

CAE Working Paper #08-11

**Strategic Analysis of Petty Corruption with an Intermediary**

by

Ariane Lambert-Mogiliansky

Mukul Majumdar

and

Roy Radner

December 2008

# Strategic Analysis of Petty Corruption with an Intermediary

Ariane Lambert-Mogiliansky\*, Mukul Majumdar\*\*,  
and Roy Radner\*\*\*

\*PSE, Paris School of Economics

\*\*Economics Department, Cornell University

\*\*\*Stern School of Business, New York University

December 4, 2008

## Abstract

This note reports part of a larger study of “petty corruption” by government bureaucrats in the process of approving new business projects. Each bureaucrat may demand a bribe as a condition of approval. Entrepreneurs use the services of an intermediary who, for a fee, undertakes to obtain all of the required approvals. In a dynamic game model we investigate (1) the multiplicity of equilibria, (2) the equilibria that are "socially efficient," and (3) the equilibria that maximize the total expected bureaucrats' bribe income. We compare these results with those for the case in which entrepreneurs apply directly to the bureaucrats.

## 1 Introduction

This note is part of an on-going project to study, from a game-theoretic point of view, the phenomenon of *petty corruption* by government bureaucrats. A typical example involves the process of obtaining government approval of a new business activity (called here a *project*) to make sure that it complies with the laws on workers' safety, safe construction standards, environmental hazards, etc. The entrepreneur organizing the project must obtain the approval of each of a set of bureaucrats (called here the *track*), often in a prescribed order. Each bureaucrat may demand a bribe as a condition of approval. Although each of the bribes may be small (hence the adjective "petty"), the total amount of the bribes may be so large that the project is no longer profitable. In this case, a project that would otherwise be economically viable will not be undertaken. Thus the system of bribes not only transfers money from the entrepreneurs to the bureaucrats, but also results in a dead weight loss corresponding to the total value of the projects that are not undertaken. For many economies, such losses have been recognized as substantial. (For an extended introduction to

this topic, and references to the related literature, see (Lambert-Mogiliansky, Majumdar, and Radner, 2007), hereafter referred to as LMRa.)

In our published work thus far we have focused on the case in which the entrepreneurs apply directly to the bureaucrats for approval of their project. We consider a situation in which a sequence of entrepreneurs have the opportunity to apply in turn to the track. Each entrepreneur's project has a value that will be realized if the project is approved. This value is known to the entrepreneur, but not to the bureaucrats in the track, nor to the other entrepreneurs. We formulate a dynamic game that captures the main features of this situation, and in the context of this game we investigate (1) the existence of an equilibrium of the game, (2) the multiplicity of equilibria, (3) the characterization of the equilibria that are "socially efficient" in a particular sense, and (4) the characterization of the equilibria that maximize the total expected bribe income of the bureaucrats.

However, it is often the case that entrepreneurs use the services of an intermediary who, for a fee, undertakes to obtain all of the required approvals. In this note we consider a variation of the above analysis for a model in which entrepreneurs apply, if at all, through a single intermediary (Section 2.) In Section 3 we compare the results to those for the model with no intermediary. In Section 4 we provide some thoughts about various extensions of the model. In particular, one important extension would be to model the enforcement of the anti-corruption laws, and the possible corruption of the bureaucrats' supervisors. This topic is related to some of the work of Leo Hurwicz; see Hurwicz (1993, 1998). Section 5 contains some brief bibliographic notes, and the list of references.

## 2 Formal Models

As noted above, the new theoretical results in this note concern a dynamic game in which a sequence of entrepreneurs apply through an intermediary to a track of bureaucrats for approval of their projects. However, to put the model in the context of the broader research agenda, we start by presenting a model of a one-stage game in which a single entrepreneur applies directly to the track of bureaucrats, without any intermediation. We shall follow that with a description of the one-stage game with an intermediary, and then an analysis of a full-blown dynamic game with an intermediary.

[Note: The material on the model of the one-stage game without an intermediary is taken from (LMRa).]

### 2.1 A One-Stage Game without an Intermediary

The players in the one-stage game consist of a single entrepreneur (EN) and a single *track* of  $N$  bureaucrats (BUs), with  $N \geq 2$ , arranged in a specific sequence. In order to get her project approved, EN must apply to and *obtain approval from each of the BUs in the prescribed order* (i.e.  $BU_1$  first, then  $BU_2$

etc.). If the project is rejected by any one BU, the game ends and EN does not proceed further in the track.

