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Petty Corruption: A Game-Theoretic Approach

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Petty corruption: A game-theoretic approach

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The paper explores a game-theoretic model of petty corruption involving a sequence of entrepreneurs and a track of bureaucrats. Each entrepreneur's project is approved if and only if it is cleared by each bureaucrat. The project value is stochastic; its value is observed only by the entrepreneur, but its distribution is common knowledge. Each bureaucrat clears the project only if a bribe is paid. The bribe for qualified projects ("extortion") and unqualified projects ("capture") may differ. We identify the nature and welfare implications of different types of equilibria under appropriate technical assumptions on the structure of the game.

Key words corruption, repeated games, hold up, extortion, capture

JEL classification C73, D61, D73, O12, O17

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1 Introduction[1]

The primary aim of this paper is to develop game theoretic models of "petty" corruption that occurs when private citizens (as e.g. investors or business managers) have to deal with relatively low level bureaucrats in getting approvals of specific privileges (e.g. registration of a new firm as a legitimate business activity that meets the regulatory standards or approval of loans from state-owned banks, getting a passport or a driver's license).[1] The basic ingredients of corruption (emphasized by Klitgaard 1988), government monopoly, discretion in interpreting "laws" or "proper procedure" and a lack of direct accountability, appear in many ways in different contexts, and typically result in "small" bribes (e.g. "speed money" and "tea money") for completing a transaction within a reasonable time-span. It has been duly stressed that the total impact of all these "small" or "petty" bribes on the efficiency and welfare of an economy may indeed be substantial. As with the case of other types of corruption, petty corruption has been described as anti-poor, anti-development, anti-growth, and

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[1] Corruption is defined as the use of public office for private gains. Petty corruption is often distinguished from "grand" corruption and "influence peddling". Countries vary significantly in the pervasiveness and types of corruption, and the literature on various aspects of corruption is huge. See, in particular, the collection by Elliott (1997) and the book by Rose-Ackerman (1999) for comprehensive lists of references. Andvig (1991), Bardhan (1997) and Lambsdorff (2001) are also useful surveys. The annual reports from Transparency International available at its website also provide valuable insights.

anti-investment, and it creates an environment that undermines the legitimacy of the state. We do not make any attempt to review and do justice to the immense literature on corruption that provides valuable insights into so many facets of the problem across the countries at different stages of economic development. A relatively small part of the literature uses formal models, and our task is to encourage further exploration in this direction.[2]

Two types of intervention by bureaucrats should be distinguished. First, applications that are "qualified" or "legitimate" may be disapproved if bribes are not paid; we refer to this as "extortion" or "hold up". Second, in return for bribes, applications that fail the relevant regulatory standards may be approved; we refer to this as "capture". Therefore, the bribes may depend upon the "quality" of the projects. The first type has been widely discussed by policy-makers in developing countries. "Competing bureaucracies, each of which can stop a project from proceeding, hamper investment and growth around the world, but especially in countries with weak governments . . . " (Shleifer and Vishny 1993, p. 615; see also Rose-Ackerman 1999). The second type helps sustain activities that may pose danger to public safety or even national security (e.g. unsafe buildings, import and transportation of drugs, and explosives (see Vittal in Gupta 2001), and challenge the rule of law.

In our general model, entrepreneurs may apply, in sequence, to a "track" of two or more bureaucrats for approval of their projects. Each entrepreneur (EP) has a project that has a specific (expected present) value V that would be realized if the project were approved. This value is known to the entrepreneur, but not to the bureaucrats. The entrepreneur must apply to each bureaucrat (BU) in the track in a prescribed order, and his or her project is approved if and only if every bureaucrat in the track approves it. Each bureaucrat may demand a bribe as a condition of approval. To make the model precise, we assume that the entrepreneurs arrive at a fixed rate of one per "period" and that their successive project values are independent and identically distributed with a probability distribution that is common knowledge to all the players. In fact, our simplifying assumption is that V is uniformly distributed over [0,1]. At any step in the period the entrepreneur may refuse to pay the bribe, in which case he or she leaves the process and the value of the project is not realized, although the enterpreneur loses the total amount of bribes paid plus application and qualification costs incurred up to that point. If the project is approved by the entire track, the entrepreneur receives the value of the project, minus the total amount of bribes paid and the total cost of qualification and application. The payoff to a bureaucrat in that period is the amount of the bribe he or she receives, if any, and the bureaucrat's total payoff in the game is the expected sum of discounted bribes he or she receives. (The discount factor is the same for all the BU.)

As a first step in the development of a formal model, we make some simplifying assumptions. First, to *qualify* a project (at least "on paper") the entrepreneur incurs a cost rV where $r > 0$. There is also an application cost $c \geq 0$, the same for each of the bureaucrats. Moreover, realistic or not, we also assume the following about the information that the players have about the actions of other players:

1. Players remember their own actions and those of the players they transact with.

[2] Given the space limitation, we do not review the formal models. See Lui (1985, 1986), Banerjee (1997), Burget and Che (2004), Compte *et al.* (2004), Marjit *et al.* (2000) and Mookherjee and Png (1992, 1995).

2. Within any single period, no bureaucrat knows the bribes demanded by the other bureaucrats.
3. Every player learns the actions of the other players in previous periods, perhaps with some delay.

Recall that a (Bayes–Nash) equilibrium of the corresponding game is a profile of strategies such that no player can increase his or her expected payoff by unilaterally changing his or her strategy. Our goal is to describe the Bayes–Nash equilibria of the corresponding game. (We restrict attention to equilibria with undominated strategies.) With some additional technical assumptions we can show:

1. In a one-period version of the game (only one entrepreneur), there is no equilibrium in which the project is approved with positive probability. However, there is an equilibrium in which the entrepreneur refuses to apply to even the first bureaucrat in the track, no matter what the value of his or her project (in [0, 1], the range of its probability distribution). The latter is called a *null-strategy-profile equilibrium.*
2. In the sequential version of the game, if the bureaucrats are sufficiently patient (discount factor close enough to unity), then there are many equilibria, and we describe a large family of them that we call *trigger strategy profile* (TSP) equilibria. In a TSP, there is a target (or "normal") bribe profile. If a bureaucrat deviates from the target bribe profile, this triggers (after some delay) a reversion to the null-strategy-profile equilibrium for a prescribed number of periods (the punishment phase), after which the players return to their normal behavior.
3. For any given profile of strictly positive normal bribe demands, if the bureaucrats are sufficiently patient, then there is a TSP equilibrium that sustains those bribes (a "folk theorem"). Hence, for a sequence of bureaucrats' discount factors approaching 1, there is a corresponding sequence of TSP equilibria that approach "social efficiency".

We also examine in more detail two "extreme" TSP equilibria. First, a *second-best equilibrium* minimizes, in the set of *all equilibria of the sequential game*, the total social loss caused by the bribe system, including any social loss caused by the approval of an unqualified project. We show that:

4. Among the second-best equilibria there is one that is a TSP. (In other words, the second-best social loss can be attained with a TSP equilibrium.)
5. In a second-best TSP equilibrium, the farther along the bureaucrat is in the track, the larger will be his or her bribe.
6. If the track is replaced by a single bureaucrat, then the second-best equilibrium bribe total will be *smaller.* The other extreme equilibria maximize the total expected bribe income within the family of TSP equilibria; call such a TSP equilibrium a *bribe-income-maximizing equilibrium* (BIME). We show that in a BIME, no project is qualified. Call a bribe profile *first-best for the bureaucrats* if it maximizes the total bribe income without the constraint that it be an equilibrium. We show that, if a BIME is not first-best for the bureaucrats, then (corresponding to results (5) and (6) above):
7. In a BIME, the farther along the bureaucrat is in the track, the larger will be his or her bribe.

