CAE Working Paper #03-02

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January 2003

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Abstract

It has been argued by Turunen-Red and Woodland that, on only mild empirical assumptions, any system of non-distorting international transfers can be replaced by an "equivalent set" of distorting national tariffs without disturbing the international allocation of resources. This remarkable claim is disputed on the ground that the assumptions required for equivalence are not mild, that in particular they rule out quite plausible models of Arrow-Debreu type, and on the further ground that, in a context of non-uniqueness, a set of tariffs which supports an initial transfer-ridden equilibrium might also support several quite different equilibria. An alternative proposition is proposed.

Keywords: Lumpsum redistribution, non-lumpsum redistribution, equivalent tariffs, Motzkin's theorem
JEL Classification Numbers: F10, F11

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1. Introduction

Consider a unique world-wide transfer-ridden but otherwise Arrow-Debreu free-trade equilibrium, which will be referred to as the *initial equilibrium*. It has been claimed recently that, subject to a "mild restriction" on the matrix of equilibrium net exports (each column composed of the net exports of a single country), the transfers can be replaced by a set of *equivalent tariff* vectors, one vector for each country, such that there exists a new equilibrium, tariff-ridden but transfer-free, with new world prices but the initial domestic prices, with the initial matrix of net exports and with the initial vector of national utilities; see Turunen-Red and Woodland (1999). Subject to the "mild restriction", the international redistribution of income effected by lumpsum and non-distorting international transfers can be equally-well effected by carefully-chosen distorting taxes on international trade.[1]

The claim is quite remarkable. For, in tandem with the Second Welfare Theorem, it implies that any improvement in technology or in endowments, in whatever country, can be converted into a world-wide Pareto-improvement either by a cooperative program of international aid or by the cooperative reform of tariffs, the program of aid driving the world economy along its new contract locus to a Pareto-superior initial equilibrium and then giving way to the reformed tariffs. The claim is also surprising for it comes hard on the heels of a considerable debate concerning the relative merits of lumpsum and non-lumpsum compensation of those who are harmed by the introduction of free international trade. That debate had been provoked by the claim of Dixit and Norman (1980) that if economy-wide gains from free trade can be assured by a scheme of intra-national lumpsum compensating transfers then they can also be assured by a set of carefully chosen commodity taxes. In the course of that debate it emerged that the equivalence of lumpsum and non-lumpsum compensation holds only under special conditions; see Kemp and Wan (1986, 1993, 1999), Dixit and Norman (1986), Hammond and Sempere (1995) and Wan (1997).

In the present paper it will be shown by example that the "mild restriction" imposed by Turunen-Red and Woodland rules out economies with no extravagant (non-Arrow-Debreu) features but for which, nevertheless, equivalent tariff vectors do not exist. It will be further noted that (*a*) tariffs that support an initial transfer-ridden allocation might also support disparate additional allocations and that (*b*) the analytical method adopted by Turunen-Red and Woodland, based as it is

on Motzkin's theorem of the alternative, excludes from consideration all of the additional allocations, implying that, in a context of non-uniqueness, their method is inappropriate.

More constructively it will be shown that, under plausible assumptions concerning the relative numbers of countries, traded commodities and primary factors of production, there is no presumption for or against tariff equivalence.

2. The Turunen-Red-Woodland proposition

Let us denote by $\boldsymbol{X}$ the matrix of net exports in an initial free-trading but transfer-ridden equilibrium. From Motzkin's theorem of the alternative, *either* (*a*) the set of inequalities $\boldsymbol{X\lambda>0}$ has a solution *or* (*b*) there is a semi-positive solution to the system of equations $\boldsymbol{pX=0}$; see Motzkin (1936) or Mangasarian (1969). Turunen-Red and Woodland assume that (*a*) is not satisfied by $\boldsymbol{X}$; this is the "mild restriction" embodied in their Assumption (*A*). On the basis of that assumption, Turunen-Red and Woodland can be sure that there exists a semi-positive $\boldsymbol{p}$, say $\boldsymbol{p}^\dagger$, such that $\boldsymbol{p}^\dagger \boldsymbol{X}=\boldsymbol{0}$.

