

The paper is organized as follows. In the next section, I develop the general formula for the limiting case of sequential bargaining when the aggregate economy swithches randomly between two states. Also in Section 2, I discuss what I call the benchmark case - the simple splitting-the-cake problem. In Section 3, two examples are introduced: a labor market model with bargaining (that can be seen as a special case of the benchmark case), and a random matching model of money. Section 4 is reserved for concluding remarks.

2 The Bargaining Theory

2.1 General Rule

Consider the following economic problem. Two agents, 1 and 2, are bilaterally matched; out of this encounter there is some mutually beneficial surplus that can be produced; agents have to decide how to split that surplus between them. This is the typical bargaining situation studied by Nash and Rubinstein. Let $x \in \mathbb{R}$ represent a decision variable for the agents determining how the surplus is divided. Agent 1 has an instantaneous payoff function $u_1(x;\Theta)$, where Θ represent the set of parameters that influence, for example, the size of the total surplus to be distributed.[2] Agent 2's payoff function is given by $u_2(x;\Theta)$. Assume $u_i \in C^2$, $i = 1, 2$. Let u_1 be increasing and concave in x for every Θ. Also, u_2 is decreasing in x and concave. Agent i discounts the future at rate r_i, $i = 1, 2$. Agents derive zero utility from no trade as well as from obtaining no surplus out of the match. Hence, $x \in [\underline{x}, \overline{x}]$, where $x = \overline{x}$ corresponds to the situation where all the surplus goes to agent 1 and $x = \underline{x}$ when all the surplus goes to agent 2. Note that the values of Θ, $\underline{x}$, $\overline{x}$ may be changing through time.

The idea is to consider the solution to the *alternating offers* bargaining game (see Rubinstein, 1982) and then study the limit of this solution when the time period between offers goes to zero. If Θ is constant through time, then it is well known that the limit of the unique subgame perfect equilibrium outcome of the alternating offers game is the Nash solution with bargaining power and threat points that depend on the details of the specific game (Binmore, 1987). If Θ_t follows a smooth nonconstant path, then the Coles-Wright (1998) solution applies. However, we will consider here the case where Θ_t takes a finite number of different possible values and *jumps* among them according to states of the economy that are determined, for example (although not necessarily), by a sunspot variable.[3] In fact, the main interest will be in the market equilibrium of an economy where agents get paired through a matching process and bargain to split mutual benefits of that match[4]. In this sense, Θ_t may represent the state of the aggregate economy that for some reason determines the value of the match (see the examples in the next section).

The alternating offers bargaining procedure operates as follows. First,

[2]Note that these are not parameters of the utility function. They represent, in a way, the state of the economy where these agents interact.

[3]Coles and Wright (1998) assume that the partial derivative of the payoff function with respect to time is bounded. This need not be satisfied in the present set up. With the aggregate environment experiencing only discrete jumps over a finite number of possible states, derivatives are only evaluated at a finite number of points. In this manner, convergence of sequences of partial derivatives is not anymore an issue as it was in Coles and Wright.

[4]This is in the spirit of Rubinstein and Wolinsky (1985), although they allow for the possibility of exogenous breakdowns.

agent 1 makes an offer $y(t)$ that may depend on time because Θ_t does. Agent 2 either accepts or rejects the offer. If agent 2 accepts then the game ends and the payoffs vector is given by $[u_1(y(t);\Theta_t), u_2(y(t);\Theta_t)]$.[5] If agent 2 does not accept agent 1's offer, then a period Δ of time goes on and at time $t+\Delta$ agent 2 gets to make an offer $z(t+\Delta)$. Agent 1 accepts or rejects that offer, and the game goes on in that manner. Assume that Θ_t, which - as was said before- can be thought of as the aggregate state of the economy where these two agents interact, follows a stochastic dynamic process that switches back and forth between two possible values (the next section provides several examples of this phenomenon). Clearly, between t and $t+\Delta$, the economy may transit from one state to another. This may then result in a more favorable situation for one of the two agents who - aware of this possibility- will act accordingly when bargaining.

More specifically, assume the aggregate state of the economy depends on a bivariate random variable $s \in \{s_a, s_b\}$ that follows a Poisson process with transition rates given by $[\pi_{a,b}, \pi_{b,a}]$.

As is usual in this type of problem, there exist reservation values $[\widehat{x}_1^s, \widehat{x}_2^s]$ such that if the economy is in state s, agent 1 accepts an offer x from agent 2 whenever $x \geq \widehat{x}_1^s$ and agent 2 accepts an offer x from agent 1 whenever $x \leq \widehat{x}_2^s$.[6] Also, from the properties of the payoff functions the best offer of an agent is always the reservation value of her partner, i.e., $y(t) = \widehat{x}_2^s$ and $z(t) = \widehat{x}_1^s$.

The equilibrium reservation values in each state s satisfy the following two equations,

$$u_1(\widehat{x}_1^s;\Theta_s) = \frac{1}{1+r_1\Delta}\left[(1-\Delta\pi_{ss'})u_1(\widehat{x}_2^s;\Theta_s) + \Delta\pi_{ss'}u_1(\widehat{x}_2^{s'};\Theta_{s'})\right], \qquad (1)$$

$$u_2(\widehat{x}_2^s;\Theta_s) = \frac{1}{1+r_2\Delta}\left[(1-\Delta\pi_{ss'})u_2(\widehat{x}_1^s;\Theta_s) + \Delta\pi_{ss'}u_2(\widehat{x}_1^{s'};\Theta_{s'})\right], \qquad (2)$$

where $s,\ s' = s_a, s_b$ and $s \neq s'$. Clearly, $\widehat{x}_1^s$ and $\widehat{x}_2^s$ will be functions of Δ. To simplify notation, I choose not to write down the dependence on Δ explicitly but it should be kept in mind for the upcoming arguments.

[5] Actually, the equilibrium payoff in state Θ_s may depend on the payoff in state $\Theta_{s'}$ too, as it will be seen in the examples of the next section. However, we will keep this simplified notation for clarity of exposition.

[6] Only history independent strategies are considered in the equilibrium to be studied. Also note that I dropped the t argument from the offer functions because the only source of dynamics will be the switching states of the aggregate economy indicated with superscript.