Here is the complete description of the extensive form of the game. Let  $V$  denote the project's potential value, which is uniformly distributed on some closed interval, which we may normalize to be  $[0, 1]$ . The probability distribution of  $V$  (the "prior") is common knowledge, but the realized value of  $V$  is known only to EN.

If and when EN applies to  $BU_n$  she incurs a cost  $c > 0$ . For convenience of exposition, this cost is assumed to be the same for all BUs. The cost  $c$  is known to *all* the players. If EN applies to  $BU_n$ , let  $b_n \geq 0$  denote the bribe demanded by him. The project is approved if and only if the bribe is paid. The bribe is demanded on a "take-it-or-leave-it" basis, so that if EN refuses to pay the bribe the game ends. It is assumed that the BUs do not observe the bribes demanded by the other BUs.

Let  $a_n = 1$  or  $0$  according as EN does or does not apply to  $BU_n$ . If she does apply, she incurs the application cost  $c > 0$  and *then* learns the magnitude  $b_n$  of the bribe demanded by  $BU_n$ . Let  $p_n = 1$  or  $0$  according as EN does or does not pay the bribe. Note that if  $a_n = 0$  or  $p_n = 0$  then the game is over. Thus, if  $a_n = 0$  we have  $p_n = 0$  and if  $p_n = 0$  then  $a_m = 0$  for all  $m > n$ .

Call the part of the game in which EN faces  $BU_n$  the *n*th step ( $n = 1, \dots, N$ ). The action taken by EN in step  $n$  is the pair  $(a_n, p_n)$ . The action taken by  $BU_n$  in step  $n$  is, of course,  $b_n$ .

For  $n \geq 1$ , let  $H_n$  denote the history of the game through step  $n$ , i.e., the sequence of actions taken by all players through step  $n$ . A strategy for EN is a sequence of functions,  $\alpha = \{A_1, P_1, \dots, A_N, P_N\}$ , which determine EN's actions according to:

$$a_n = A_n(V, H_{n-1}), \quad (1)$$

$$p_n = P_n(V, H_{n-1}, a_n, b_n). \quad (2)$$

(Here  $H_0$  denotes an exogenous constant, the "prehistory of the game.")

Since  $BU_n$  does not know the magnitudes of any previously demanded bribes, his strategy for the game is the magnitude of the bribe he demands,

$$b_n \geq 0.$$

To complete the description of the game, we must describe the players' payoff functions. The payoff for  $BU_n$  is the bribe he demands, if it is paid, i.e.,

$$U_n = p_n b_n. \quad (3)$$

The payoff for EN is the value of the project if the project is approved, less the sum of the application costs and bribes paid (whether or not the project is completely approved). Thus EN's payoff is

$$U_0 = p_N V - \sum_{1 \leq n \leq N} (a_n c + p_n b_n). \quad (4)$$

Finally, without loss of generality, assume:

$$0 < Nc < 1. \tag{5}$$

(Otherwise, no project would be profitable.)

It may be of interest to quote here some results about equilibria of this game, although they will not be used in what follows. As usual, a Bayes-Nash *equilibrium* of the game is a profile of strategies such that no player can increase his or her expected pay-off by unilaterally changing his or her strategy. A strategy is (weakly) *undominated* if there is no other strategy that yields the player as high a payoff for all strategy profiles of the other players, and a strictly higher payoff for some strategy profile of the other players. For our first result, we shall confine ourselves to equilibria in undominated strategies. The first result is that *there is no equilibrium in undominated strategies in which the project is approved -with positive probability.* (See [LMRa], as well as [Lambert, Majumdar, and Radner, 2008], hereafter referred to as LMRb.) We note that for  $N \geq 2$ , the theorem is valid even when  $c = 0$ .