Turunen-Red and Woodland then abandon the transfers and seek a new tariff-ridden equilibrium in which all countries are just as well off as in the initial equilibrium. Provisionally, they accept $\boldsymbol{p}^\dagger$ as the new vector of world prices and choose the tariff vectors of the trading countries so that, given $\boldsymbol{p}^\dagger$, domestic prices must settle at their initial levels. They then note that if domestic prices and utilities are unchanged so will be all excess supplies; and that if excess supplies are unchanged then, since $\boldsymbol{p}^\dagger \boldsymbol{X}=\boldsymbol{0}$, $\boldsymbol{p}^\dagger$ may be confirmed as the new vector of equilibrium world prices.

That completes our summary of the Turunen-Red-Woodland argument. In Section 3 we shall present a simple example involving pure exchange and homothetic preferences but with no extraordinary features. In the example there are no equivalent tariffs. The example therefore fails to satisfy Assumption (*A*) and casts doubt on the mildness of that assumption.

3. Assessment

As noted in the Introduction, our assessment of the Turunen-Red-Woodland proposition rests on our misgivings concerning the plausibility of their "mild restriction" on the matrix of equilibrium net exports and concerning the relevance of their Motzkin-based analytical method in a

context of multiple world equilibria. The implausibility of the "mild restriction" is established by means of a simple example of Arrow-Debreu type which nevertheless does not satisfy the "mild restriction" and does not yield a set of equivalent tariffs.

3.1 An example

We propose a world of just three trading countries: *A*, for *ANZUS* (Australia, New Zealand and the United States); *O*, for *OPEC*; and *F*, for the *FIS* (Federation of Independent States). Each country behaves like a single consumer, possibly because each has in place a scheme of lumpsum compensation which ensures that either all inhabitants of that country benefit or all suffer when the country is disturbed by policy or other changes to its environment.

There are two commodities, food and oil, with oil serving as numeraire. Neither commodity is produced; each is available in fixed world supply. Country *A* is the sole supplier of food, with an endowment which, by choice of units, is equal to 1. Countries *F* and *O* are the only sources of oil, with endowments which, by choice of units, are equal to ¼ and ¾, respectively.

The utility function of *A* is

$$u^A = x^{2/3} y^{1/3} \tag{1a}$$

where *x* and *y* denote *A*'s consumption of food and oil respectively. Similarly

$$u^F = (x')^{1/3} (y')^{2/3} \tag{1b}$$

and

$$u^0 = (\mathrm{x}'')^{1/3} (\mathrm{y}'')^{2/3} \tag{1c}$$

where x' and y' denote *F*'s consumption of food and oil, respectively, and x'' and y'' denote *O*'s consumption of food and oil, respectively. We have chosen homogeneous utility functions for convenience; general homothetic functions would have been inconvenient but equally decisive.

Let us consider an initial world equilibrium in which *A* and *F* are free traders but *O* imposes an import duty. By construction, the equilibrium consumption vector of *A* is taken to be $(x,y)=(2/3, 1/5)$. Since *A* trades freely, the equilibrium world price ratio must be equal to *A*'s marginal rate of substitution at (2/3, 1/5); that is,

$$p^0 = 2y/x = 2(1/5)/(2/3)=3/5 \tag{2}$$

Given the price ratio p^0, the consumption vector of *F*, which also trades freely, is $(x', y') = (5/36, 1/6)$. By market clearance, the consumption vector of *O* must then be (7/36, 19/30). To support consumption at that point, the domestic price ratio in *O* must be $y''/(2x'') = 57/35,$ implying a rate of tariff of $[(57/35) - (3/5)]/(3/5) = 12/7.$