Let $h^s(\Delta) \equiv \widehat{x}_1^s - \widehat{x}_2^s$. It is shown in **Appendix 6.1** that $h^s(\Delta) = O(\Delta)$ (i.e., $h^s(\Delta)/\Delta \to c \in \mathbb{R}$ as $\Delta \to 0$). Then, using the Taylor expansion of $u_i(\widehat{x}_1^s; \Theta_s)$, $i = 1, 2$, around $\widehat{x}_2^s$, from (1) and (2) one obtains,

$$\begin{aligned} r_1 u_1(\widehat{x}_2^s; \Theta_s) &= -(1 + r_1\Delta)\left[u_1'(\widehat{x}_2^s; \Theta_s)\frac{h^s(\Delta)}{\Delta} + \frac{o(\Delta)}{\Delta}\right] + \\ &\quad + \pi_{ss'}\left(u_1(\widehat{x}_2^{s'}; \Theta_{s'}) - u_1(\widehat{x}_2^s; \Theta_s)\right), \end{aligned} \tag{3}$$

$$\begin{aligned} r_2 u_2(\widehat{x}_2^s; \Theta_s) &= u_2'(\widehat{x}_2^s; \Theta_s)\frac{h^s(\Delta)}{\Delta} + \frac{o(\Delta)}{\Delta} + \\ &\quad + \pi_{ss'}\left(u_2(\widehat{x}_2^{s'}; \Theta_{s'}) - u_2(\widehat{x}_2^s; \Theta_s) + O(\Delta)\right). \end{aligned} \tag{4}$$

Then, after some substitutions and taking limits when $\Delta \to 0$,[7] the following result obtains.

Proposition 1 *When the state of the aggregate economy, Θ, shows stochastic dynamics over two possible states $[\Theta_{s_a}, \Theta_{s_b}]$, then $\{\widehat{x}^s(t),\ s = s_a, s_b\}$, the limiting (as $\Delta \to 0$) spliting rule from bargaining , satisfies the following equations:*

$$\begin{aligned} &\left[r_1 u_1(\widehat{x}^s; \Theta_s) + \pi_{ss'}\left(u_1(\widehat{x}^s; \Theta_s) - u_1(\widehat{x}^{s'}; \Theta_{s'})\right)\right]\frac{1}{u_1'(\widehat{x}^s; \Theta_s)} \\ = &\ \frac{1}{-u_2'(\widehat{x}^s; \Theta_s)}\left[r_2 u_2(\widehat{x}^s; \Theta_s) + \pi_{ss'}\left(u_2(\widehat{x}^s; \Theta_s) - u_2(\widehat{x}^{s'}; \Theta_{s'})\right)\right] \end{aligned} \tag{5}$$

for $s \neq s'$; $s, s' \in \{s_a, s_b\}$.

Remark 2 *One possible motivation for the dynamics imposed on Θ is the case where the aggregate economy uses a sunspot variable as a coordination devise between two feasible equilibrium situations (see the following section for some illustrations). In this case then, note that when $\Theta_{s_a} = \Theta_{s_b}$, i.e. when sunspots don't matter at the level of the aggregate economy, we have $\widehat{x}^{s_a} = \widehat{x}^{s_b}$ and equations (5) reduce to the standard Nash Bargaining solution.*

[7] Note that when $\Delta \to 0$ it becomes irrelevant who makes the first offer ($h(\Delta) \to 0$ as $\Delta \to 0$).

Remark 3 *Note that (1) and (2) form a system of four equations in four unknowns,* $\widehat{x}_1^{s_a}$, $\widehat{x}_2^{s_a}$, $\widehat{x}_1^{s_b}$, $\widehat{x}_2^{s_b}$. *Assuming that* $\widehat{x}_1^{s_a} > \widehat{x}_1^{s_b}$ *and* $\widehat{x}_2^{s_a} > \widehat{x}_2^{s_b}$ *(just a matter of labeling), if a solution to this (generally nonlinear) system exists, then* $\widehat{x}_1^{s_a} < \widehat{x}_2^{s_a}$ *and* $\widehat{x}_1^{s_b} < \widehat{x}_2^{s_b}$*, which yields an Immediate Trade Equilibrium (with no delay in the bargaining process). An important condition for this to happen is that the transition probabilities* $[\pi_{ss'}, \pi_{s's}]$ *are small enough.*

Remark 4 *Suppose there is an individual rationality constraint for agent 2 given by* $u_2(y(t);\Theta_t) \geq I(\Theta_t)$ *where* I *is a function of* Θ_t*. This constraint translates into* $\widehat{x} \in [0, \widetilde{x}]$*. Then, for the constraint to be binding in state* s*, we need*

$$u_2(\widetilde{x};\Theta_s) \geq \frac{1}{1+r_2\Delta}\left[(1-\Delta\pi_{ss'})u_2(\widetilde{x};\Theta_s) + \Delta\pi_{ss'}u_2(\widehat{x}_1^{s'};\Theta_{s'})\right] \tag{6}$$

in place of (2). And (5) becomes

$$\begin{aligned} &\left[r_1u_1(\widetilde{x};\Theta_s) + \pi_{ss'}\left(u_1(\widetilde{x};\Theta_s) - u_1(\widehat{x}^{s'};\Theta_{s'})\right)\right]\frac{1}{u_1'(\widetilde{x};\Theta_s)} \\ \leq\ & \frac{1}{-u_2'(\widetilde{x};\Theta_s)}\left[r_2u_2(\widetilde{x};\Theta_s) + \pi_{ss'}\left(u_2(\widetilde{x};\Theta_s) - u_2(\widehat{x}^{s'};\Theta_{s'})\right)\right]. \end{aligned} \tag{7}$$

In summary, if when $\widehat{x}^s = \widetilde{x}$ *we have that (7) holds, then we obtain that the solution for the bargaining problem is constrained.*

Remark 5 *Suppose that the state of the aggregate economy can take values in the finite set* $S = \{s_a, s_b, s_c, ...\}$*. In this more general case, equation* (5) *above, becomes*

$$\begin{aligned} &\left[r_1u_1(\widehat{x}^s;\Theta_s) + \sum_{s'\in S}\pi_{ss'}\left(u_1(\widehat{x}^s;\Theta_s) - u_1(\widehat{x}^{s'};\Theta_{s'})\right)\right]\frac{1}{u_1'(\widehat{x}^s;\Theta_s)} \\ =\ & \frac{1}{-u_2'(\widehat{x}^s;\Theta_s)}\left[r_2u_2(\widehat{x}^s;\Theta_s) + \sum_{s'\in S}\pi_{ss'}\left(u_2(\widehat{x}^s;\Theta_s) - u_2(\widehat{x}^{s'};\Theta_{s'})\right)\right], \end{aligned} \tag{8}$$

for $s, s' \in S$.

2.2 Benchmark Case: Splitting the Pie

One of the most studied examples of bargaining situations is the case where two agent with linear utility functions meet, produce a surplus (a pie) of size

P and then have to decide how to split it between the two (see for example Rubinstein (1982)). Let x be the part of the pie that goes to agent 1, and $P - x$ the corresponding share for agent 2. Consider a situation where the size of P depends on a random state variable taking two possible values (s_a or s_b) and following a Poisson process with transition rates $[\pi_{ab}, \pi_{ba}]$. Note that as it stands, the uncertainty in the present case is essentially *intrinsic*. This makes perfect sense because - as was said before- the formula derived in Section 1 is applicable to general situations (including, but not exclusively, the sunspots case). Moreover, in the next section it will be shown how the phenomenon described here can be closely related to a sunspot equilibrium of a particular aggregate economy. In the decentralized exchange economies that we shall study in Section 3, the sunspot effect is in a way exogenous to the match (once in the match, agents take as given that the economy where they interact is subject to sunspots effects).