The second result concerns a family of equilibria in which *no EN applies to the first BU, and hence no project is ever approved.* A strategy profile in this family will be called a *null strategy profile* (NSP). A particular NSP is characterized by  $N$  parameters,  $b'_n$ ,  $n = 1, \dots, N$ . The parameter  $b'_n$  represents the bribe that EN expects  $BU_n$  to demand, and it is also the bribe that  $BU_n$  plans to demand. These parameters satisfy the conditions:

$$\begin{aligned} 0 &< b'_n < 1; \text{ for } n < N, & (6) \\ \max \left\{ 1 - c, \frac{1}{2} \right\} &< b'_N < 1. & (7) \end{aligned}$$

The *EN's* strategy is: for  $1 \leq n \leq N$ , EN applies to  $BU_n$  only if the value of her project is as large as the sum of the *expected* cost of completing the track, whereas she pays the *actual* bribe demanded only if the value of her project is as large as the sum of this actual bribe and the cost of completing the track if the remaining BUs demand their planned bribes.

The strategy of  $BU_n$  is: If EN applies to him, demand the bribe  $b'_n$ .

As is common in game-theoretic analyses, we wish to confine attention to equilibria in which the strategies are in some sense "credible," which involves examining the behavior of the system "off the equilibrium path." To this end, for the purpose of the following theorem we find it convenient to replace the requirement that strategies be undominated by a condition that we call *admissibility*, which is in some sense more demanding, but also somewhat more complex to state. First, *a strategy of  $BU_n$  is admissible* if the bribe demanded is strictly between zero and one. *A bribe profile is admissible* if each BU's strategy is admissible. *A strategy for EN is admissible* if it is a best response to *some* admissible bribe profile.

For a BU, we alter somewhat the definition of undominated strategy. An admissible strategy for  $BU_n$  is *quasiundominated* if there is no other admissible

strategy for him that yields him as high a payoff for all admissible strategy profiles of the other players, and a strictly higher payoff for some admissible strategy profile of the other players.

Finally, *an equilibrium strategy profile is admissible* if EN's strategy is admissible, and each BU's strategy is admissible and quasiundominated.

**Theorem 1** *Suppose that,*

$$\text{for each } n, 1/2 < b'_n < 1;$$

*then the corresponding NSP is an admissible equilibrium, and for every value of  $V$ , EN does not apply to  $BU_1$ .*

In the model of the dynamic game without an intermediary (which will not be described here formally), a large class of equilibria (including second-best equilibria) are sustained by threats to revert to an NSP equilibrium if any player defects ("Nash reversion"). This topic will be taken up briefly in Section 3.

## 2.2 A One-Stage Game with an Intermediary

In the one-stage game with an intermediary, the EN does not apply directly to the track of BUs, but instead, she applies, if at all, through an intermediary, IN, who charges a fee, say  $f$ . IN then applies to the track of BUs, playing a role analogous to that of EN in the previous subsection. The intermediary's fee is actually paid only upon approval of the project. EN knows the fee before making the decision. If EN does not apply to IN, the game is over, and every player's payoff is zero. If EN applies to IN, then IN then begins visiting each BU in the prescribed order. Let  $a_n = 1$  or  $0$  according as IN does or does not apply to  $BU_n$ . If he does apply, he incurs a cost  $k > 0$ , and then learns the magnitude  $b_n$  of the bribe demanded by  $BU_n$ . If IN pays the bribe, the project is approved by  $BU_n$ , and IN goes on to the next BU (if there are any remaining). If IN does not pay the bribe demanded by  $BU_n$ , the game ends, and the payoffs are as follows: For  $m < n$ ,  $BU_m$  gets a payoff  $b_m$ , IN loses the application costs and bribes already incurred, and the other players get zero. If the project is approved by all the BUs, then it is approved by the track. In this case, EN's payoff is the value of the project less IN's fee, IN's payoff is the fee less the application fees and bribes that he has paid, and each BU's payoff is the bribe that he has been paid. Again, it is assumed that the BUs do not observe the bribes demanded by the other BUs.

More formally, We shall make a corresponding adaptation of the notation for the players' actions. Let  $a = 1$  or  $0$  according as EN does or does not apply to IN,  $a_n = 1$  or  $0$  according as IN does or does not apply to  $BU_n$ , and  $p_n = 1$  or  $0$  according as IN does or does not pay the bribe demanded by  $BU_n$ . From the above description of the game,

$$a = 0 \Rightarrow a_1 = 0. \tag{8}$$

$$a_n = 0 \Rightarrow p_n = 0, \quad (9)$$

$$p_n = 0 \Rightarrow a_m = 0 \text{ for all } m > n. \quad (10)$$

Let  $p = p_N$ . Note that the project is approved if and only in  $p = 1$ . The payoffs to EN, IN, and  $BU_n$  are, respectively,

$$\begin{aligned} U_0 &= p[V(t) - f], \\ W &= pf - \sum_{1 \leq n \leq N} [a_n k + p_n b_n], \\ U_n &= p_n b_n, \quad 1 \leq n \leq N. \end{aligned} \quad (11)$$

As noted in the previous subsection, in [LMRa and LMRb] the analysis of the dynamic game with no intermediary makes use of null-strategy-profile (NSP) equilibria of the one-stage game. However, in the analysis of the sequential game with an intermediary (in the following subsection) we do not make use of any information about equilibria of the one-stage game, and so we omit that here.