Thus, by construction, the initial equilibrium is at e^0 in Figure 1. Because of *O*'s tariff, the allocation of e^0 is inefficient. By eliminating the tariff and introducing a suitable scheme of inter-country *GMG* compensation based on the initial consumption of the three countries, the world economy could be moved to $e^{\#}$, a Pareto-improving point on the world contract locus. (In Figure 1, C^j indicates the consumption of country *j, j=A,F,O.*) Since initially *F* was free riding on the aggressive behaviour of *O*, in the new equilibrium *F* would enjoy more than one-third of the utility of *O*. Indeed it may be calculated that, at $e^{\#}$, the consumption vectors of *A*, *F* and *O* are

$$\{x^{\#}, y^{\#}\} = \{(2/3)[(2/3) + 1/(5p^{\#})],\ (1/3)[(2/3)p^{\#} + (1/5)]\} \qquad (3a)$$

$$\{x,^{\#'}\ y^{\#'}\} = \{(1/3)[(5/36) + 1/(6p^{\#})],\ (2/3)[(5/36)p^{\#} + (1/6)]\} \qquad (3b)$$

and

$$\{x^{\#''}, y^{\#''}\} = \{(1/3)[(7/36) + 19/(30p^{\#})],\ (2/3)[(7/36)p^{\#} + (19/30)]\} \qquad (3c)$$

respectively. In market-clearing equilibrium,

$$x^{\#} + x^{\#'} + x^{\#''} = 1 \qquad (4a)$$

and

$$y^{\#} + y^{\#'} + y^{\#''} = 1 \qquad (4b)$$

Substituting from (3) into either (4*a*) or (4*b*) and solving for $p^{\#}$, we find that

$$p^{\#} = 9/10 \qquad (5)$$

Hence, substituting for $p^{\#}$ in (3), we arrive at the equilibrium world consumption matrix

$$\boldsymbol{C}^{\#} = \begin{bmatrix} 16/27 & 35/324 & 97/324 \\ 4/15 & 7/36 & 97/180 \end{bmatrix}$$

where the first, second and third columns relate to *A*, *F* and *O*, respectively, and where food consumption is recorded in the first row, oil consumption in the second. Making use of (6), it can be verified that, at $e^{\#}$, *F* consumes more than one-third of *O*'s consumption of each commodity.

Suppose now that the three countries seek to find the tariff vector that is equivalent to the scheme of *GMG* compensation. They will fail in their quest. For the *MFN* clause ensures that *F* and *O* will be treated equally, so that in any tariff-ridden final equilibrium *F* will enjoy exactly one-third of *O*'s utility. Alternatively, from (6) and the endowments of the three countries, one can calculate that the matrix of equilibrium exports in the compensated equilibrium is

$$\boldsymbol{X}=\begin{bmatrix} 11/27 & -35/324 & -97/324 \\ -4/15 & 1/18 & 19/90 \end{bmatrix}$$

and that the equation

$$\begin{bmatrix} p_1 & p_2 \end{bmatrix} \boldsymbol{X} = 0$$

has no semi-positive solution.

That completes the presentation of our example. In the example, countries outnumber commodities. However similar examples can be constructed without that special feature. Thus one might suppose the country *A* is endowed with food and clothing, both of which are consumed by all three countries.

3.2. Non-uniqueness

Any attempt to establish tariff equivalence must confront the possibility that an international equilibrium, whether transfer-ridden but tariff-free or transfer-free but tariff-ridden, lacks uniqueness. Let us focus on a particular transfer-ridden but tariff-free initial world equilibrium and on a particular set of tariff vectors which can support the initial allocation in the absence of transfers. Then it must be recognised that the tariffs might also support several other transfer-free allocations, each quite different from that of the initial equilibrium. Moreover this problem can appear even in the familiar two-by-two case; see, for example, Kemp and Wan (2002). The Turunen-Red-Woodland Motzkin-based method simply excludes all other equilibrium allocations and world prices. It therefore must be judged to be inadequate in a context of multiple equilibria.

4. An alternative approach

We have argued by example that the key assumption introduced by Turunen-Red and Woodland is not well described as "mild" and have suggested that their appeal to Motzkin's theorem may be inappropriate. In the present section we offer an alternative approach to the problem posed by Turunen-Red and Woodland and argue that if, realistically, $N>M>2$ then there is no presumption for or against tariff equivalence. We begin by considering three familiar special cases.