8. If the track is replaced by a single bureaucrat, then the total bribes in the BIME will be *larger*.
Finally:
9. If the bureaucrats are sufficiently patient, then there is a BIME that is first-best for the bureaucrats.

The conclusions (6) and (8) are relevant to an oft-proposed policy recommendation that in any jurisdiction the track of multiple bureaucrats should be replaced by a single "window" to which the entrepreneur can apply. Our analysis shows that *one cannot conclude that such a change will unambiguously result in an increase in social efficiency* (see Section 4.1).

Our results also enable us to assess another much-advocated reform, the rotation of bureaucrats among tracks. The multiplicity of equilibria again make the results ambiguous. However, our analysis suggests that, under certain plausible circumstances the *rotation policy will actually result in a decrease in social efficiency* (see Section 4.2).

We conclude with a few suggestions for generalizing our model in different directions.

The brief remarks in the present paper on background of the theoretical model of the present paper, and on the policy implications of our analsysis, draw heavily on the more extensive discussion in (Lambert-Mogiliansky *et al.* 2007). Also, some of the theoretical results in the present paper are announced there, without proofs. However, those results deal only with the case of extortion (hold-up), not capture.

2 Formal model

2.1 One bureaucrat

To introduce some of the main ideas of the model, including hold-up and capture, we begin with the special case of a track with a single bureaucrat. We first analyze a one-stage game with one entrepreneur, and then go on to the analysis of the case of multiple bureaucrats.

2.2 A one-stage game

Consider a model with one entrepreneur (EP) and one bureaucrat (BU). The entrepreneur has a project with a potential value, V, which is a random variable with a uniform distribution on the unit interval. The entrepreneur learns the value of the project at the beginning of the game. The bureaucrat never learns the value, but does know its distribution. In order to realize this value, the project must be approved by BU. Formally, to qualify for approval, EP must fulfill certain requirements, at a cost $R(V)$, which is subtracted from the value. We assume that *this cost is proportional to the project value*:

$$R(V) = rV, \qquad r > 0. \tag{1}$$

(However, more general formulations would be amenable to analysis.) The parameter r is given exogenously. In addition, in order to apply to BU, EP must incur a cost, $c \geq 0$. In some cases it will turn out to matter whether this cost is zero or strictly positive.

It is important to point out that our specification (1) of a qualification cost falls short of a realistic description of the process of approval and implementation of investment projects in many contexts. An EP typically incurs some cost in preparing a proposal that is "qualified on paper" (and submits an estimate of costs for upholding the regulatory standards); but the more substantial part of the cost is incurred during the phase of actual construction, which is often subject to further on-site inspections. In (1) we shorten this process by assuming that the EP will implement the exact proposal qualified on paper. In contrast, an unqualified project approved through bribery will not result in any subsequent cost to the EP (fines/settlement of liability claims etc.). A more elaborate model would be needed to capture these intricacies, but we do not expect that our theoretical results would be significantly affected.

The bueaucrat can demand a bribe before approving the project; this is a "take-it-or-leave-it" demand. Furthermore, BU can approve the project even if it does not meet the formal qualification. If EP pays the bribe demanded by BU, the project will be approved, whether or not it is formally qualified. We shall assume that

$$c + r < 1, \tag{2}$$

which guarantees that, *if no bribes were demanded, and only qualified projects were accepted*, then with strictly positive probability it would be profitable to qualify some projects and apply for approval. Recall that V is uniformly distributed on $[0, 1]$.

Here is the timeline of events:

Step 0 "Nature" determines V according to its distribution, and EP learns its magnitude.
Step 1 EP decides whether to qualify the project and whether to apply to BU for approval.
Step 2 If EP applies, BU observes whether the project is qualified, and then demands a (nonnegative) bribe.
Step 3 If EP pays the bribe, the project is approved; otherwise it is not approved, EP derives no value from it, and loses any previously incurred costs.

Let

$$\begin{aligned} q &= 1 \text{ or } 0 \text{ according as EP does or does not qualify the project,} \\ a &= 1 \text{ or } 0 \text{ according as EP does or does not apply to BU,} \\ b &= \text{the bribe demanded by BU } (b \geq 0), \\ p &= 1 \text{ or } 0 \text{ according as EP does or does not pay the bribe.} \end{aligned} \tag{3}$$

Without loss of generality, assume that $a = 0 \Rightarrow p = 0$, and that $a = 0 \Rightarrow q = 0$.

A *strategy for EP* is a triple (Q, A, P) of functions that determine his or her actions as a function of his or her information at the corresponding step; therefore,

$$\begin{aligned} q &= Q(V), \\ a &= A(V), \\ p &= P(V, b). \end{aligned} \tag{4}$$

A strategy for BU is a function B that determines the bribe demanded as a function of whether the project is qualified:

$$b = B(q). \tag{5}$$

The payoffs to EP and BU are, respectively,

$$U_0 = -qrV - ac + p(V - b), \tag{6}$$

$$U_1 = pb. \tag{7}$$

As usual, an *equilibrium* of the game is a pair of strategies, one for each player, such that no player can increase his or her expected payoff by *unilaterally* changing his or her strategy. A strategy is (weakly) *undominated* if there is no other strategy that yields the player as high a payoff for all strategy profiles of the other players, and a strictly higher payoff for some strategy profile of the other players. We shall confine ourselves to equilibria in undominated strategies.

Theorem 1 *If* $c > 0$, *then there is no equilibrium in which the probability that a project is approved is strictly positive.*

For the complete proof, see the Appendix. The basic idea is that if EP applies to BU, this provides BU with information about the value of the project, which in turn influences the size of the bribe that he or she demands. Suppose that EP expects BU to demand a bribe μ for a qualified project and θ for an unqualified project, and the (gross) value of the project is V. There are 3 cases.

Case 1 If $rV + \mu > \theta$ and $V > \theta$, then EP will apply with an unqualified project. In this case, BU infers that $rV + \mu \geq \theta$ and $V > \theta$. (Note that if $rV + \mu = \theta$, then EP is indifferent between qualifying and not qualifying.) BU will then demand a bribe, say b, that maximizes his or her expected bribe income, conditional on V satisfying the last two inequalities. In computing this optimal bribe demand, BU takes account of the fact that EP will pay the bribe b if and only if $V > b$, because the cost of application is "sunk". Using Lemma 1 (below), one can show that in this case BU's *optimal bribe strictly exceeds* θ. Hence it was not rational for EP to expect BU to demand the bribe θ.

Case 2 If $rV + \mu < \theta$ and $V > \theta$, then EP will apply with a qualified project. A similar argument shows that in this case BU's *optimal bribe strictly exceeds* $rV + \mu$.

Case 3 If neither Case 1 nor Case 2 holds, then EP will not apply.

In summary, the proof of the theorem shows that, *whatever bribes* EP *expects, if* EP *applies then* BU's *optimal bribe will be larger.* It follows that there is no equilibrium in which EP will apply.