### 2.3 A Dynamic Game with an Intermediary

We now turn to the main results of this note, which concern a *dynamic* game with an intermediary. In this dynamic game, there is a sequence of entrepreneurs,  $EN(t)$ ,  $t = 0, 1, 2$ , *ad inf*, a single intermediary, IN, and a single track of bureaucrats,  $BU_1, \dots, BU_N$ . Entrepreneur  $EN(t)$  has a project with a potential value  $V(t)$ . Each of the entrepreneurs, in succession, plays the one-stage game described in the preceding subsection with the same intermediary and bureaucrats. We extend the notation of the one-stage game to the dynamic game in the obvious way, so that the actions of the players in the game of stage  $t$  are  $f(t)$ ,  $a(t)$ ,  $a_n(t)$ ,  $p_n(t)$ ,  $b_n(t)$ , and we let  $p(t) = p_N(t)$ . These actions must satisfy the constraints corresponding to (8)-(10). The one-stage payoffs corresponding to (11) are  $U_0(t)$ ,  $W(t)$ ,  $U_n(t)$ , respectively. The dynamic-game payoff for  $E(t)$  is  $U_0(t)$ , and the dynamic-game payoffs for IN and the BUs are their respective total discounted one-stage payoffs, with a common discount factor,  $\delta$ . We assume that

$$\{V(t)\} \text{ are independently and uniformly distributed on } [0, 1]; \quad (12)$$

$$k > 0, \quad (13)$$

$$Nk < 1. \quad (14)$$

(The last assumption is needed to keep the game from being trivial.)

Finally, to complete the description of the dynamic game, we need to specify the information available to each player at each stage of the game. Recall that we have assumed that, within any one stage, the BUs do not observe the bribes demanded by the other BUs. Regarding the information that players have about the history of previous stages, we make the following assumption:

**Assumption I.** *The players in any one period know nothing about the history of the game in previous periods other than the previous history of the transactions in which they directly participated.*

Even with this draconian assumption, we shall show that - because of the presence of the long-lived intermediary - if the players' discount factor is close enough to unity, then there is a family of equilibria for which there is a positive probability that an EP will apply to the intermediary, who will pay the bribes demanded, and the entrepreneur and the intermediary will both retain a positive surplus. However, in contrast with the "Folk-Theorem-like" result for the game without an intermediary, the equilibrium set may be bounded away from social efficiency when the discount factor approaches unity.

Since the entrepreneurs  $EN(t)$  are different for different stages  $t$ , the dynamic game that we have just described is not, strictly speaking, a "repeated game." Nevertheless, because of the assumption that the project values,  $V(t)$ , are independently and identically distributed, the method of analysis and the results will bear some similarity to those for repeated games. Hence we shall, with some abuse of terminology, refer to the dynamic game as the *supergame*.

We shall now describe a particular family of strategy profiles of the supergame. The equilibria in this family will be indexed by a vector of parameters,

$$\beta = (f, b_1, \dots, b_N). \quad (15)$$

In what follows, in equilibrium,  $f$  will be the fee charged by IN, and  $b_n$  will be the bribe demanded by  $BU_n$ . Define

$$C = \sum_{n=1}^N (k + b_n). \quad (16)$$

Thus  $C$  is IN's total cost if the project is approved by the entire track. The vector  $\beta$  is required to satisfy the constraints:

$$b_n > 0, \quad 0 \leq n \leq N, \quad (17)$$

$$C < 1. \quad (18)$$

In addition,

$$f = \frac{1 + C}{2}. \quad (19)$$

Note that

$$C < f < 1. \quad (20)$$

Given  $\beta$ , the players' strategies are determined as follows.  $EN(t)$  applies to IN if and only if the value of her project exceeds IN's fee, i.e.,

$$a(t) = 1 \Leftrightarrow V(t) > f(t). \quad (21)$$

IN always charges the same fee:

$$f(t) = f, \text{ for all } t. \quad (22)$$



Say that IN is *open in period  $t$*  if in period  $t$  he applies to the BUs on behalf of EN( $t$ ) when requested; otherwise, we shall say that IN is *closed*. IN is open in period 0. Say that BU $_n$  *defects* in period  $t$  if  $b_n(t) > b_n$ . If IN applies to BU $_n$  in period  $t$ , then he pays the bribe demanded if and only if BU $_n$  does not defect.