Case (*i*) Let us begin with the simplest case of two countries and two commodities($M=N=2$). The matrix $\boldsymbol{X}$ of exports in the compensated free trade equilibrium contains terms which may be positive or negative. Market clearance ensures that each row of $\boldsymbol{X}$ sums to zero, implying that (*a*) each row contains terms of opposite sign but equal magnitude and (*b*) that the columns of $\boldsymbol{X}$ and therefore the rows of $\boldsymbol{X}$ are collinear. It follows that there exists a two-dimensional non-null vector $\boldsymbol{p}$ such that $\boldsymbol{pX}=\boldsymbol{0}$. If and only if each column of $\boldsymbol{X}$ contains terms of opposite sign, $\boldsymbol{p}>\boldsymbol{0}$ and equivalent tariffs exist. However it is also possible that $\boldsymbol{X}$ contains columns both terms of which are of the same sign (as when the recipient country chooses to import both commodities and the donor country chooses to export both commodities). In that sub-case, $\boldsymbol{p}$ must contain terms of opposite sign; $\boldsymbol{p}$ cannot be semi-positive and equivalent tariffs do not exist.

Case (*ii*) Consider the next case (familiar from Section 3) in which three countries trade in two commodities($M=3$, $N=2$). In this case, the rows of $\boldsymbol{X}$ are not necessarily collinear. If in fact the rows are not collinear then equivalent tariffs do not exist. If the rows are collinear then there exists a non-null $\boldsymbol{p}$ such that $\boldsymbol{pX}=\boldsymbol{0}$. If and only if each column of $\boldsymbol{X}$ contains terms of opposite sign, $\boldsymbol{p}>\boldsymbol{0}$ and equivalent tariffs exist. However it is also possible that $\boldsymbol{X}$ contains a column both terms of which are of the same sign. In that sub-case, $\boldsymbol{p}$ must contain terms of opposite sign; that is, $\boldsymbol{p}$ cannot be semi-positive and equivalent tariffs do no exist. This reasoning applies whenever $M>N=2$.

Case(*iii*) Suppose finally that three countries trade in three commodities ($M=3=N$). As in the first case considered, the columns of $\boldsymbol{X}$, and therefore its rows, are necessarily collinear. Hence there exists a non-null $\boldsymbol{p}$ such that $\boldsymbol{pX}=\boldsymbol{0}$. If any column of $\boldsymbol{X}$ contains only negative terms or

only positive terms then $\boldsymbol{p}$ must contain terms of opposite sign; that is, $\boldsymbol{p}$ cannot be semi-positive and equivalent tariffs do not exist. If no column contains only negative or only positive terms then there may or may not exist a semi-positive $\boldsymbol{p}$ and equivalent tariffs. Moreover this reasoning applies whenever $M = N > 2$; indeed it applies whenever $M \leq N,\ N > 2$.

The above calculations confirm that equivalent tariffs may or may not exist. More important, they provide us with the insight that the existence of equivalent tariffs is sensitive to the relative numbers of commodities and countries and to the possibility that some country has only negative or only positive exports. We now try to build on those insights by adding to them a small dose of casual empiricism.

Suppose then that $2<M<N$ and that each country imports at least one commodity and exports at least one commodity. These assumptions might well be accepted as "mild". Moreover, as already noted, the assumptions ensure that the rows of $\boldsymbol{X}$ are collinear and that each column of $\boldsymbol{X}$ contains at least one positive and one negative term. However they do not ensure the existence of equivalent tariffs; nor do they ensure the non-existence of equivalent tariffs Neither outcome is generic.[2]

In this section, we have assumed that each element of $\boldsymbol{X}$ is either positive or negative. The section could be reworked to accommodate the possibility that some elements are zero; the conclusions would change only in minor detail. If a whole row were null (a non-traded commodity), that row could be discarded without loss.

Footnote

1 Turunen-Red and Woodland allow for the possibility that there are tariffs in the initial situation. However they do not rule out the special case of free trade.