It is easy to see how the problem just presented fits perfectly in the analysis of Section 2. The payoff functions are given by $u_1(x;\Theta_s) = x$ and $u_2(x;\Theta_s) = P^s - x$, $s = s_a, s_b$; and equations (5) are in this case,

$$r_1 x^s + \pi_{ss'}(x^s - x^{s'}) = r_2(P^s - x^s) + \pi_{ss'}\left[P^s - x^s - (P^{s'} - x^{s'})\right], \qquad (9)$$

for $s, s' = s_a, s_b$, and $s \neq s'$.

- **Result 1:** Suppose that $P^{s_a} = P^{s_b} = P$. Then, $x^{s_a} = x^{s_b} = [r_2/(r_1 + r_2)]P$ and if $r_1 = r_2$, then they will split the pie in halves. These are the usual results of traditional Nash Bargaining Theory.

- **Result 2:** Suppose that $P^{s_a} \neq P^{s_b}$ and $r_1 = r_2$. Then $x^{s_a} = P^{s_a}/2$ and $x^{s_b} = P^{s_b}/2$, i.e. in spite of the dynamics in the size of the pie, they still split it in halves (see Theorem 3 in Coles and Wright (1998) for an analogous result).

- **Result 3:** Suppose that $P^{s_a} \neq P^{s_b}$ and $r_1 \neq r_2$, then

$$x^{s_a} = \frac{r_2}{r_1 + r_2} P^{s_a} + \pi_{ab}\left[\frac{r_2 - \left(\frac{r_1+r_2}{2}\right)}{\left(\frac{r_1+r_2}{2}\right) + \pi_{ab} + \pi_{ba}}\right](P^{s_b} - P^{s_a}) \qquad (10)$$

and

$$x^{s_b} = \frac{r_2}{r_1 + r_2} P^{s_b} - \pi_{ba}\left[\frac{r_2 - \left(\frac{r_1+r_2}{2}\right)}{\left(\frac{r_1+r_2}{2}\right) + \pi_{ab} + \pi_{ba}}\right](P^{s_b} - P^{s_a}). \qquad (11)$$

If, for example, $P^{s_b} > P^{s_a}$ and $r_2 > r_1$, then $x^{s_a} > [r_2/(r_1 + r_2)]P^{s_a}$ and $x^{s_b} < [r_2/(r_1 + r_2)]P^{s_b}$. The equilibrium payoff of agent 1, who is relatively more patient, is higher in the "small pie" state because she would be willing to wait for the change in state relatively longer than agent 2 (this raises her reservation value, which is what she will end up getting in equilibrium). Similarly, when the economy is in state s_b, agent 1 will be eager to close a deal in the current situation: the risk of loss from the switch in states the following Δ-period is relatively more important for this agent (she is more patient, thus she cares relatively more about future losses).

From this analysis, one can see that when the discount rates differ among agents, the disparity in payoff is either accentuated by the *variability* in the size of the pie, as in the "small pie" state s_a (as $x^{s_a} > [r_2/(r_1 + r_2)]P^{s_a} > (1/2)P^{s_a}$), or dissipated, as in the "big pie" state s_b (as usually $(1/2)P^{s_b} < x^{s_b} < [r_2/(r_1 + r_2)]P^{s_b}$ when π_{ba} is relatively small). Put in the context of macro-economies, the prediction from this theory would be that one should expect more disparate 'surplus-spliting' conditions during recessions when bargaining is a substantial component of the transaction mechanisms operating in the economy.[8] The present discussion should serve as a preamble for the first example of the next section, which constitutes a special case of this problem, but embedded in what it would turn out to be a more comprehensive setup.

3 Application: The Sunspots Case

Bargaining situations are characteristic of decentralized exchange economies where agents get bilaterally matched to trade. Similarly, as it was said in the introduction, this type of economies are natural environments for the study of sunspot equilibria. In consequence, we find that relating these two phenomena with the intention of illustrating the use of the new concepts developed in the paper is a natural step.

In what follows, I will present a pair of examples of model economies where bargaining and sunspots are important elements for the analysis. The formula obtained in Proposition 1 will be used to represent the solution of the bargaining procedure and its consequences will be discussed to evaluate

[8] See more on this at the end of the first example on Section 3.

the relevance of explicitly considering the effect of sunspots on the bargaining outcomes.

The first application is based on an extensive literature on trade externalities developed after the seminal work by Peter Diamond on the possible macroeconomic consequences of Search Equilibrium analysis (see Diamond (1982), Howitt and McAfee (1988, 1992) and the references therein). Those models typically generate multiple equilibria and sunspot equilibria; however, the bargaining process is usually not explicitly modeled (although it is clearly implicit in the analysis; see Drazen, 1988).[9] The present paper, in a way, intends to start filling in this hole of the mentioned literature.

The second application is a random matching monetary economy in the spirit of Kiyotaki and Wright (1989). The second generation of this type of models have considered price determination through a bargaining procedure within each monetary match. One interesting result out of those papers is the possibility of multiple and sunspot monetary equilibria (see Shi (1995), Trejos and Wright (1995), and Ennis (1998)). Following the analysis of the previous section, it becomes apparent that to study sunspot equilibria in these models it is necessary to take into account the potential effects of extrinsic uncertainty on the bargaining outcomes. This is pursued in the second part of this section.

3.1 Wage Bargaining and Sunspots

3.1.1 The Model

The first example in this section constitutes a complete description of an economy where a large number of workers interact with firms to accomplish production and explicitly bargain over wages. One of the distinguishing characteristics of this economy will be the presence of the trade externality introduced by Diamond (1982). In fact, the present economy is a simplified version of the one in Howitt and McAfee (1988), but in which I analyze *explicitly* the bargaining process that determines wages. In this economy, the trade externality is the driving force for the existence of multiple equilibria produced by a typical coordination failure situation. This property of the

[9] Mortensen (1999), in a model with similar characteristics than the ones considered in this paper, uses explicit Nash bargainig with the bargaining power of agents changing through time following an exogenous rule associated with certain indicators of the aggregate state of the economy (e.g., labor market tightness).

economy is what allows for the possibility that sunspots affect equilibrium allocations. Although the model is stylized, it will illustrate how the bargaining rule obtained in Section 2 can be introduced in a specific model economy and demonstrate some of its crucial effects.