With the specification of the strategy profile given thus far, EN( $t$ ) will apply to IN if and only if  $V(t) > f$ . If EN( $t$ ) does apply, then IN will apply to each BU in the track, pay the bribes  $b_n$ , and the project will be approved. We shall complete the specification of the strategies by describing what happens if the above specification is violated at some node in the game tree. Say that a *shock* occurs in period  $t$  at BU $_n$  if he defects in period  $t$ , and IN *does* pay the bribe demanded. When a shock occurs in a period, then IN remains open for the remainder of that period, but remains closed for *all* succeeding periods. Furthermore, if a shock occurs in a period at BU $_n$ , then in all subsequent periods the latter will demand a bribe equal to  $b'_n$ , which we shall specify below. Otherwise, BU $_n$  demands the bribe  $b_n$  specified in the vector  $\beta$  until his next defection, if any. We shall call this entire strategy profile the  $\beta$ -INSP.

It remains to specify the *threat bribes*,  $b'_n$ , and for this we need some additional notation. For each  $n$  ( $1 \leq n \leq N$ ), define:

$$P_n = k + \sum_{m < n} (k + b_m), \quad (23)$$

$$S_n = \sum_{m > n} (k + b_m). \quad (24)$$

Thus  $P_n$  is IN's total "sunk cost" in the current period preceding his decision whether to pay the bribe  $b_n(t)$ , and  $S_n$  is the cost of applying and paying the  $\beta$ -INSP bribes to the succeeding BUs in the track. Note that

$$P_n + b_n + S_n = C. \quad (25)$$

The threat bribe  $b'_n$  is defined as

$$b'_n = f - S_n - \frac{P_n}{2}. \quad (26)$$

Note that

$$b'_n - b_n = f - S_n - \frac{P_n}{2} - b_n \geq f - C > 0.$$

When the players use the  $\beta$ -INSP strategy profile, if IN applies to the track of BUs, then his one-period payoff (net revenue) equals

$$R = f - C = \frac{1 - C}{2}.$$

Note that, by (18),  $R > 0$ .

**Theorem 2** Under Assumption I, if

$$\frac{\delta}{1-\delta} > \frac{2(1+C)}{(1-C)^2}, \quad (27)$$

then a  $\beta$ -INSP strategy profile satisfying (17) - (19) is an equilibrium.

**Proof.** It is immediate that EN( $t$ )'s strategy is a best response to those of the other players. Given the strategies of the other players, if in period  $t$  IN demands a fee equal to  $b$ , his expected net profit in that period is

$$(b-C) \Pr(V > b) = (b-C)(1-b).$$

This is maximized when  $b = f = (1+C)/2$ , and the corresponding net profit is

$$\pi = \frac{(1-C)^2}{4}. \quad (28)$$

Now suppose that BU $_n$  defects in period  $t$ ; without loss of generality, we may take  $t = 0$ . If IN does not pay the bribe, then he loses  $P_n$  in period 0, but his expected total discounted payoff in the remaining periods is

$$\frac{\delta\pi}{1-\delta},$$

so his net expected discounted payoff is

$$\frac{\delta\pi}{1-\delta} - P_n.$$

On the other hand, if he did pay the bribe, then his payoff in period 0 would be

$$f - P_n - b_n(0) - S_n.$$

However, in all succeeding periods BU $_n$  would demand the bribe  $b'_n$ , and IN's per-period total cost of getting the project approved would be

$$\begin{aligned} P_n + b'_n + S_n &= P_n + b_n + S_n \\ &= P_n + f - S_n - \frac{P_n}{2} + S_n \\ &= f + \frac{P_n}{2} \geq f; \end{aligned}$$

hence it would be optimal for IN not to apply in the succeeding periods, and his payoffs would be zero. Hence it is optimal for IN not to pay the "defection bribe"  $b_n(0) > b_n$  if

$$\frac{\delta\pi}{1-\delta} - P_n > f - P_n - b_n(0) - S_n,$$

or

$$\frac{\delta}{1-\delta} > \frac{f - b_n(0) - S_n}{\pi}. \quad (29)$$

Observe that

$$\frac{f - b_n(0) - S_n}{\pi} < \frac{f}{\pi} = \frac{2(1+C)}{(1-C)^2}, \quad (30)$$

which verifies the inequality in the Theorem.