As a supplement to Theorem 1, we also consider the case in which there is no cost of application. We shall show that *if* $c = 0$ *then the only equilibria are those of the following form: As above, define* μ *and* θ *by*

$$\mu = B(1), \theta = B(0);$$

then

$$\begin{aligned} A(V) &= 1 \text{ and } Q(V) = 0 \text{ for all } V, \\ P(V) &= 1 \text{ if and only if } V > b, \\ \theta &= 1/2, \\ \frac{1-r}{2} &\leq \mu < 1. \end{aligned} \tag{8}$$

We note that the following lemma (proved in Section 5.1) is an important step in the proofs of these results, because it helps derive the properties of BU's best-response mapping.

Lemma 1 *Suppose that EP has applied, and BU infers that* $h \leq V \leq k$, *where* $0 \leq h < k \leq 1$; *then BU's optimal bribe is*

$$b = \max\left\{\frac{k}{2}, h\right\}. \tag{9}$$

2.3 A track of bureaucrats

Consider the case in which there is a single EP who has a project value V and faces a *track* of N bureaucrats ($N \geq 2$) arranged in a specific order. To get the project approved, the EP must apply to and get approval or clearance from each BU in the prescribed order. If the project is rejected by any BU, the game ends, and the EP does not proceed farther in the track. That the project value V is uniformly distributed over the unit interval is common knowledge; but only the EP observes the realized value at the beginning of the game. No bureaucrat knows the bribes demanded by other bureaucrats.

We sketch the rather obvious modifications of the notation by using the appropriate subscript. First, the EP decides whether to qualify the project ($q = 1$) or not ($q = 0$). Of course, the EP will not qualify if he or she does not intend to apply to BU_1. Let $a_n = 1$ or 0 according as the EP does or does not apply to $\text{BU}_n (n \geq 1)$. If the EP applies to BU_n, he or she incurs a cost $c \geq 0$ (assumed to be the same for all bureaucrats), and then learns the magnitude of b_n, the bribe demanded by BU_n. Let $p_n = 1$ or 0 whether the EP does or does not pay the bribe, respectively. Note that if $a_n = 0$ or $p_n = 0$ for any $n \geq 1$, the game is over. Naturally, if $a_n = 0$ then $p_n = 0$, and if $p_n = 0$, then $a_m = 0$ for all $m > n$.

Denote by step 0 the initial part of the game in which the EP decides whether to qualify the project, and call the part of the game in which the EP faces BU_n the n-th step ($n = 1, \ldots, N$). The action of the EP at step 0 is q, and at the n-th step is a pair (a_n, p_n). The action of BU_n is b_n (the convention is applied even for steps at which $a_n = 0$).

For $n \geq 1$, let H_n denote the history of the game through step n (i.e. the sequence of actions taken by all the players through n). A strategy for the EP is a sequence, $\alpha = \{Q, A_1, P_1 \ldots A_n, P_n\}$, of functions that completely specify the EP's actions at each step according to:

$$\begin{aligned} q &= Q(V), \\ a_n &= A_n(V,\ H_{n-1}), \\ p_n &= P_n(V,\ H_{n-1},\ a_n,\ b_n). \end{aligned} \tag{10}$$

Recall that it is assumed that no bureaucrat knows the bribes demanded by other bureaucrats. Therefore, BU_n's bribe depends only on the quality of the project: his or her strategy is a function B_n that determines his or her bribe demand according to

$$b_n = B_n(q).^3 \tag{10'}$$

To complete the formal description of the game we describe the players' pay-offs. The pay-off of BU_n is the bribe, if paid:

$$u_n = p_n b_n. \tag{11}$$

The pay-off for the EP is the value of the project if approved minus the sum of the application costs and bribes paid (whether or not the project is completely approved). Therefore, the EP's pay-off is:

$$u_0 = p_N V - \sum_{1 \le n \le N} (a_n c + p_n b_n). \tag{11'}$$

Finally, assume that

$$0 \le Nc < 1. \tag{12}$$

The main result paralled to the case of a single bureaucrat is now stated. We should perhaps stress that with $N \ge 2$ bureaucrats, the following result is valid when $c \ge 0$.

Theorem 2 *There is no equilibrium in which the project is approved with positive probability.*

Here is an informal sketch of a "proof". For an equilibrium with undominated strategies, the bribes b_n demanded by BU_n must be all strictly positive. Hence, if the EP ever applies to the *last* bureaucrat $BU_N (N \ge 2)$, the bureaucrat knows that the EP has already incurred a positive cost *even when* $c = 0$. This corresponds to the case of a single BU with $c > 0$, and our earlier result applied to $BU_N (N \ge 2)$ leads to the conclusion of Theorem 2.

In what follows we shall simply focus on

$$0 < Nc < 1. \tag{12'}$$

[3] We shall also write $b_n = (\mu_n, \theta_n)$, where $\mu_n = B_n(1)$ and $\theta_n = B_n(0)$ (see (8)). We shall indulge in other abuses of notation when the meaning is clear from the context!

It is useful to have a formal statement of the EP's best response to any strategy profile of the BU. To this effect write

$$\begin{aligned} \theta &= \sum_{n=1}^{N} B_n(0), \\ \mu &= \sum_{n=1}^{N} B_n(1), \end{aligned} \tag{13}$$

$$\begin{aligned} \gamma_n(q) &= \sum_{m \geq n} [c + B_m(q)] \\ &= (N - n + 1)c + \sum_{m \geq n} B_m(q), \end{aligned} \tag{14}$$

$$\gamma(q) = \gamma_1(q). \tag{15}$$

Lemma 2 *Let BU_n's strategy be B_n, $n = 1, \ldots N$. Then the EP's best response is:*

$$a_1 = 1 \Leftrightarrow V > \min_q \left\{ \frac{\gamma(q)}{1 - qr} \right\}, \tag{16}$$

$$q = 1 \Leftrightarrow a_1 = 1 \textit{ and } \frac{\gamma(1)}{1 - r} < \gamma(0), \tag{17}$$

$$a_n = 1 \Leftrightarrow V > \gamma_n(q), \quad n > 1, \tag{18}$$

$$p_n = 1 \Leftrightarrow V > b + \sum_{m > n} [c + B_m(q)]. \tag{19}$$

Our next step is to construct a family of strategy profiles, to be called *null strategy profiles* (NSP), and to show that a NSP is an equilibrium in a precise sense.

A particular NSP is characterized by parameters $b'_n(q)$, where $q = 0$, 1 and $n = 1, \ldots,$ N. Here $b'_n(q)[\equiv B'_n(q)]$ represents the bribe that the EP expects BU_n to demand, and also the bribe that BU_n plans to demand. These bribes are required to satisfy the following conditions. Let M be a positive number such that

$$\max(1 - c, \ 1/2) < M < 1; \tag{20}$$

then

$$\begin{aligned} 0 &< b'_n(q) < 1, \text{ for } q = 0, 1; \quad n < N, \\ b'_N(q) &= M, \text{ for } q = 0, 1. \end{aligned}$$

The EP's strategy is given by Lemma 2, and BU_n's strategy is to demand $b'_n(q)$.