Consider an economy with a large number of identical firms and a large number of identical households. There are three tradable objects: output, homogeneous labor services, and money (it will be a pure accounting device). Firms' receipts are instantaneously transferred to their owners and workers and must be used for purchasing output from other firms during the current period. The market for output is perfectly competitive, but firms incur a transaction cost to operate in the market. I will consider an *iceberg type* transaction cost, i.e. it takes the form of output used up in the sales process. Hence, a firm employing n units of labor will have a net revenue of

$$R(n, \overline{n}) = [1 - \sigma(\overline{n})]f(n), \tag{12}$$

where $f(n)$ is the firm's production function and $\sigma(\overline{n})f(n)$ is the transaction cost. Assume $\sigma(\bullet)$ is a continuous decreasing function. The trade externality comes through $\sigma(\overline{n})$, which depends upon the aggregate employment (per firm) $\overline{n}$: the higher the general level of employment in the economy, the easier is to sell goods and therefore the lower is the transaction cost $\sigma(\overline{n})f(n)$.

At every moment in time, a firm can potentially get matched with a household according to a Poisson process with arrival rate β. After the match is formed, households work and get paid by the firm. They use the proceeds to buy goods for consumption. Additionally, they experience disutility from labor, $v(n)$.

Under this setup, workers and firms develop bilateral relationships when they get matched. The standard approach for this situation is to assume that they will bargain over the surplus from the match. Let x be the payment to the workers from the negotiations. Define V_i to be the value for agent i of being unmatched waiting for a potential arrival. It is not hard to show that $V_1(\overline{n}) = (\beta/r_1)[R(\overline{n}, \overline{n}) - x(\overline{n})]$ and $V_2(\overline{n}) = (\beta/r_2)[x(\overline{n}) - v(\overline{n})]$ in equilibrium -where r_j, $j = 1, 2$ are the time discount rates for firms and workers respectively, and $x(\overline{n})$ is the equilibrium wage bill when aggregate employment (per firm) is $\overline{n}$.

One well accepted bargaining procedure is the Nash bargaining solution. The predictions of this solution concept are equivalent to the limiting outcomes in the alternating offers Rubinstein game (Binmore, 1987). Then, assuming the firm and the worker Nash bargain over labor n and payroll x implies that (n, x) solves the following problem,

$$\max_{x,n}[R(n,\overline{n}) - x + V_1(\overline{n})]^{\theta}[x - v(n) + V_2(\overline{n})]^{1-\theta}$$
$$\text{subject to } \; x + V_2(\overline{n}) \geq v(n) \quad \text{and} \quad x \leq R(n,\overline{n}) + V_1(\overline{n})$$

From the first order conditions (assuming an interior solution),

$$x^* = (1-\theta)[R(n^*,\overline{n}) + V_1(\overline{n})] + \theta[v(n^*) - V_2(\overline{n})], \tag{13}$$

and,

$$R_1(n^*,\overline{n}) \equiv (1-\sigma(\overline{n}))f'(n^*) = v'(n^*). \tag{14}$$

Note that, as expected, the Nash solution is "efficient" (given $\overline{n}$, n^* maximizes net surplus $R(n,\overline{n})+V_1(\overline{n})-v(n)+V_2(\overline{n})$). The weight θ represent the relative bargaining power of firms. If one thinks of Nash bargaining as a simplified approximation to the solution of the Rubinstein game, then $\theta/(1-\theta)$ is given by the ratio of workers' and firms' discount rates (r_2/r_1). Considering the net payoff x_N and z_N, for workers and firms respectively, we see that

$$x_N^* = x^* - v(n^*) + V_2 = (1-\theta)[R(n^*,\overline{n}) + V_1 - v(n^*) + V_2] \tag{15}$$

and,

$$z_N^* = R(n^*,\overline{n}) + V_1 - x^* = \theta[R(n^*,\overline{n}) + V_1 - v(n^*) + V_2]; \tag{16}$$

i.e., θ of the surplus from the match goes to the firm and $(1-\theta)$ to the workers. If both have the same discount factors(i.e. $\theta = 1/2$), it can be shown that $x^* = [R(n^*,\overline{n}) + v(n^*)]/2$ when $\overline{n} = n^*$, and they split *net* surplus in half (see **Result 1** in the Benchmark Case for a direct analogy). This result will become important further ahead in the paper when we study the sunspot equilibrium with equal discount factors. Finally, note that n^* is independent of the spliting rule x^*.

Definition 6 *A certainty equilibrium for this economy is given by* $\{(\overline{n}, n^*, x^*), (V_1, V_2)\}$ *such that:* ***1)*** (n^*, x^*) *satisfy (13) and (14);* ***2)*** $\overline{n} = n^*$*;* ***3)*** $V_1 = (\beta/r_1)[R(\overline{n},\overline{n}) - x^*]$ *and* $V_2 = (\beta/r_2)[x^* - v(\overline{n})]$.

It should be apparent that the trade externality can generate multiple equilibria in this economy. In fact, the higher the aggregate employment in the economy, the easier it becomes to sell the produced goods and therefore the higher is the marginal productivity of labor. This implies higher optimum levels of employment at the firm level and possibly a "high" employment equilibrium (n_H, n_H, x_H). Inversely, for low levels of $\overline{n}$, the marginal productivity of labor is low and this can result in a low actual employment equilibrium levels n_L (see **Figure 1**). Clearly, where the economy is in any period depends exclusively on where the agents are coordinated to be.

3.1.2 Sunspot Equilibrium

Assume that there are multiple certainty equilibria and that expectations over the aggregate employment level in the economy follow a bivariate Poisson process with transitions rates $\{\pi_{LH}, \pi_{HL}\}$. In other words, the agents in the economy coordinate themselves in either (low or high employment) certainty equilibria according to a bivariate *sunspot* random variable. Assume also that no matter which state the economy is in, when a firm and a household are matched, they produce. However, if one considers the limiting solution of the alternating offers bargaining process, the outcome of the negotiations now differs (in general) from that in a certainty equilibrium.[10] At the moment of the negotiations, agents take into account that the economy might switch at any time to the other state, affecting the payoffs obtainable from the match. There is in fact another important effect of sunspots on the equilibrium quantities in the economy; the value of being unmatched waiting for an arrival V_i also depend on the dynamic properties of the aggregate economy. However, in this paper we shall be specially interested in the first of the mentioned effects.[11]

[10] Note that if one assumes *myopic* behavior by the agent in a match, then a fix exogenous rule for spliting the surplus obtains. This is what it is done in much of the previous literature (see Drazen, 1988). Under that assumption sunspot equilibria of this model constitute nothing more than a trivial randomization over certainty equilibria. However, endogenizing the spliting rule through explicit *sophisticated* bargaining (as in Section 2) will be shown to derive in new possible observable equilibrium outcomes. See Shell (1987) for a general discussion on the importance of this issue for the study of sunspot equilibrium.