In any period,  $BU_n$  should not demand strictly less than  $b_n$ . On the other hand, if he demands more than  $b_n$ , then his payoff will be zero in that period, without any future benefit.

Finally, it is straightforward to verify that no strategy in a  $\beta$ -INSP strategy profile satisfying (17) - (19) is dominated, even weakly. For the BUs, this depends on the bribes  $b_n$  being strictly positive. This completes the proof of the Theorem.

**Remark 1.** In this family of equilibria, IN's fee is determined by the vector of bribes,  $(b_1, \dots, b_N)$ , and the multiplicity of this vector is constrained by (17)-(18). As the discount factor approaches unity, the "most efficient" equilibria in the family approach an outcome with zero bribes ( $b_1 = \dots = b_N = 0$ ) and the smallest fee,  $f = (1 + Nk)/2$  (see Section 3 for definitions and a discussion of alternative concepts of efficiency).

**Remark 2.** With higher bribes, IN receives lower payoffs, and the BUs receive higher payoffs up to a point. One can verify that the total expected discounted payoffs of all the BUs together is maximized when the total of the bribes is

$$\sum_{n=1}^N b_n = \frac{1 - Nk}{2}. \quad (31)$$

**Remark 3.** In the version of the  $\beta$ -INSP presented here, once the IN is closed, he is closed *forever*. This corresponds to what Aumann has called a "grim punishment strategy." Alternatively, IN could remain closed for some predetermined number of periods, (a parameter of IN's strategy), corresponding to what Aumann has called a "relenting punishment strategy." For this latter kind of strategy, one can prove results similar to those presented here, provided the punishment period is "long enough."

### 3 Comparison with the Case of No Intermediary.

In (LMRa) and (LMRb) we studied a model of a dynamic game with no intermediary, based on the one-stage game described in Section 2.1. We shall refer to that as the "NoIN model." We shall not describe the NoIN model in any detail, but just highlight the main differences in the assumptions and results compared to the model of Section 2.3, the "IN model."

The most important difference in the assumptions (aside from the absence of an intermediary) concerns the information that players have about the history of previous stages. For the NoIN model, we made the following assumption:

**Assumption N.** *At the beginning of period  $t$ , all the current players know the history of all defections (if any) in all periods up to and including period*

$t-1$ . In addition, all players know the history of the transactions in which they directly participated.

Observe that Assumption N is related to Assumption I by the addition of the first sentence, and hence is strictly stronger.

Stemming from this difference in Assumptions N and I is an important difference in the results. To state the relevant results for the NoIN model, we first give a heuristic description of a *trigger strategy profile* (TSP). There is a vector,  $(b_1, \dots, b_N)$ , of *normal bribes*. We shall say that  $BU_n$  *defects* in a particular period if in that period he demands a bribe that strictly exceeds his normal bribe. If and when a defection takes place, then *all players will play the null-strategy-profile of the one-stage game* (see Section 2.1) in all subsequent periods. (By Assumption N, if a defection occurs, all the players learn about it at the beginning of the next period.) Hence, once a defection occurs all players receive a zero payoff in all subsequent periods. If a TSP forms an equilibrium of the NoIN dynamic game we shall call it a *TSP equilibrium* (TSPE). In a TSPE, each bureaucrat always demands his normal bribe, and each project with a potential value that exceeds the total costs of application and normal bribes is approved.