2 A more detailed argument in support of this statement may be obtained from the authors.

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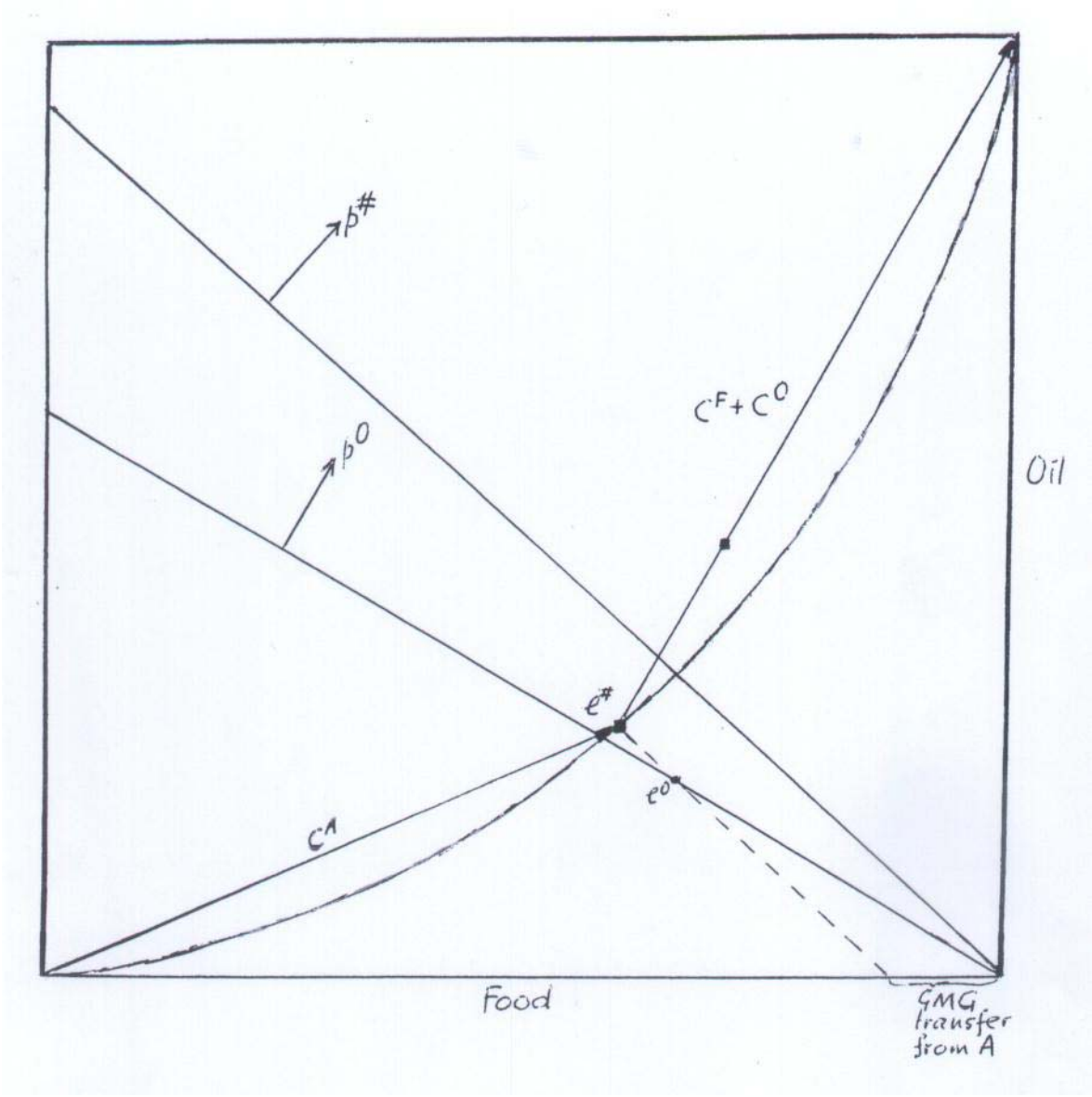

Figure 1