[11] The value functions for firms and households when sunspots matter are given by the system

$$V_{1i} = \frac{\beta}{r_1}\left[(R_i(n_i, n_i) - x_i) + \pi_{ij}(V_{1j} - V_{1i})\right],$$

Let superscript s denote quantities on the sunspot equilibrium. Clearly, production efficiency still obtains. Therefore, n_L^s and n_H^s solve versions of (14).[12] Using the general proposition from Section 2 of the paper one can determine the payroll x^s in each state. The version of equation (5) in the current setup is,

$$r_1[R(n_i^s, \overline{n}_i^s) - x_i^s + V_{1i}^s] + \pi_{ij}[R(n_i^s, \overline{n}_i^s) - x_i^s + V_{1i}^s - \left(R(n_j^s, \overline{n}_j^s) - x_j^s + V_{1j}^s\right)] =$$

$$= r_2(x_i^s - v(n_i^s) + V_{2i}^s) + \pi_{ij}[x_i^s - v(n_i^s) + V_{2i}^s - \left(x_j^s - v(n_j^s) + V_{2j}^s\right)], \quad (17)$$

where $i, j = L, H$; $i \neq j$ and n_i^s solves $R_1(n_i^s, \overline{n}_i^s) = v'(n_i^s)$ (efficient bargaining). Let $\theta = r_2/(r_1 + r_2)$. The equations determining the equilibrium values of x_i^s are

$$x_i^s = (1-\theta)\left[R(n_i^s, n_i^s) + V_{1i}^s\right] + \theta\left[v(n_i^s) - V_{2i}^s\right] +$$

$$+\Phi\left[R(n_j^s, \overline{n}_j^s) + V_{1j}^s - \left(v(n_j^s) - V_{2j}^s\right) - \left(R(n_i^s, \overline{n}_i^s) + V_{1i}^s - \left(v(n_i^s) - V_{2i}^s\right)\right)\right], \quad (18)$$

for $i, j = L, H$; $i \neq j$, $\Phi = [\pi_{ij}(1-2\theta)]/[r_1 + r_2 + 2(\pi_{ij} + \pi_{ij})]$.

These equations should be compared with those in (13). Note that when the discount rates of firms and workers are the same the bargaining power index $\theta = 1/2$, $\Phi = 0$ and the third term in the sum disappears. Further calculations shows that $x_i^s = [R(n_i^s, n_i^s) + v(n_i^s)]/2$, and the solution is 'immune' to the existence of sunspots (both effects due to sunspots, the change on the value functions and the change in the bargaining rule, wash out when $r_1 = r_2$). In any other case, the existence of sunspot fluctuations affects the spliting rule from bargaining. As was said before, the net effect is the result of the interaction of two channels through which sunspots influence the state

$$V_{2i} = \frac{\beta}{r_2}\left[(x_i - v_i(n_i)) + \pi_{ij}(V_{2j} - V_{2i})\right],$$

where $i, j = L, H$; $i \neq j$.

[12] It can easily be shown that the equilibrium payoff for both the firm and the workers is increasing in net surplus $R(n, n) - v(n)$. Therefore, the partners in the match will agree to maximize that surplus and production efficiency will obtain.

of the match. On one hand, the value functions are different according to the expected dynamics of the overall macro economy. On the other hand, and most important to this paper, is the effect of sunspots over the process of negotiations.[13] This is represented by the last term in expression (18). In particular, if $r_2 < r_1$, i.e. if workers are more impatient than managers and if the total surplus from the match is bigger in the high employment equilibrium situation H, then x_L^s tends to be lower in equilibrium. The more patient side in the negotiation is more willing to delay during bad times and this increases its bargaining power. One could say that the theory presented here predicts a tendency to lower payrolls as a proportion of total revenue during a slump (especially when there exists a perception among agents that the economy would, with considerable probability, recover from the current depression). Similarly, x_H^s tends to be higher. During good times, firm discount less future losses and they are eager to close a deal before the economy switches to the bad state, sacrificing in this way some of their bargaining power. Note finally that this two implications (lower x_L^s and higher x_H^s) tend to increase the variance of workers payoff under the effect of sunspots. The wage bill is a lower proportion of a low surplus during bad times and a higher proportion of a high surplus during good times. However, our simplified structure implies that workers are risk neutral in income and hence, that this extra variability does not derive into additional welfare loses for the household.

3.2 Monetary Equilibrium, Bargaining and Sunspots

The last example presented in this section consists of a random matching economy with money *à la* Kiyotaki and Wright (1991). In fact, the present example should be regarded as an extension of Trejos and Wright (1995) analysis to the case of sunspot equilibrium. Although Shi (1995) already proved (using arguments of continuity) the existence of sunspot equilibria in this model, their actual characteristics are somewhat unexplored (see Ennis (1998) for a characterization of some of the Steady State properties of these sunspot equilibria). This example shows how equation (5) can help us understand some of the effects that excess volatility may have in the functioning

[13] It is worth noting that if one includes a risk of breakdown in the bargaining arrangement, i.e. a threat point equal to the value of being unmatched waiting for an arrival, then the value functions dropped out of the formula for x and only this effect of sunspots over the process of negotiations remains.

of this type of monetary economy.

The model follows closely that in Trejos and Wright (1995) (see also Shi (1995) and Ennis (1998)).[14] Time is continuous. A unit measure of infinitely-lived agents get matched every period according to a Poisson process. There is specialization in production and consumption (no agent consumes what she produces). With probability y, a double coincidence of wants takes place between two matched agents. Agents derive utility $u(q)$ from consumption and disutility $c(q)$ from producing. Money is indivisible and agents have either 0 or 1 unit of it at every point in time. Goods are non-storable and divisible. After production, agents have to consume to be able to produce again. Let $M \in (0,1)$ be the measure of agents holding one unit of money at the initial time. These assumptions imply the invariance of the distribution of money holdings: at every moment in time there is a fraction M of agents holding money and a fraction $(1-M)$ of agents with a production opportunity.

There are two classes of matches that originate trade: a monetary match and a barter match. In a *monetary match*, an agent with one unit of money meets an agent with an opportunity to produce the good that the former wants. They then decide how much of the good will be exchanged for the unit of money through a bargaining procedure. In a *barter match* none of the agents has money but there is a double coincidence of wants (one agent wants what the other can produce and vice versa). In this case, traded quantities are also determined through bargaining.

Assume that $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all $q \geq 0$ and that $c(0) = 0$, $c'(q) > 0$ and $c''(q) \geq 0$ for $q \geq 0$ (without loss of generality, I will assume $c(q) = q$ when convenient). Also assume there is a $\widehat{q} > 0$ such that $u(\widehat{q}) = c(\widehat{q})$.