Before stating the next theorem, we define a concept of "economic efficiency," or "efficiency" for short. Recall that the cost of application of each entrepreneur  $EN(t)$  to each bureaucrat in the track is  $c > 0$ . We shall call an outcome of the game *efficient* if every project whose potential value exceeds  $Nc$  is approved. In particular, in any equilibrium whose outcome is efficient the bribes must be zero. One can prove that, with some additional technical conditions:

**Theorem 3** *If the normal bribes are strictly positive and satisfy the constraint*

$$\sum_{n=1}^N (c + b_n) < 1, \quad (32)$$

*then there exists  $\delta^* < 1$  sufficiently large such that, for all  $\delta \geq \delta^*$ , the TSP is an equilibrium of the supergame. Furthermore, as the discount factor approaches unity, there is a sequence of TSP equilibria whose outcomes approach efficiency.*

The conclusion of this last theorem contrasts with that of 2, since in any INSP equilibrium the fee is, by (16) and (19), at least  $(1 + Nk)/2$ , which by (14) is strictly greater than  $Nk$ . Hence, if  $k \geq c$  then the outcome of an INSP equilibrium is not efficient. On the other hand, if  $k < c$ , then for  $\delta$  sufficiently close to unity there are TSP equilibria that are efficient. In other words, if IN can apply to the track at lower cost than the ENs can, then the outcome of an INSP equilibrium can be efficient if the fee and the bribes are not too high.

However, if  $k < c$  (which would seem to be the "normal" case), it could be argued that, in the definition of efficiency,  $Nc$  should be replaced by  $Nk$ . With this definition of efficiency, even as  $\delta \rightarrow 1$  the outcome of INSP equilibria are bounded away from efficiency.

## 4 Concluding Remarks

It would be desirable to extend the IN model in a number of directions that are beyond the scope of this note. Here are some possible and interesting extensions:

1. Include the option for the ENs to apply directly to the track of BUs. However, if the ENs did not obtain information about the bribes demanded in previous periods, it would be difficult for them to sustain reasonable supergame equilibria on their own. Furthermore, it would be natural to assume that the ENs' application cost is higher than that of the IN, so that it seems likely that any equilibria that involved direct applications by the ENs would be dominated by equilibria in which the IN did the applying.

2. Consider multiple intermediaries, and competition among them. Multiple intermediaries might differ in the "quality" of the service that they provide, e.g., speed of approval, probability of detection of bribery, etc.

2. Introduce some structure of anti-corruption enforcement, including supervisors as players.

3. Vary the information structure concerning the history of the game and the values of the projects.

4. Consider more general distributions of project values, and even more general processes of entrepreneur arrival.

5. Analyze the effects of various "reform" proposals.

(Regarding the last extension in the context of the NoIN model, see [LMRa and LMRb].)

### Acknowledgments

The authors are grateful to Pranab Bardhan, Francois Bourguignon, Prajit Dutta, and Jayanta Roy for helpful discussions on the subject of this entire project. As usual, we are solely responsible for any errors or bad ideas in this note.

## 5 Bibliographic Notes and References

A thorough review of the literature on the present topic is beyond the scope of this note. There are only a few published game-theoretic analyses of intermediaries and corruption. The most recent appears to be by Hasker and Okten (2007), which contains references to the previous literature. We also note the paper by Bayar (2005). Indeed, we are not aware of any theoretical analyses of petty corruption, with or without intermediaries, using a dynamic-game model. For references to the more general topic of corruption, and to game-theoretic analyses in particular, see (LMRa) and Lambsdorff (2007). For insightful observations on the general topic of enforcement, see (Hurwicz, 1993, 1998).

## References

- Bayar, G. (2005), "The Role of Intermediaries in Corruption," *Public Choice*, 122, 277-298.
- Hasker, K., and C. Okten (2007), "Intermediaries and Corruption," to appear in *J. of Economic Behavior and Organization*.
- Hurwicz, L. (1993), "Implementation and Enforcement in Institutional Modeling," in W. A. Barnett, et al, eds., *Political Economy: Institutions, Competition, and Representation*, Cambridge U. Press, Cambridge, 1993.
- Hurwicz, L. (1998), "But Who Will Guard the Guardians?," Economics Dept., U. of Minn., May 13, 1998 (unpublished).
- Lambert-Mogiliansky, A., M. Majumdar, and R. Radner (2007), "Strategic Analysis of Petty Corruption: Entrepreneurs and Bureaucrats" *J. of Development Economics*, 83, 351-367.
- \_\_\_\_\_ (2008), "Petty Corruption: A Game-Theoretic Approach," *J. of Economic Theory*, 4, 273-297.
- Lambsdorff, J. G. (2007), *The Institutional Economics of Corruption and Reform*. Cambridge University Press, Cambridge, UK.