As is common in game theoretic analysis, we wish to confine attention to equilibria in which the strategies are in some sense "credible", and this involves examining the behavior of the system off an equilibrium. To this end, it is convenient to replace the requirement that the equilibrium strategies be undominated by a condition that we call *admissibility*, which is in some sense more demanding, but also more complex to state formally. First, we say that a strategy of BU_n is admissible if the bribes are strictly between 0 and 1. A bribe profile is admissible if each BU's strategy is admissible. A strategy for the EP is *admissible* if it is a best response to some admissible bribe profile.

For a BU we alter somewhat the definition of an undominated strategy. An admissible strategy for BU_n is *quasi-undominated* if there is no other strategy that yields him or her at least as high a pay-off for all admissible strategy profiles of the other players, and a strictly higher pay-off for some admissible strategy profile of the other players.

Finally, a strategy profile is admissible if the EP's strategy profile is admissible, and each BU's strategy profile is admissible and quasi-undominated. We can now state the following:

Theorem 3 *Suppose that for each* $n \geq q\ 1$,

$$1/2 < b'_n(q) < 1, \quad q = 0, 1;$$

then the corresponding NSP is an admissible equilibrium and for every value of V, the EP does not apply to BU_1.[4]

Two points need to be stressed. First, there may be other equilibria of the one-stage game in which the probability that the EP's project is approved is zero. Second, in an NSP equilibrium, each player has a zero pay-off. This property of an NSP equilibrium will be important in the framework of the repeated game, in which the threat of reverting to an NSP will, under certain conditions, deter a BU from deviating from "cooperative-like" behavior.

3 A "repeated" game with multiple bureaucrats and a sequence of entrepreneurs

We shall see that a transition from a static 'one-stage' game to a repeated game opens up new possibilities.

3.1 A sequence of entrepreneurs

Suppose that the one-stage game of the preceding section is repeated in an infinite sequence of periods, with a succession of EP with independent and identically distributed (i. i. d.) project values, but the same track of BU. With a slight abuse of standard terminology, we shall call this game the *supergame*. Strictly speaking, the supergame is not a repeated game,

[4] This theorem is a generalization of theorem 2 in Lambert-Mogiliansky *et al.* (2007), which deals only with the case of extortion. Since the extension of the proof to include the phenomenon of hold-up is straightforward, it is omitted here.

because the EP change from period to period. However, because the project values are i. i. d., the method of analysis and the attainable results are similar to those of a strictly repeated game. In particular, repeating the one-stage null-strategy profile (admissible) equilibrium in every period, regardless of the history of play in the previous periods, is an (admissible) equilibrium of the supergame.

Let $V(t)$ denote the potential project value of EP(t), the EP in period t $(t = 0, 1, 2, \ldots)$. The values $V(t)$ are independent and uniformly distributed over the unit interval (and this is *common knowledge*). Only EP(t) knows the realized value of $V(t)$. In each period, EP(t) and the (same) track of BUs play the game described in the last section. With an obvious corresponding notation, action variables in period t are $q(t)$, $a_n(t)$, $p_n(t)$ and $b_n(t)$. A player's strategy determines in each period t his or her actions as a function of his or her information about the past history of the game. The pay-off of each EP(t) is defined as in the one stage game $(11')$. The pay-off of each BU$_n$ is the sum of the discounted one period pay-offs, where the constant discount factor (assumed to be the same for all the bureaucrats) is denoted by δ, $0 < \delta < 1$.

In the equilibria to be studied here, the bribes demanded on the equilibrium path are called *normal bribes*. On the equilibrium path, an EP applies if his or her project value exceeds the application cost plus the normal bribes, having chosen optimally between qualifying and not qualifying the project. Once the EP has applied and learned about the bribe, he or she pays if the project value exceeds the bribe. Call this EP's *normal behavior*. We say that a BU *defects* in a particular period if he or she demands a bribe that strictly exceeds the normal bribe. If and when the EP learn that a defection has taken place, then all the players will play the NSP of the one-stage game for T periods (where T, possibly infinite, is a parameter of the strategy profiles). These T periods constitute a *punishment phase*. After the punishment phase is over (if ever) the players will return to their normal behavior until the next defection, if any. Of course, on the equilibrium path there is no defection and no punishment phase. Because a defection (eventually) triggers a punishment phase, we follow a standard terminology and call such a strategy profile a trigger strategy profile (TSP). Following Aumann's terminology if T is finite, then the TSP is *relenting*, whereas if T is infinite, the TSP is *grim.*

If the bribe demanded for a qualified project is zero, but the bribe demanded for an unqualified project is *sufficiently large*, then all qualified projects for which $V > rV + c$ will be approved, and no bribe will be paid. We shall call this the *economically efficient outcome*.

The set of equilibria of the supergame depends on what information the players have about the previous history of the game. To this effect, we make the following assumption:

Assumption 1 *At the beginning of period t, all the current players know the history of all defections (if any) in all periods up to and including the period $t - D$, where $D \geq 1$ is an exogenously given parameter. All players know the history of the transactions in which they participated.*

We retain the assumption (see $(12')$)

$$0 < Nc < 1, \tag{21}$$

and c is the *same* for all EP(t).

The normal bribes $B_n(q)$ demanded for qualified and unqualified projects satisfy

$$B_n(q) > 0$$
$$\gamma(q) = \sum_n [c + B_n(q)] < 1. \tag{22}$$

The normal behavior of each EP is given by Lemma 2 (see (16)–(19)). Finally, after any defection by some BU is known (with a D-period lag) to others, the players play the NSP for T periods, and then return to their normal strategy profile until the next defection, if any.

Theorem 4 *For any TSP satisfying the preceding conditions there exist δ^* and T^* sufficiently large such that, for all $\delta \geq \delta^*$ and $T \geq T^*$, the TSP is an admissible equilibrium of the supergame. Furthermore, as the discount factor approaches unity, there is a sequence of TSP admissible equilibria that approach economic efficiency.*

PROOF: See the Appendix. □

3.2 Extreme equilibria of the repeated game

In this subsection we examine the properties of two classes of "extreme" equilibria of the repeated game with multiple bureaucrats, within the class of TSP equilibria studied in the previous section. The equilibria in the first class, called "second-best", maximize the social surplus in the set of all equilibria. If the social cost caused by the approval of unqualified projects is sufficiently large, then in these equilibria all projects for which the entrepreneur applies are qualified, provided that the discount factor is sufficiently close to one. We prove two results about second-best equilibria. First, the normal bribes are increasing in the order of the bureaucrats in the track. Second, replacing the track with a single bureaucrat increases the social surplus. The equilibria in the second class maximize the total expected bribe income of the bureaucrats. For these equilibria (BIME), no project is qualified. For discount factors close enough to one, it is possible for a BIME to achieve the maximum expected bribe income that would be possible if the bureaucrats were able to collude. We show that, if this "collusive outcome" is not attainable as a BIME: then (i) the normal bribes are increasing in the order of the bureaucrats in the track; and (ii) replacing the track with a single bureaucrat increases the expected bribe income, and decreases social surplus. We shall comment on the implications of these results in Section 4 below.

3.2.1 second-best equilibria

By a *second-best equilibrium* (SBE) is an equilibrium that minimizes (in the set of all equilibria) the loss in expected total social surplus due to the system of bribes. The outcome implemented in a SBE will be called a *second-best outcome*. We shall show that a second-best outcome can be implemented by a grim TSP equilibrium; that is, with a TSP for which the punishment period lasts forever ($T = \infty$).