Let V_0 be the value of being a producer (prior to a match) and V_1 the value of holding money (also prior to a match). In a steady-state (non-sunspots) monetary equilibrium with $q \leq \widehat{q}$, (V_0, V_1) will satisfy

$$rV_o = \Omega + M \ (V_1 - V_0 - c(Q)) \tag{19}$$

and

$$rV_1 = (1-M) \ [u(Q) + V_0 - V_1], \tag{20}$$

[14] For a good discussion of the general assumptions underlying the structure of the economy, see Wallace (1997).

where $\Omega = (1-M)\ y\ [u(q^*)-c(q^*)]$ is the barter payoff $(u'(q^*) = c'(q^*))$. Note that from these two equations one can solve for (V_1, V_2) as functions of Q, the quantity of goods for which a unit of money is exchanged in equilibrium.

When one takes the Nash solution as the bargaining rule for both types of matches, it is well known (see Shi (1995) and Trejos and Wright (1995)) that this model presents two alternative monetary equilibria, a high price $(1/Q^l)$ constrained equilibrium and a low price unconstrained equilibrium.[15]

In this paper, however, I am interested in studying sunspot equilibria and the implementation of the bargaining rule introduced in Section 2. Shi (1995) showed the existence of sunspot equilibria in this type of economy following ideas first developed in Wright (1994). Assume the economy switches from the high to the low price state, and back, following a Poisson process with transition rates $\{\pi_{ss'}, \pi_{s's}\}$, where s (s') indicates that the economy is in the low (high) price state.

In principle, for a sunspot equilibrium, the value of being a seller or of being a buyer will depend on the current state of the economy. Hence, one can show that the value functions are now given by

$$rV_o^k = \Omega^k + M(V_1^k - V_o^k - c(Q^k)) + \pi_{kh}(V_o^h - V_o^k), \quad (24)$$

$$rV_1^k = (1-M)\ [u(Q^k) + V_0^k - V_1^k] + \pi_{kh}(V_1^h - V_1^k), \quad (25)$$

for $k, h = s, s'$, $k \neq h$, and where Ω^k comes from the payoff in a barter match in state k. Again note that these value functions are in fact functions of $(Q^s, Q^{s'})$, the inverse of the state contingent equilibrium price level of the aggregate economy (outside the specific match).

When agents meet in a mutually beneficial match (either barter or monetary), they will bargain over production. As explained in Section 2, these

[15] The Nash bargaining problem in a monetary match is

$$\max[V_0(Q) + u(q)][V_1(Q) - c(q)] \quad (21)$$

subject to

$$V_1(Q) - c(q) \geq V_0(Q) \quad (22)$$

$$V_0(Q) + u(q) \geq V_1(Q) \quad (23)$$

where Q is the quantity exchanged in monetary trades that predominates in the aggregate economy, and q is the quantity to be exchanged in this particular match. Restrictions (22) and (23) are individual rationality constraints for sellers and buyers, respectively. In the high price constrained equilibrium, restriction (22) is binding.

negotiations will be influenced by the fact that agents know that the economy is switching states over time and that the current state is probably only temporary.

Consider first a barter match. Define

$$J_i^s = V_0^s - V_0^{s'} + u(q_i^s) - u(q_i^{s'}) - (c(q_j^s) - c(q_j^{s'})), \tag{26}$$

for $i, j = 1, 2$; $i \neq j$ and q_i as the quantity agent i acquires from the match. In this case, equation (5) in state s takes the form

$$[V_0^s + u(q_i^s) - c(q_j^s) + \frac{\pi_{ss'}}{r} J_i^s] \frac{1}{u'(q_i^s)} = \frac{1}{c'(q_i^s)} [V_0^s + u(q_j^s) - c(q_i^s) + \frac{\pi_{ss'}}{r} J_j^s], \tag{27}$$

for $i, j = 1, 2$; $i \neq j$. A similar pair of equations determine the quantities traded in a barter match when the state of the economy is s'.

Claim 7 *For small enough transition rates* $[\pi_{ss'}, \pi_{s's}]$*, barter quantities traded are independent of sunspots.*

Proof. See **Appendix 6.2**.[16] ■

From the above Claim one can conclude that $\Omega^s = \Omega^{s'} = (1 - M)\ y\ [u(q^*) - c(q^*)]$, as in the *non*sunspots equilibria.

For the monetary match, agents bargain only over the quantity of the good that will be changed for the indivisible unit of money in the trade arrangement. Then, equation (5) in this case takes the form

$$[V_0^k + u(q^k) + \frac{\pi_{kl}}{r} \left(V_0^k + u(q^k) - V_0^l - u(q^l)\right)] \frac{1}{u'(q^k)} =$$

$$= \frac{1}{c'(q^k)} [V_1^k - c(q^k) + \frac{\pi_{kl}}{r} \left(V_1^k - c(q^k) - V_1^l + c(q^l)\right)], \tag{28}$$

$k, l = s, s'; k \neq l$. Therefore, the results of the bargaining procedure will be given by the solution to (28) subject to $V_1^k - c(q^k) \geq V_0^k$ and $V_0^k + u(q^k) \geq V_1^k$, $k = s, s'$. It can be easily shown that the latter constraint is binding only

[16] At the intuitive level, this turns out to be a natural result. Even when sunspots matter, if two agents meet with a double coincidence of wants, they constitute identical sides in a negotiation process and one would expect them to obtain identical payoff.

when the former is. So, in order to track a solution to the bargaining problem only the sellers' constraint (the agent with the production opportunity) is relevant. Finally, note that a version of (7) holds in the constrained situation (see the third **Remark** at the end of Section 2).

Define $\Delta_j^{s's} \equiv V_j^{s'} - V_j^s = -\Delta_j^{ss'}$, $j = 0, 1$ (these are functions of the equilibrium price levels $(Q^s, Q^{s'})$) and $T(Q) = [(r + M)(1 - M)u(Q) - \Phi c(Q)]u'(Q) - [\Phi u(Q) - M(r+1-M)c(Q)]c'(Q)$ with $\Phi = r(1+r) + M(1-M)$. Consider the following system of equations (a reformulation of the equilibrium versions for (28)),

$$\begin{aligned} T_k^*(Q^k, Q^l) \; = \; & T(Q^k) + [(1-M)u'(Q^k) - (1+r-M)c'(Q^k)](\Omega + \pi_{kl}\Delta_0^{lk}) + \\ & +[(r+M)u'(Q^k) - Mc'(Q^k)]\pi_{kl}\Delta_1^{lk} + \\ & +[u'(Q^k)\left(c(Q^l) - c(Q^k) - \Delta_1^{lk}\right) - \\ & \quad - c'(Q^k)\left(u(Q^k) - u(Q^l) - \Delta_0^{lk}\right)](1+r)\pi_{kl} \end{aligned} \tag{29}$$

with $k, l = s, s'; k \neq l$. When there are no sunspots and no barter possibilities $(\Omega = 0)$ the equilibrium values of Q are given by solutions to $T(\widehat{Q}) = 0$ (there is a unique monetary equilibrium in this case, see Shi (1995) and Trejos and Wright (1995)). In the case with barter $(\Omega > 0)$ and no sunspots, there are two monetary equilibria, one low price unconstrained equilibrium that solves $T_k^*(Q, Q) = 0$, and a high price constrained equilibrium that satisfies the seller's rationality constraint with equality. Clearly, for values of $\{\pi_{ss'}, \pi_{s's}\}$ small enough, a sunspot equilibrium can exist switching from the constrained to the unconstrained equilibrium (this is essentially the result of the existence proof in Shi (1995)). When agents act myopically during the bargaining process (the traditional Nash solution applies), the last term in the RHS of equation (29) disappears and the rest of the analysis proceeds in the same manner. In general, one would like to identify in this last term the same type of effect that we observed in the two previous examples when the 'sophisticated' formulae (5) were used to obtain the bargaining outcomes. In this model, one should expect that in the low price state s, the agent holding a unit of money in the match will be eager to close the deal. Therefore, she would get relatively lower quantities in equilibrium (relatively smaller Q^s than when using the myopic rule). These would require the last term in (29) to be negative. Although this cannot be shown in general, Q^s being close enough to q^* will guaranty the result.[17] In the following example, we present

[17]The buyer fear that with certain probability the economy will switch to the low quan-

a situation where 'sophisticated' bargaining does lower the value of money in the low price state (as it would be expected).

Example 8 *Let* $u(Q) = 2Q^{\frac{1}{2}}$, $M = \frac{1}{2}$, $y = 0.2$ *and,* $r = 0.01$. *Also let* $\pi_{ss'} = \pi_{s's} = 0.05$. *Note that* $q^* = 1$ *and, therefore,* $\Omega = 0.1$. *In this economy, there exist two* non*sunspot monetary equilibria, with the equilibrium quantities for the monetary matches given by* $Q_l = 0.011172$ *in the constrained equilibrium and* $Q_h = 0.9051$ *in the unconstrained one. Note that both quantities are* ex-post *Pareto inefficient because* $q_0^* = \arg\max_Q V_o(Q,Q) = 0.9611$.[18] *There is also a sunspot equilibrium with* $\left(Q^s, Q^{s'}\right) = (0.6365, 0.002880)$. *The last term of (29) in the low price state s is given by*

$$[u^{'}(Q^s)\left(c(Q^{s'}) - c(Q^s) - \Delta_1^{s's}\right) - $$
$$- c^{'}(Q^s)\left(u(Q^s) - u(Q^{s'}) - \Delta_0^{s's}\right)](1+r)\pi_{ss'} = -0.0360337,$$

negative, as conjectured before. The quantities for the sunspot equilibrium with myopic bargaining are $\left(\widetilde{Q}^s, \widetilde{Q}^{s'}\right) = (0.8582, 0.001742)$, *which reflects the fact that the buyer obtains lower quantities in state s if she recognizes the possibility that the sunspot state might change while she is already engaged in the bargaining process.*

As a final comment, note that the nature of the system given by equations (29) opens up the possibility for other sunspot equilibria.[19]

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tity state s', lowering the value of money and causing her a capital loss. This press for the quantities traded to go down. However, under certain conditions, the seller also experience a net loss from a switch to state s', because to get any positive payoff she has to become a buyer, and when the economy switches to the low quantity state, this potential future payoff also reduces. However, note that this type of loss is suffered by both sellers and buyers, and then it is their relative weight, given by the values of $u'(Q^s)$ and $c'(Q^s)$, what determines the effect over quantities.

[18] For the implications of this see Ennis (1998).

[19] Perhaps this potentially highly nonlinear system in the space $(Q^s, Q^{s'})$ has, for example, three solutions (it is reflexive to the 45^o line) in the area for unconstrained quantities. Then one can construct a sunspot equilibrium with two of the unconstrained quantities. Though this is yet to be studied, the example just discussed shows that for this to happen the utility and cost functions have to have certain specific nonlinerarity properties (not present in that example).

4 Conclusion

I develop here a formula for the limiting solution to the alternating-offers bargaining game (as the time interval between offers goes to zero) in an economy subject to a specific type of stochastic dynamics. The aggregate state variable follows a Poisson process defined over a finite set of possible values. The formula is relatively simple and intuitive. Although this "sophisticated" solution has strong similarities with the traditional Nash bargaining, there are important differences. Mainly, we have that as agents anticipate the switch of states, they modify their reservation values for the closure of a deal during negotiation. This in turn alters the final outcome from the game.

To suggest the broad applicability of these results, I present a set of examples of economies for which this bargaining rule applies. In particular, a great deal of effort is dedicated to demonstrate the implications of this novel solution concept in decentralized exchange economies in which aggregate variables are not sticky in that they can *jump* while *sunspots* realizations serve to coordinate agents among possible equilibrium outcomes. It is clear from the exposition that sunspot equilibrium is only one of the many possible applications of this (sophisticated) bargaining solution concept. In fact, shocks to fundamentals that follow the specified stochastic dynamics can be handle -with a slight modification in our formula- to determine the equilibrium bargaining outcomes.

As a first example of how important the new bargaining solution may be we present a benchmark case, the traditional 'splitting-the-cake' problem. In that case, when the size of the cake is relatively small and there is a given probability that it will get larger in the near future, the impatient agent gets a smaller share from negotiations. This example is primarily partial equilibrium and the uncertainty over the size of the cake is exogenously assumed (and in a way, purely *intrinsic*).

In the third section of the paper we present a pair of fully specified economies where agents get matched and bargain over splitting a surplus that they jointly generate. For the first application, I introduced a complete economy with bilateral production matching and wage bargaining. In this economy, a trade externality generates the possibility of sunspot equilibria due to a coordination failure. One of the main ideas illustrated by this application is that the effect of sunspots over bargaining strongly depends on the agents' relative positions in the match. If both agents are symmetric in other ways, then differences in discount factors play a very important role. In particular, when the discount rates are the same (so that agents have equivalent bargaining power), the bargaining outcomes are immune to sunspots and the sunspot equilibria are trivial randomizations over certainty equilibria. However, when the discount rates differ, similar conclusions to those in the 'spliting-the-cake' problem are reached. For example, if workers are more impatient than entrepreneurs, one predicts that lower wage bills as a proportion of income obtain during slumps in anticipation of sunspot dynamics.

When agents have asymmetric positions in the bargaining match, the implications of the theory are less apparent, but still consequential. The second application in the paper is an attempt to handle one of these cases. It consists of a monetary random matching economy with bargaining and sunspots. The bargaining solution method proposed in the paper is applied to this example in an effort to shed new light on the possible effects upon state prices of the existence of sunspot fluctuations. Two types of matches need consideration in this case, barter matches (double-coincidence of wants and no money) and monetary matches (single coincidence of wants with one sided money). Again, the players' relative position for negotiation is determinant. It is shown that in the barter match, where agents have very symmetric positions, sunspots do not alter the bargaining solution. However, in monetary matches agents are fundamentally different (one is a seller and the other is a buyer), and this situation, the value of money in the low-price equilibrium outcome is reduced by the consideration of 'sophisticated' forward-looking bargainers (due to potential capital losses associated with delays).