The loss in expected surplus can come from two sources. First, the system of bribes enlarges the set of projects that are not approved but would otherwise be economically profitable. Second, unqualified projects may be approved if the corresponding bribes are paid, resulting in some social loss. The social costs due to the approval of unqualified projects have not played any role in the formal analysis, because it has been assumed that the players do not take such costs into account in their own calculations. In what follows, these costs will not be modelled explicitly. Instead, it is assumed that the social costs of approving unqualified projects are sufficiently large so that, for large enough discount factors, all projects that are undertaken in a second-best equilibrium will be qualified. With this assumption, the net value of qualified projects (net of the cost of qualification), will be uniformly distributed on the interval $[0, 1-r]$. Hence, by a change of scale, this is equivalent to a model in which the cost of qualification is zero, and the bribes demanded for unqualified projects are "sufficiently large". This simplifies the analysis considerably.

Before proceeding to the detailed analysis of second-best equilibria, we explain informally why second-best outcomes can be attained in the class of grim TSP equilibria, which we shall denote by $\mathcal{G}$. First, each EP participates for a single period, and he or she has no short-run incentive to deviate from his or her equilibrium strategies in a TSP equilibrium (neither in a normal phase nor in a punishment phase). Second, although the BUs have a short-run incentive in a normal phase, they are deterred by the threat of triggering a switch to a punishment phase. Furthermore, if the punishment phase lasts forever, then a BU's payoff is zero forever once a defection is detected (after D periods). However, zero is the lowest payoff that a BU can receive, and so the threat of triggering a switch to an infinitely long punishment phase is "maximal". (Technically, zero is each BU's *maximum payoff*.) Hence, an equilibrium that minimizes the social loss in the set $\mathcal{G}$ also minimizes the social loss in the set of all equilibria. We note that there may be second-best equilibria that are not in $\mathcal{G}$, but in what follows we confine our attention to those in $\mathcal{G}$.

Recall that if all projects are qualified, then (see Appendix) the social loss is an increasing function of the total of the normal bribes, $B \equiv \mu_1 + \mu_N$.

Now suppose that BU_n considers defecting at some period, which we may take to be period 0. If he or she demands a bribe b, then an EP with project value V will pay the bribe if and only if V exceeds b plus the total of the remaining application costs and normal bribes in the track. Given the vector $\boldsymbol{\mu} = (\mu_1 \ldots, \mu_N)$ of normal bribes, one can calculate the optimal one-period defection payoff, υ_n; that is, the maximum one-period payoff that BU_n can obtain if he or she defects. On the other hand, if BU_n does not defect, and he or she and all other players continue to play their equilibrium strategies, then BU_n will obtain each period a payoff of, say, u_n^*. As in the proof of Theorem 4, BU_n will be deterred from defecting if

$$\frac{\upsilon_n}{u_n^*} \leq g(\delta, \infty, D),$$

where

$$g(\delta, \infty, D) = \frac{1}{1-\delta^D}$$

(recall that $T = \infty$). We can rewrite this constraint as

$$u_n^* - k\upsilon_n \equiv f_n(\boldsymbol{\mu}) \geq 0, \tag{23}$$

where

$$k \equiv 1 - \delta^D. \tag{24}$$

Hence, *for a second-best equilibrium in* $\mathcal{G}$, the vector $\boldsymbol{\mu}$ *of bribes minimizes the total bribe,* $B \equiv \mu_1 + \cdots + \mu_N$, *subject to the constraints,*

$$f_n(\boldsymbol{\mu}) \geq 0, \quad n = 1, \ldots, N. \tag{25}$$

In what follows, we shall refer to these last constraints as the *incentive constraints*.

A first question is: which incentive constraints, if any, are binding in an SBE?

Lemma 3 *For any second-best equilibrium, if k is sufficiently small, then all the incentive constraints are binding.*

As a corollary of the preceding lemma we have:

Theorem 5 *For any second-best equilibrium, if k is sufficiently small, then the normal bribes* μ_n *are strictly increasing in n.*

Lemma 3 can also be used to determine the effect of replacing the track of *N* bureaucrats with a "single window" of one bureaucrat, but with an application cost *Nc*.

Theorem 6 *If the track of N bureaucrats is replaced by a "single window" of one bureaucrat, but with an application cost Nc, then the total second-best equilibrium bribe is strictly reduced.*

It is interesting to consider the special case in which all of the bribes in the track are constrained to be equal. For example, this might be the case if the BU had a social norm of "equal treatment". Let b denote the common normal bribe. An "optimal equal-treatment equilibrium" minimizes b in $\mathcal{G}$. The argument leading to Theorem 5 implies that only the incentive constraint for the last BU will be binding. From this it is straightforward to verify that the optimal common normal bribe minimizes μ subject to the constraint

$$4\mu(1 - Nc - Nb) - k = 0. \tag{26}$$

The smaller root of this equation is

$$b = \frac{(1 - Nc) - [1 - Nc)^2 - kN]^{1/2}}{2N}, \tag{27}$$

which decreases to zero as k decreases to zero. Furthermore, μ^* is decreasing in the application cost, c (for sufficiently small k) and equals

$$\mu^{**} = \frac{1 - [1 - kN]^{1/2}}{2N}$$

when $c = 0$. Finally, it is easily verified that the total normal bribe, Nb^*, is larger than the total normal bribe in a SBE, which is consistent with Theorem 6.

Finally, we point out that the entire analysis of this subsection could be carried out for the case in which, T, the length of the punishment phase, is finite (and fixed). Therefore, a *third-best equilibrium* would minimize the social loss in the set of all TSP equilibria (with T fixed, finite or infinite). One would obtain the same results corresponding to Lemma 3 and Theorems 5 and 6, except that "second-best" would be replaced by "third-best", and k would be given by

$$k = g(\delta,\ T,\ D) = \frac{1 - \delta^{D}}{1 - \delta^{D+T}}. \tag{28}$$

Of course, third-best equilibria would not be second-best unless T were infinite.

3.2.2 Bribe-income-maximizing equilibria

A BIME is an equilibrium that maximizes, *in the class of TSP equilibria*, the total expected bribe income of the BU. Notice that in this definition we do not restrict the trigger strategies to be grim. Because the potential profit available to an entrepreneur (before subtracting any bribes) is maximized when his or her project is not qualified, the potential bribe income is also maximized when no project is qualified. Therefore, in a BIME the bribes $B_n(1)$ demanded for qualified projects will be large enough so that no entrepreneur will qualify his or her project. Let $\theta_n = B_n(0)$ be the normal bribe demanded by BU_n for an unqualified project, and let

$$B = \sum_{n} \theta_n. \tag{29}$$

Let L denote the total expected bribe income of the BU. If the BU could commit to a bribe profile they might be able to obtain a total expected bribe income larger than what they could obtain in a TSP equilibrium. One can show that the maximum such "collusive" expected bribe income is obtained when $B = (1 - Nc)/2$.

Again, let υ_n be the maximum one-period payoff that BU_n can obtain if he or she defects, and u_n^* be his or her normal one-period payoff. Corresponding to (25), the incentive constraints are

$$\begin{aligned} f_n(\boldsymbol{\theta}) &= u_n^* - k\upsilon_n \geq 0, \quad n = 1, \ldots, N, \\ \boldsymbol{\theta} &= (\theta_1, \ldots, \theta_N). \end{aligned} \tag{30}$$

Corresponding to Lemma 3 we have:

Lemma 4 *For any BIME, if $B \neq (1 - Nc)/2$ and if k is sufficiently small, then all the incentive constraints are binding.*

We shall call a BIME *binding* if all of the incentive constraints are binding. Corresponding to Theorem 5 we have:

Theorem 7 *For any binding BIME, if k is sufficiently small, then the normal bribes θ_n are strictly increasing in n.*

The theorem that corresponds to Theorem 6 (for second-best equilibria) has, in a sense, the *opposite* conclusion, namely that, in the case of a binding BIME, replacing a track of N BU with a single BU and the same total application cost *increases* the total expected bribe income. However, the logic is similar in that, in both cases, replacing a track of N BU with a single BU relaxes the incentive constraints.

Theorem 8 *For any binding BIME with $N > 1$ bureaucrats, replacing the track with a single BU and the same total application cost increases the total expected bribe income.*

Finally, if k is small enough, but still positive, then the BU can attain in equilibrium the optimal collusive expected bribe income.

Theorem 9 *If k is sufficiently small, then there is a BIME in which the total expected bribe income is the maximum possible, and $B = (1 - Nc)/2$.*

4 Implications and extensions

In this section we comment on the implications of our results for two proposed reforms, and briefly mention some directions for further research. For a fuller discussion of this material, see Lambert-Mogiliansky *et al.* (2007).

4.1 Two policy implications

Theorems 6 and 8 shed some light on the potential value of a proposed reform called called the "single-window policy" or "one-stop shop". Although there are several versions of this reform, a common feature is that the entrepreneur meets with only a single bureaucrat. Our analysis suggests that the single-window policy need not always lead to a reduction in the bribe burden. Theorem 6 shows that a switch to a single window (with the same total cost of application) in the second-best equilibrium does reduce the total amount of the bribe, and hence, reduces the social loss. However, Theorem 8 shows that this switch will *increase* the social loss in the bribe-income-maximizing equilibrium, unless the bribe income is already the maximum possible that could be obtained through collusion of the bureaucrats. Of course, there may be reasons to recommend the single-window policy that are not reflected in our present model, such as the possible increase in the "transparency" of the approval process and the greater likelihood of detecting illegal activities.

Our model enables us to assess another proposed reform, the rotation of bureaucrats among tracks. The standard (informal) argument in favor of such measures is that they reduce the opportunities for corrupt practices based on long-standing relationships. However, note that the extreme form of rotation (i.e. a change of bureaucrats every period), corresponds to the one-stage version of our model (Section 2), in which the system of bribes causes the process of project approval to break down completely. However, the "long-standing relationship" corresponds to the repeated-game model of Section 3, with a multiplicity of equilibria in which both project approval and bribery take place. It is beyond the scope of this paper to discuss the many possible rotation policies but here is one that can be analyzed explicitly in the framework of our model. Suppose that each track has a constant per-period probability, π, of being replaced by another track with different bureaucrats. Therefore, $\pi = 0$ gives us our model of the infinitely repeated game with one track, and $\pi = 1$ gives us our model of the one-stage game. One can show that increasing this probability is formally equivalent to reducing the discount factor. This, in turn, increases the mediating parameter k (28). Because there is a multiplicity of equilibria for each value of k, the effect of decreasing k is ambiguous without a rule that selects a unique equilibrium for each value of k. For example, if we focus on, say, the bribe-income-maximizing equilibria, then an increase in π leads to a decrease in social loss. In contrast, in the third-best (and, hence, second-best) equilibria an increase in π leads to an increase in social loss.

4.2 Some possible extensions

We conclude with a couple of suggestions for future research. First, note that the assumption that the project value V is uniformly distributed over [0,1] has been used at several steps in our proofs. It will be of interest to extend our analysis to more general cases and identifying relevant properties of the density function of V. Second, the model itself can be enriched (and, from anecdotal evidence, made more realistic) by introducing other agents: like intermediaries who facilitate the payment of bribes and make enforcement of laws more difficult. Finally, it would be useful to formally model supervision and detection.

Appendix

Proofs of Theorem 1 and Lemma 1

We first prove Lemma 1. EP will pay the bribe b iff $V > b$. The conditional distribution of V, given that $h \leq V \leq k$, is uniform on the interval $[h, k]$. BU's optimal bribe maximizes $b \Pr\{V > b | h \leq V \leq k\}$, and

$$b \Pr\{V > b | h \leq V \leq k\} = \begin{cases} b, & 0 \leq b \leq h, \\ b(k-b)/(k-h), & h \leq b \leq k, \\ 0, & b \geq k. \end{cases}$$

An elementary calculation now yields the conclusion of the lemma. □

To prove Theorem 2.1 (in the case in which $c > 0$), we first calculate EP's best response to a strategy, B, of the BU. If EP were to apply, then he or she would prefer to qualify the project if and only if (iff) $rV + \mu \leq \theta$, or

$$Q(V) = 1 \text{ if } V \leq \frac{\theta - \mu}{r}, \tag{31}$$

$$Q(V) = 0 \text{ if } V > \frac{\theta - \mu}{r}. \tag{32}$$

The entrepreneur does apply if and only if the project's value exceeds the sum of the relevant costs, that is, if and only if

$$V > c + Q(V)(rV + \mu) + [1 - Q(V)]\theta. \tag{33}$$

If EP has applied, and BU demands a bribe b, then EP pays iff $V > b$. (Any costs previously incurred by EP are "sunk".)

Case 1 $0 < (\theta - \mu)/r < 1$.

From the preceding properties of EP's best response, we infer that EP applies iff

$$\begin{aligned} V &\leq \frac{\theta-\mu}{r} \text{ and } V > rV + c + \mu, \text{ (in which case } q = 1 \text{) or} \\ V &> \frac{\theta-\mu}{r} \text{ and } V > c + \theta, \text{ (in which case } q = 0 \text{);} \end{aligned}$$

which is equivalent to

$$\begin{aligned} \frac{c+\mu}{1-r} &< V \leq \tfrac{\theta-\mu}{r}, \text{ or} \\ V &> \max\left\{c+\theta, \frac{\theta-\mu}{r}\right\}. \end{aligned} \tag{34}$$

One easily verifies that either

$$0 < \frac{c+\mu}{1-r} < c + \theta < \frac{\theta-\mu}{r} < 1, \tag{35}$$

or

$$0 < \frac{\theta-\mu}{r} < c + \theta < \frac{c+\mu}{1-r} < 1, \tag{36}$$

and that the first set of inequalities is equivalent to

$$(1-r)\theta > \mu + rc. \tag{37}$$

It is useful to divide Case 1 into two subcases.

Case 1.1

$$0 < \frac{c+\mu}{1-r} < c + \theta < \frac{\theta-\mu}{r}.$$

In this case, EP's best response satisfies:

$$\begin{aligned} V &< \frac{c+\mu}{1-r} \Rightarrow q = a = 0, \\ \frac{c+\mu}{1-r} &< V < \frac{\theta-\mu}{r} \Rightarrow q = a = 1, \\ \frac{\theta-\mu}{r} &< V \Rightarrow q = 0 \text{ and } a = 1. \end{aligned} \tag{38}$$

Case 1.2

$$0 < \frac{\theta-\mu}{r} < c + \theta < \frac{c+\mu}{1-r}.$$

In this case, EP's best response satisfies:

$$\begin{aligned} V &< c + \theta \Rightarrow q = a = 0, \\ V &> c + \theta \Rightarrow q = 0 \text{ and } a = 1. \end{aligned} \tag{39}$$

Case 2 $\theta \leq \mu$.

In this case, for all $V > 0, rV + \mu > \theta$, and, hence, $Q(V) = 0$. Therefore, EP's best response satisfies the same condition as in Case 1.2.

Case 3 $\theta - \mu \geq r$.

In this case, for all $V < 1, rV + \mu < \theta$, and, hence, $Q(V) = 1$. Hence EP's best response satisfies:

$$\begin{aligned} V &< \frac{c+\mu}{1-r} \Rightarrow q = a = 0, \\ V &> \frac{c+\mu}{1-r} \Rightarrow q = a = 1. \end{aligned} \tag{40}$$

We now examine the possibility of an equilibrium in each of the above cases.

Case 1.1 If $a = q = 1$, then

$$\frac{c+\mu}{1-r} < V < \frac{\theta-\mu}{r},$$

and so, by Lemma 1, the optimal bribe is

$$\max\left\{\frac{\theta-\mu}{2r}, \frac{c+\mu}{1-r}\right\}.$$

If $a = 1$, $q = 0$, then

$$V \geq \frac{\theta-\mu}{r},$$

and so the optimal bribe is

$$\max\left\{\frac{1}{2}, \frac{\theta-\mu}{r}\right\}.$$

Hence, for an equilibrium in this region of the (μ, θ) space,

$$\mu = \max\left\{\frac{\theta-\mu}{2r}, \frac{c+\mu}{1-r}\right\}, \tag{41}$$

$$\theta = \max\left\{\frac{1}{2}, \frac{\theta-\mu}{r}\right\}. \tag{42}$$

However, from the first of the above equations,

$$\mu < \frac{\mu}{1-r} \leq \max\left\{\frac{\theta-\mu}{2r}, \frac{c+\mu}{1-r}\right\},$$

unless $c = \mu = 0$. However, if $\mu = 0$, then from the second of the above equations,

$$\theta < \frac{\theta}{r} \leq \max\left\{\frac{1}{2}, \frac{\theta}{r}\right\},$$

unless $\theta = 0$. However, $\mu = \theta = 0$ yields BU a payoff of zero, against any strategy of EP, and, hence, is weakly dominated. Therefore, there is no equilibrium in this region, *even if* $c = 0$.

Case 1.2 In this case, $q = 0$. If $a = 1$, then $V > c + \theta$, and, again by the lemma, the optimal bribe is

$$\max\left\{\frac{1}{2}, c+\theta\right\},$$

which strictly exceeds θ unless $c = 0$. Hence, no equilibrium exists in this region unless $c = 0$.

Case 2 In this case, as in Case 1.2, $q = 0$, and if $a = 1$, then $V > c + \theta$. It follows, as in Case 1.2, that no equilibrium exists in this region unless $c = 0$.

Case 3 In this case, $q = 1$. If $a = 1$, then

$$V > \frac{c+\mu}{1-r},$$

and an argument like that in Case 1.1 shows that there is no equilibrium unless $c = \mu = 0$. However, if $\mu = 0$, then by the definition of Case 3,

$$\frac{\theta}{r} \geq 1,$$

which implies that,

$$rV + \mu = rV < \theta, \qquad \text{for all } V < 1.$$

Hence, $a = 1$ for all $V < 1$, so that BU's payoff is zero with probability one. Hence, in this region there is no equilibrium in undominated strategies, even if $c = 0$. *This concludes the proof of the first part of the theorem with* $c > 0$.

It remains to consider the second part of the theorem, with $c = 0$. Because application is costless, EP will apply, for any strictly positive value of the project. Any strategy (μ, θ) of BU satisfying

$$\begin{aligned} \theta &= 1/2, \\ \frac{1-r}{2} &\leq \mu < 1 \end{aligned}$$

falls under Cases 1.2 or 2, and, hence, EP's best response requires that $Q(V) = 0$ for all V. EP will pay the bribe demanded if and only if it is strictly less than V. Hence, we have shown that the strategy of EP described by the theorem is a best response to BU's strategy.

Because $Q(V) = 0$ for all V, the fact that the project is not qualified carries no information about the magnitude of its value. Hence, by the lemma, the optimal bribe is

$$\theta = \frac{1}{2}. \tag{43}$$

Suppose now that EP applies with a qualified project (i.e. $a = q = 1$). Because this behavior is contrary to EP's strategy, we must describe what beliefs BU infers about the value of V. We shall suppose that BU believes that $h \leq V \leq k$, for some numbers h, k. If

$$\frac{1-r}{2} \leq \mu < \frac{1}{2},$$

take $h = 0$, $k = 2\mu$; then by Lemma 1, BU's optimal bribe is μ. However, if

$$\frac{1}{2} \leq \mu \leq 1,$$

then take $h = \mu$, $k = 1$, and again BU's optimal bribe is μ. This concludes the proof of Theorem 1. □

Proof of Theorem 4

The proof of Theorem 4 uses calculations that are standard in the published literature, so we provide only a sketch. Let u_n^* denote BU$_n$'s expected one period pay off during a normal phase and V_n denote the least upper bound for the expected payoff to BU$_n$ from a defection, say in period 0. Then one can calculate that a defection is deterred if

$$(1 + \delta + \cdots + \delta^{D-1})v_n \leq (1 + \cdots + \delta^{D+T-1})u_n^*,$$

or,

$$u_n/u_n^* \le \frac{1-\delta^{D+T}}{1-\delta^D} \equiv g(\delta, T, D).$$

Note that

$$\lim_{\substack{\delta\to 1\\ T\to\infty}} g(\delta, T, D) = \infty,$$

and this limit is approached monotonically. Hence, for all T "sufficiently large" and δ "sufficiently close to unity", BU_n does not defect.

Note that if μ tends to 0 and θ remains sufficiently large, then all EPs who apply will qualify their projects. More precisely, qualification will be preferred by all EPs if $0 > r + \mu$, and in this case an EP will apply with a qualified project if

$$V > rV + c + \mu,$$

or

$$V > \frac{c+\mu}{1-r}.$$

One can verify that if μ tends to 0 and θ remains large, the ratio u_n^*/v_n tends to zero. However, for any finite ratio v_n/u_n^*, there is a pair (δ, T) such that the corresponding TSP is an admissible equilibria.

Second-best equilibria: Proofs of Lemma 3, Theorem 5, and Theorem 6

Recall that b_n is the *normal bribe* demanded by BU_n in an equilibrium in $\mathcal{G}$. Recall, too, that in a second-best equilibrium all active projects are qualified. Hence, without loss of generality, by a change in scale one can take $r = 0$, and assume that $\theta_n > \mu_n$. Define:

$$\begin{aligned} C_n &= \sum_{m\ge n} (c+\mu_n), \quad 1 \le n \le N, \\ C_{N+1} &= 0, \\ C &= C_1. \end{aligned}$$

Assume

$$C < 1. \tag{44}$$

Suppose that BU_n considers defecting at some period, which we may take to be period 0. If he or she demands a bribe b, then an EP with project value V will pay the bribe if and only if $V > b + C_{n+1}$. Hence, BU_n's optimal defection demand, b'_n, maximizes

$$b\Pr\{V > b + C_{n+1} | V > C\},$$

which is equal to

$$\begin{aligned} b, \ \text{ if } b + C_{n+1} &\le C; \\ b\frac{(1-b-C_{n+1})}{(1-C)}, \ \text{ if } C &\le b + C_{n+1} \le 1, \\ 0, \quad \text{if } 1 &\le b + C_{n+1}. \end{aligned} \tag{45}$$

The second line is maximized when

$$b = \frac{1 - C_{n+1}}{2},$$

so

$$b'_n = \max\left\{C - C_{n+1}, \frac{1 - C_{n+1}}{2}\right\}. \tag{46}$$

Assume that

$$Nc < \frac{1}{2}; \tag{47}$$

then for $\mu_1, \ldots, \mu_N$, sufficiently small,

$$b'_n = \frac{1 - C_{n+1}}{2}. \tag{48}$$

Henceforth, we shall assume that equation (47) is satisfied, which implies that $b'_n > C - C_{n+1}$.

In this case, BU_n's maximum expected one-period defection payoff is

$$\upsilon_n = \frac{(1 - C_{n+1})^2}{4}. \tag{49}$$

However, BU_n's normal expected one-period payoff is

$$u_n^* = \mu_n(1 - C), \tag{50}$$

since $\mathrm{EP}(t)$ will apply only if $V(t) > C$. Hence, BU_n is deterred from defection if and only if

$$\begin{aligned} \upsilon_n &\leq g(\delta, D, \infty)u_n^*, \text{ where} \\ g(\delta, D, \infty) &= \frac{1}{1 - \delta^D}. \end{aligned}$$

Define

$$\begin{aligned} f_n(\boldsymbol{\mu}) &= \mu_n(1 - C) - \left(\frac{k}{4}\right)(1 - C_{n+1})^2, \\ \boldsymbol{\mu} &= (\mu_1, \ldots, \mu_N), \\ k &= 1 - \delta^D. \end{aligned} \tag{51}$$

Assume that δ and D are such that

$$0 < k < 1, \tag{52}$$

which, for any $D < \infty$, is true for δ sufficiently close to 1. The expected value of approved projects is

$$\int_C^1 \upsilon d\upsilon = \frac{1 - C^2}{2}.$$

Hence, *a second-best vector* $\boldsymbol{\mu}$ *of bribes minimizes C subject to*

$$f_n(\boldsymbol{\mu}) \geq 0, \quad n = 1, \ldots, N. \tag{53}$$

Recall that these last inequalities are called the *incentive constraints.*

To prove Lemma 3, suppose, to the contrary, that $\boldsymbol{\mu}$ is the vector of normal bribes for a SBE, and that for some i, $f_i(\boldsymbol{\mu}) > 0$. Let $\mathbf{h} = (h_1, \ldots, h_N)$ be a vector such that

$$\begin{aligned} \sum_1^N h_n &= 0, \\ h_n &\neq 0, \quad n = 1, \ldots, N, \end{aligned} \tag{54}$$

and define

$$y_n(\mathbf{h}) = \mu_n + h_n, \quad n = 1, \ldots N, \tag{55}$$

$$y(\mathbf{h}) = [y_1(\mathbf{h}), \ldots, y_N(\mathbf{h})]. \tag{56}$$

Note that C remains unchanged as $\mathbf{h}$ changes.

Recall that in an equilibrium with undominated strategies every normal bribe must be strictly positive. Hence, because the functions f_N are continuous, for every sufficiently small k there exists $\mathbf{h}$ sufficiently small (in the Euclidean metric, and satisfying (54), such that

$$f_n[y(\mathbf{h})] > 0, \quad n = 1, \ldots, N. \tag{57}$$

Because $\mu_n > 0$ for every n, $C > Nc$. Hence, there exists a vector $\mathbf{z} = (z_1, \ldots, z_N)$ sufficiently close to $y(\mathbf{h})$ such that

$$\begin{aligned} z_n &> 0, \quad n = 1, \ldots, N, \\ f_n(z_n) &\geq 0, \quad n = 1, \ldots, N, \\ \sum_1^N z_n + Nc &< C. \end{aligned}$$

Hence, z is the vector of normal bribes in a TSP equilibrium with a smaller social loss than that of $\boldsymbol{\mu}$, so $\boldsymbol{\mu}$ could not be a SBE. □

We turn to the proof of Theorem 5. If all the incentive constraints are binding, then from (51) and (53), for every n

$$f_n(\boldsymbol{\mu}) = \mu_n(1 - C) - \left(\frac{k}{4}\right)(1 - c_{n+1})^2 = 0, \tag{58}$$

and, hence,

$$\mu_n = \frac{k(1 - C_{n+1})^2}{4(1 - C)}. \tag{59}$$

For an undominated equilibrium, every normal bribe must be strictly positive; hence, C_n is strictly decreasing in n (even if $c = 0$), and so the conclusion of the theorem follows from the preceding equation.

We prove Theorem 6 now. Recall that B is the total of the normal bribes,

$$B = \sum_n \mu_n.$$

Note that $C = B + Nc$. From equation (53), the total bribe, B, in a SBE of the track with N bureaucrats satisfies

$$B(1 - C) = B(1 - Nc - B) = \left(\frac{k}{4}\right) \sum_n (1 - C_{n+1})^2.$$

Recall that $C_{N+1} = 0$, then the incentive constraint for a single bureaucrat, with an application cost of Nc is

$$\mu(1 - Nc - \mu) \geq \frac{k}{4}.$$

Note that, for the track of N BUs,

$$\sum_n (1 - C_{n+1})^2 > (1 - C_{N+1})^2 = 1.$$

Hence, a single bribe equal to B satisfies the single BU's incentive constraint with a strict inequality. Therefore, the single BU can demand a bribe strictly less than B without violating his incentive constraint, which concludes the proof of the theorem. □

Bribe-income-maximizing equilibria

Let $\theta_n = B_n(0)$ be the normal bribe demanded by BU_n for an unqualified project, and let

$$B = \sum_n \theta_n, \quad C = B + Nc. \tag{60}$$

Without loss of generality we may assume that $C < 1$. The normal expected bribe income of BU_n, and the total expected bribe income are, respectively,

$$u_n^* = \theta_n(1 - C), \tag{61}$$

$$L = B(1 - C). \tag{62}$$

L is maximized when $B = (1 - Nc)/2$.

Again, the optimal defection bribe for BU_n is given by equation (46), which is

$$b_n' = \max\left\{C - C_{n+1}, \frac{1 - C_{n+1}}{2}\right\}. \tag{63}$$

In this case, however, we cannot rule out either of the two expressions on the right-hand side. The corresponding maximum expected one-period bribe income from defection for BU_n is given by:

$$\upsilon_n = \max\begin{cases} u_n' &= (C - C_{n+1})(1 - C), \\ \upsilon_n'' &= \left(\frac{1}{4}\right)(1 - C_{n+1})^2. \end{cases} \tag{64}$$

Again, as in (53), the incentive constraints are

$$\begin{aligned} f_n(\boldsymbol{\theta}) &= u_n^* - k\upsilon_n \geq 0, \quad n = 1, \ldots, N, \\ \boldsymbol{\theta} &= (\theta_1, \ldots, \theta_N). \end{aligned} \tag{65}$$

With these formulas, the proofs of Lemma 4 and Theorems 7 – 9 are straightforward, and are omitted. □

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