5 References

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6 Appendix

6.1.- In what follows, it is shown that $h^s(\Delta) \equiv \widehat{x}_1^s(\Delta) - \widehat{x}_2^s(\Delta)$ converges to zero at rate Δ as Δ approaches zero. First note that from (1),

$$u_1(\widehat{x}_1^s;\Theta_s) - u_1(\widehat{x}_2^s;\Theta_s) = -\Delta r_1 u_1(\widehat{x}_1^s;\Theta_s) + \Delta\pi_{ss'}[u_1(\widehat{x}_2^{s'};\Theta_{s'}) - u_1(\widehat{x}_2^s;\Theta_s)]. \tag{30}$$

Clearly, from this expression, we have

$$u_1(\widehat{x}_1^s;\Theta_s) - u_1(\widehat{x}_2^s;\Theta_s) \to 0 \tag{31}$$

as $\Delta \to 0$. Since $u_1(\bullet;\Theta_s)$ is continuous and strictly increasing, this implies that we have $\widehat{x}_1^s(\Delta) \to \widehat{x}_2^s(\Delta)$. As a consequence, $h^s(\Delta) \to 0$ as $\Delta \to 0$. It is not difficult to see that $\widehat{x}_i^s$ converges ($i = 1, 2$ and $s = s_a, s_b$). Then, by the continuity of u_1 again,

$$\begin{aligned} \lim_{\Delta\to 0} \frac{u_1(\widehat{x}_1^s;\Theta_s) - u_1(\widehat{x}_2^s;\Theta_s)}{\Delta} &= -r_1 u_1(\widehat{x}^s;\Theta_s) + \pi_{ss'}[u_1(\widehat{x}^{s'};\Theta_{s'}) - u_1(\widehat{x}^s;\Theta_s)] \\ &= \gamma_1^s \in \mathbb{R}. \end{aligned} \tag{32}$$

We also know that

$$\lim_{h^s(\Delta)\to 0} \frac{u_1(\widehat{x}_2^s + h^s(\Delta);\Theta_s) - u_1(\widehat{x}_2^s;\Theta_s)}{h^s(\Delta)} = \gamma_2^s \in \mathbb{R} \tag{33}$$

because $u_1(\bullet;\Theta_s)$ is differentiable. Finally, since $h^s(\Delta)$ is a continuous function of Δ, we can write

$$\lim_{\Delta\to 0} \frac{u_1(\widehat{x}_2^s + h^s(\Delta);\Theta_s) - u_1(\widehat{x}_2^s;\Theta_s)}{\Delta}\frac{\Delta}{h^s(\Delta)} = \gamma_2^s. \tag{34}$$

Substituting (32) in (34), we get

$$\gamma_1^s \lim_{\Delta\to 0} \frac{\Delta}{h^s(\Delta)} = \gamma_2^s, \tag{35}$$

which says that $h^s(\Delta) = O(\Delta)$.

6.2.-

Claim 9 *For small enough transition rates* $\{\pi_{ss'}, \pi_{s's}\}$*, barter quantities traded are sunspot independent.*

Proof. Let q^* solve $u'(q^*)/c'(q^*) = 1$. Note that if $q_1^k = q_2^k$, $k = s, s'$, then $J_i = 0$, $i = 1, 2$, and equations (27) become the traditional Nash bargaining rule with the unique solution q^*. Then, it suffices to show that we have $q_1^k = q_2^k$ with $k = s, s'$ and small enough transition rates. Suppose not, suppose without loss of generality that we have $q_1^k < q_2^k$ for some k and every possible $\pi_{kk'}$. From (27) we have,

$$\frac{u'(q_1^k)}{c'(q_1^k)} = \frac{c'(q_2^k)}{u'(q_2^k)} = \xi. \tag{36}$$

Now, since $q_1^k < q_2^k$, $u(q_1^k) - c(q_2^k) < u(q_2^k) - c(q_1^k)$, and we have for small enough $\pi_{kk'}$ that $u(q_1^k) - c(q_2^k) + (\pi_{kk'}/r)J_1^k < u(q_2^k) - c(q_1^k) + (\pi_{kk'}/r)J_2^k$ then we have $\xi < 1$. Also, since $u'(q_1^k)/c'(q_1^k) = \xi < 1$ implies $q_1^k > q^*$ and, since $c'(q_2^k)/u'(q_2^k) = \xi < 1$ implies $q_2^k < q^*$, we have that $q_1^k > q_2^k$; which is a contradiction. Hence $q_1^k = q_2^k = q^*$ for $k = s, s'$, which means that barter trades are independent of sunspots. ■

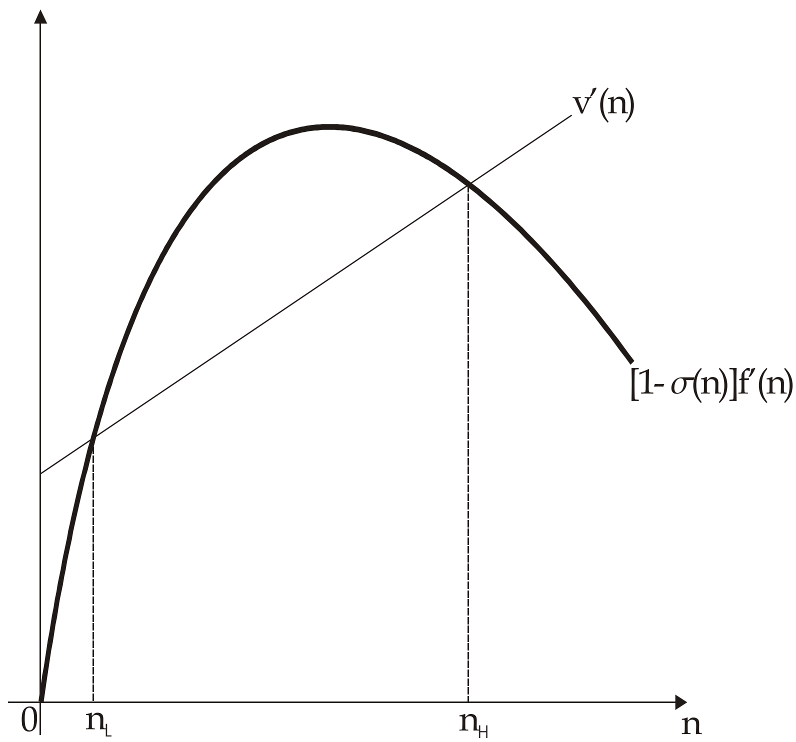

Figure 1: