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Financial Deepening and Bank Runs

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Abstract

We analyze an economy with banks and markets and uncover implications of the presence of asset markets for the run-prone banking sector. Consumers can split their endowment between a market investment and a deposit contract which admits bank runs. Banks specialize in providing ex ante liquidity insurance. Market investment acts as insurance if there is a run. Banks provide a higher degree of the liquidity insurance while facing a lower probability of a run when compared to the banks-only economy. As long as consumers invest in both the market and the deposit contract, the welfare is higher when compared to the economy with banks, or markets, alone.

JEL classification: E42, E44, G21, O16

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1   Introduction

The banking system is often viewed as the base of a financial system. It is on this base that financial markets are built. It is not surprising then that emerging economies, where financial markets are underdeveloped, tend to rely heavily on the banking system as a means of transforming savings into investment. Empirically, a close relationship between the level of the stock market and banking sector development has been documented. At the same time, banks' problems are more recurrent and insolvency is more costly in the developing world[1].

One might wonder whether the nonexistence of alternative channels to allocate funds can contribute to the instability of the banking sector. In this paper we ask how financial deepening influences the banking sector. We use the term "financial deepening" broadly to denote the development of financial markets. We model this development by introducing a possibility for economic agents to invest and trade assets in an asset market. We derive the implications of the existence of an asset market for the banks' role in liquidity provision and for the probability of a run banks face.

We develop a simple three-period equilibrium model that includes economy-wide aggregate shocks (affecting asset returns and cross-sectional distribution of preferences) and idiosyncratic preference shocks (as in Diamond and Dybvig[15]). Consumers deposit their endowments with financial intermediaries in exchange for a complex banking-financial contract. The contract specifies what part of the endowment is invested in a private market portfolio. Private portfolio can be readjusted by trading in the asset market in the interim period. Prices of assets in the market depend on the amount of liquidity in the market and thus on the aggregate state of the world. The rest of the endowment is exchanged for a demand deposit contract which promises a fixed payment independent of

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[1] See Caprio and Klingebiel [8].

the aggregate state of the world in the interim period. The reason we focus on this type of contracts is that more sophisticated contracts are not observed in practice, especially in developing countries. We refer to the demand deposit part of the banking-financial contract as a "bank".

Risk-averse consumers are *ex ante* identical. After idiosyncratic preference shocks are realized, individuals become either early consumers (who value only immediate consumption), or late consumers (who value future consumption). So there are two *ex post* types. A consumer's realized type is assumed to be private information and thus it is not possible to write type-contingent insurance contracts. Then, in a competitive market equilibrium, idiosyncratic risk is non-diversifiable. By contrast, banks can provide insurance against unobservable preference shocks by pooling depositors' resources and creating an *ex ante* cross-subsidy between *ex post* types (liquidity insurance). However, banks are subject to a run. This makes demand deposits a risky investment.

An important part of the banking-financial contract is a trading restriction. Consumers are only allowed to retrade their original portfolio holdings. This assumption allows banks to provide the *ex ante* cross-subsidy in the optimum[2]. It enables us to uncover the effects market investment has on the properties of demand deposit contracts, the probability of a run banks face, and consumers' welfare.

To evaluate the effects of the presence of the asset market on the stability of banks, we need a device that ties together changes in the deposit contract in the economy with markets and the probability of a run. For this purpose, we use the concept of a risk-factor of the run equilibrium, originally proposed by Ennis and Keister [16]. The risk-factor makes it possible to compute the probability of a run banks face in the banks-only economy and compare it to the probability of a run in the economy with banks and markets.

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[2] We discuss this assumption further in subsection 3.3.

A late consumer has to decide in the interim period whether or not to run on the bank. Running is only optimal if other late consumers run, too. This is the essence of panic-driven runs. The argument we develop here is that a late type's "incentive to run" is influenced by the possibility for consumers to invest and trade in the asset market. This is because market investment acts as insurance for consumers who deposit part of their endowment in the run-prone banking sector. In case of a run, both types of consumers are guaranteed that their wealth will be positive.

Our results can be summarized as follows. A standard finding in the bank runs literature (for banks-only economies) is that banks face a trade-off between liquidity provision and stability: The higher the cross-subsidy they create, the higher the probability of a bank run. We show by examples that the presence of the asset market *reduces* probability of a run on the banking sector thus making it more stable. At the same time, banks provide *more* liquidity when compared to the economy with banks only. With markets weakening the trade-off between the liquidity provision and the probability of a run, the banking sector "specializes" in creating the *ex ante* cross-subsidy between early and late types. In essence, banks reward risk-averse consumers for getting insurance against bank runs by investing in the market.

There is always a possibility to offer a deposit contract that is immune to runs, a so-called run proof contract. The cost of offering such a contract *ex ante* is that it provides no liquidity insurance. In banks-only economies, the run proof contract is offered once the probability of a run exceeds a certain threshold value. Below this value, the benefit of run admitting contracts (liquidity provision) outweighs the cost (possibility of a run) and runs are tolerated in equilibrium. We find in our examples that in the economy with banks and markets run proof contracts are not offered. Once the probability of a run exceeds a certain

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threshold level, consumers choose to invest in the market portfolio only. A related finding is that runs are tolerated for higher values of the probability of a run in the economy with banks and markets than in the banks-only economy. This is possible due to the existence of the insurance against bank runs provided by markets.

The *ex ante* welfare in the banking-financial economy is higher as compared to the economy with banks, or markets, alone. This result arises despite the fact that adding an additional asset (the demand deposit contract) to the incomplete asset structure does not complete markets in this economy. The driving force behind this finding is the different nature of services provided by banks and markets.

The remainder of the paper is organized as follows. In the next section, we discuss the related literature. In section 3, we describe the model. Subsequently, we analyze functioning of the asset market: optimal portfolio choice and determination of asset prices. In section 5, we examine banks' behavior (assuming the asset market is non-existent), namely properties of the deposit contract offered and when it is optimal for banks to offer a run proof contract. We introduce the concept of the risk-factor of the run equilibrium. In section 6, we analyze the economy with both banks and markets and describe the banking-financial contract. Section 7 contains some numerical examples and discussion of our main results. We conclude in section 8. For ease of exposition, all proofs are in the appendix.

2 Related Literature

This paper builds on three broad strands of literature. First strand concerns liquidity-based asset pricing. Our second building block is bank runs literature. Finally, our analysis is related to a series of papers dealing with coexistence and

interaction between banks and markets.

In modeling the asset market we follow Allen and Gale [1][3]. We introduce aggregate uncertainty. In equilibrium, price of an asset depends on the amount of liquidity in the market.

The role of banks we focus on is provision of liquidity and maturity transformation. First to address the implications of the maturity mismatch in bank's balance sheet for its stability were Diamond and Dybvig [15][4]. They showed that the simple demand deposit contract can implement the first-best (full information) allocation but it also admits a bank run equilibrium in the post-deposit game. The issue not tackled in their paper is whether consumers would be willing to deposit in the bank *ex ante*, knowing that there exists a bank run equilibrium. This problem requires solving a full pre-deposit game and introducing an equilibrium selection rule.

Peck and Shell [24] consider a broad set of possible deposit contracts and show that depositors can prefer a contract that admits sunspot-triggered bank runs in equilibrium provided that the probability of a bank run is sufficiently small. The bank can always choose a run proof contract, i.e. the contract that induces a unique no run equilibrium in the post-deposit game. Cost of choosing such a contract *ex ante* is that it provides no liquidity insurance. Ennis and Keister [16] study the effect of the possibility of a bank run on capital formation and economic growth. We adopt the concept of the risk-factor of the run equilibrium they introduced to model the strength of individuals' incentives to run. This is an important ingredient of our analysis that unveils the effect market investment has on the probability of a run. In all models above, banks face a trade-off between liquidity provision and possibility of a run[5]. By contrast,

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[3]In the paper, they examine volatility of asset prices in financial markets.

[4]See also Bryant [6].

[5]Goldstein and Pauzner (2004) reach the same conclusion in Morris and Shin's [23] framework.

presence of the asset market in our setup makes this link considerably weaker.

The strand of the literature focusing on the interaction between markets and banks and its impact on liquidity provision starts with Jacklin [21]. He allows agents in the Diamond-Dybvig model to trade bank assets at an exogenously fixed price in a competitive market. Jacklin points out that introduction of the market eliminates the ability of banks to provide liquidity by creating an *ex ante* cross-subsidy. This is due to the fact that if the banking contract continued providing liquidity insurance, a late consumer would have an incentive to make his withdrawal in period 1 and trade period-one consumption in the asset market for claims on period-two consumption. Moreover, individuals are able to achieve the only feasible, incentive compatible consumption allocation in the economy by holding assets directly and subsequently trading in the market.

Diamond [14] shows that under the assumption of the limited market participation, banks and markets can coexist. Banks' ability to create a cross-subsidy between early and late types is reduced but not eliminated (unlike in Jacklin [21]). He analyzes effects of financial development on the structure and market share of banks. Banking sector in his model is not prone to bank runs. Limited market participation leads to the illiquidity problem in the market (lower price of the asset than the one that would arise if participation was full). In this environment, existence of the banking sector can divert some liquidity demand away from the asset market and thus help improve the illiquidity problem. Our paper is looking at the interaction between banks and markets from the opposite angle. It is in the banking sector where problems can arise (bank runs). Here, the reason for consumers to invest in both banks and (indirectly) markets is acquiring liquidity insurance on one hand and buying insurance against bank runs on the other.

A recent paper by Allen and Gale [3] constructs a model of a complex finan-

cial system and uses it for evaluating government intervention and regulation of liquidity provision. Consumers give their endowments to banks and do not directly participate in the financial market. Banks have full access to a set of Arrow securities markets in period 0 and a spot market in period 1. Panic-based bank runs are ruled out by assumption. As long as markets are complete, there is no scope for welfare-improving government intervention to prevent financial crises. With incomplete markets, the outcome is "inefficient". In our model, markets are incomplete and banks are offering contracts which admit bank runs. We are able to make predictions about the probability of a run on the banking sector and show by examples that the asset market brings about a welfare improvement in the incomplete markets/incomplete contracts setup.

3   The Model

There are three periods, $t = 0, 1$, and 2 and a single homogeneous good. Consumption and asset returns are measured in terms of the good.

The economy is subject to aggregate shocks that affect return on a long-term asset, R, and aggregate demand for liquidity, λ. Let R and λ be two random variables defined on a discrete probability space. Let Θ denote the corresponding two-dimensional space of R and λ. Let $g(R)$ and $f(\lambda)$ be the marginal probability mass functions of R and λ, respectively. We assume that these two random variables are independent. Effectively, there is a finite number of aggregate states of the world, denoted by $\theta \in \Theta$, each of which is characterized by particular realizations of the aggregate demand for liquidity and the return on the long-term asset. Values of the long-term asset return R and aggregate demand for liquidity λ are realized (but not publicly revealed) at the beginning of period 1^6.

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[6] We use R and λ to denote both the random variables themselves and their realized values.

In addition to aggregate shocks, each individual is subject to idiosyncratic preference shocks, which determine his demand for liquidity (more details shortly).

3.1 Consumers

There is a $[0, 1]$ continuum of consumers, each lives for three periods. Every consumer has an endowment ω equal to 1 unit of the good in period 0.

Ex ante (as of period 0), individuals are identical. In period 1 individuals receive an idiosyncratic preference shock which will cause some of them to become "early" consumers (they only value period-one consumption, $u = u(c_1)$) and some "late" consumers (who only value period-two consumption; if they receive good early they can store it, $u = u(c_1 + c_2)$). Probability of being an early type is λ (thus, probability of being a late type is $(1 - \lambda)$). We assume $\lambda > 0$ for all realizations of λ. As is standard in the literature, we invoke "law of large numbers" convention and assume that the cross-sectional distribution of types is the same as the probability distribution λ. Hence, λ is also the fraction of consumers in population who face an urgent liquidity need in period 1. Following Peck and Shell [24], we denote the probability that the fraction of early consumers is λ conditional on a consumer's being late by $f_p(\lambda)$. Using Bayes' rule, we have that

$$f_p(\lambda) = \frac{(1 - \lambda) f(\lambda)}{\sum\limits_{\lambda} (1 - \lambda) f(\lambda)}.$$

Individuals maximize expected utility of consumption. Let $u(c_t)$ denote utility of a consumer in period t. The function $u(c)$ is twice continuously differentiable, strictly increasing, and strictly concave. Moreover, $u(0) = 0$ and the coefficient of the relative risk aversion is greater than 1 for $c \in [1, R]$ [7].

It will be clear from the context what we have in mind.

[7] An example of the utility function satisfying these properties is $u(c) = \frac{\tau_1 c}{1 + \tau_2 c}$, $\tau_1 > 0$, $\tau_2 > 1$. We will use this functional form when computing numerical examples in Sections 5 and 6. Note that we cannot use the constant-relative-risk-aversion utility function $u(c) = \frac{c^{1-\rho}}{1-\rho}$,

3.2 Asset Structure

There are two real assets in the economy: a short-term (liquid) asset and a long-term (illiquid) asset. A short-term asset is represented by the costless storage technology: it offers a return equal to 1 after one period[8]. A long-term asset yields a random return R after two periods. We assume that $R > 1$ always holds to reflect that the long-term asset is more productive than the short-term asset over the long-run. Furthermore, we assume an existence of the liquidation technology which allows to liquidate the long-term asset early, in period 1. This yields a fixed liquidation return L. We assume $L \in (0, 1)$ to reflect that early liquidation is costly. Whenever the asset market is present in the economy, the long-term asset can be sold at the market price P_1 instead of being privately liquidated.

Constant returns to scale technology allows consumers/banks to transform one unit of the good into one unit of the short-term or long-term asset in period 0. Short sales are not allowed. Asset payoffs are summarized below:

Date	$t = 0$		$t = 1$		$t = 2$
Short asset	1	$\rightarrow$	1	$\rightarrow$	1
Long asset	1	$\rightarrow$	L or P_1	$\rightarrow$	$R > 1$

3.3 Banks and Markets

Consumers always have an option to costlessly store their endowment (autarky) and thus achieve utility equal to $u(1)$. We let $\overline{u}$ denote this autarky level of utility.

Banks offer a complex banking-financial contracts in period 0. The contract

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where $\rho > 1$ is the coefficient of the relative risk aversion. This function does not satisfy $u(0) = 0$.

[8] We use the storage technology only for technical simplicity. Nothing substantial would change if the short term asset was represented by an interest-bearing, perfectly liquid security.

allows consumer to invest (part of) his endowment in a portfolio of assets, with an option to re-balance this portfolio in period 1, once the private shock is realized. Moreover, consumer chooses how much to invest in the demand deposit contract, which offers a fixed period-one return.

We assume that consumers are only allowed to retrade their original portfolio holdings. This is an important assumption. We know that in a partial equilibrium framework [21], or in a general equilibrium setup with complete markets [3], the banking allocation is weakly dominated by the market allocation when consumers participate in markets *directly*. This is because as long as the deposit contract provides an *ex ante* cross-subsidy, a late consumer has an incentive to withdraw from the bank in period 1 and sell his period-one consumption in the market in exchange for the period-two consumption (we call this phenomenon "cash-out"). By doing so he is able to achieve a higher return than the return he would get from the bank if he waited until period 2 to withdraw. Banks providing the liquidity insurance and markets cannot coexist. We find that the problem of the cash-out extends to our incomplete markets setting if we allow consumers to *directly* participate in the asset market.

There are several ways how to change the model to allow for an equilibrium coexistence between banks and markets. A notable example is an extension along the lines of Diamond [14][9]. However, possible changes introduce new mechanisms in the standard Diamond and Dybvig framework and make it harder to disentangle what role the possibility to invest in the asset market plays in stabilizing the banking sector. Instead, we choose to impose a trading rule as a part of the banking-financial contract to stay close to the traditional bank runs model.

In summary, the timing of events is as follows. In period 0, a consumer receives an endowment of 1 which he can deposit in the bank in exchange for the

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[9]See previous section for the discussion of this paper.

banking-financial contract. At the beginning of period 1, a consumer privately observes his type. Early consumers withdraw from the bank. Late consumers decide whether to withdraw in period 1 or 2. Banks pay out a promised period-one return on withdrawals. Then, there is a centralized trading in the asset market. Both early and late types adjust their market portfolio holdings. Banks trade in the market if they find it optimal to do so. Early types consume their wealth, consisting of a return on the market investment and the bank withdrawal. At the beginning of time 2, the realized return on the long-term asset R is revealed. Late types consume their wealth, consisting of a return on the market investment and the bank withdrawal.

4   Asset Market[10]

We now look at the economy with an asset market only (banks are absent). In period 0, a consumer decides how much to invest in the short-term asset and how much to invest in the long-term asset. Let α denote the fraction of his endowment invested in the short-term asset. Then, $(1 - \alpha)$ fraction is invested in the long-term asset.

At the beginning of period 1, the period-two return on the long-term asset R, the aggregate liquidity demand (fraction of early consumers in the population) λ, and consumers' idiosyncratic preference shocks are realized. Individuals privately observe the realization of the preference shock. They do not observe the realizations of R and λ. Then, assets are traded in the market. We normalize the price of period-one consumption to 1. It is easy to show that the price of the short-term asset coincides with the price of period-one consumption and is

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[10]In what follows we build on Allen and Gale [1].   The main difference between their approach and ours is that in their model, all uncertainty is resolved in the beginning of period 1. Here, the realization of the two random variables, λ and R, is *not* revealed to market participants in period 1. Thus, agents make inferences from the market price.

equal to 1^{11}. Let $P_1(\theta)$ ^{12} denote the price of one unit of the long-term asset in terms of period-one consumption in state θ. We assume that individuals know and take as given price function P_1. They submit optimal amounts of the short- and long-term assets to sell (given their expectations about the price) and price $P_1(\theta)$ clears the market. At date 0, given the expected utility consumers can achieve in period 1, they choose α optimally.

Definition 1 (Equilibrium) *Equilibrium is characterized by a price $P_1(\theta)$ that clears the asset market in state θ and period 1; optimal final consumption allocations for early and late types, respectively; and a portfolio choice $(\alpha, 1 - \alpha)$ that maximizes expected utility given P_1 in period 0.*

We first analyze asset market equilibrium in period 1. We then look at the optimal portfolio choice in period 0.

At the beginning of period 1, each individual holds a portfolio $(\alpha, 1 - \alpha)$. For now, we assume $0 < \alpha < 1^{13}$. Fraction $\lambda > 0$ of consumers becomes early and fraction $(1 - \lambda)$ becomes late. It is straightforward to show that in equilibrium, early consumers keep their initial short-term asset holdings and may choose to acquire more short-term asset using their long-term asset holdings. An early consumer prefers to sell some or all of his long-term asset holdings in the market in exchange for the short-term asset as long as $P_1(\theta) \geq L$ (since he can privately liquidate the long-term asset and gather return L per unit). Let $0 \leq b(P_1) \leq 1$ denote the optimal fraction of the long-term asset holdings to be sold. Then,

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[11] We will thus talk about the short-term asset and period-one consumption interchangeably.
[12] We use (θ) to denote dependence of the variable on the outcome for both R and λ. Henceforth, we will use (λ) to denote dependence on the outcome for λ and analogously for R.
[13] We prove this is indeed the case in Proposition 6.

an early consumer's supply is as follows

$$
b = \begin{cases} 1 \text{ if } P_1 > L \\ [0, 1] \text{ if } P_1 = L \\ 0 \text{ if } P_1 < L \end{cases} .
$$

Let S be the total amount of the long-term asset early consumers supply to the market in period 1. We have

$$
S \equiv \lambda b \left(P_1 \right) \left(1 - \alpha \right) \omega \le \lambda \left(1 - \alpha \right) \omega.
$$

It is easy to show that in equilibrium, late consumers keep their initial long-term asset holdings until period 2 and may choose to acquire more long-term asset using their short-term asset holdings. Each late consumer holds the amount equal to $\alpha \omega$ of the short-term asset. Let us denote the optimal fraction of the short-term asset holdings to be sold by $a \left(P_1 \right)$, $0 \le a \left(P_1 \right) \le 1$. The remaining part of the short-term asset holdings, $\left(1 - a \left(P_1 \right) \right) \alpha \omega \ge 0$ is reinvested for one more period. Whenever $a \left(P_1 \right) < 1$ we say that a late consumer prefers to diversify, i.e. his portfolio in period 2 consists of both the short-term and the long-term asset. Let D denote the total amount of the short-term asset supplied by the late types to the asset market. We have

$$
D \equiv \left(1 - \lambda \right) a \left(P_1 \right) \alpha \omega \le \left(1 - \lambda \right) \alpha \omega.
$$

The price that clears the market is given by

$$
P_1 \left(\theta \right) S = D,
$$

i.e. the value of period 2 claims in period 1 (supplied by the early consumers)

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has to equal to the value of period 1 claims in period 1 (supplied by the late consumers). Holding the long-term asset is risky due to the rate-of-return risk (realization of R is unknown until period 2) and the market risk (P_1 is uncertain).

We now characterize the equilibrium price function. First, note that price $P_1(\theta)$ cannot contain any information about the return on the long-term asset, R, since it is unknown to all market participants. Then, there are two possibilities: either price reveals some information about λ, or price is non-revealing. We can prove the following Lemma.

Lemma 2 *Price P_1 fully reveals λ in period 1.*

Since price P_1 fully reveals λ, we write $P_1(\lambda)$ and we have

$$P_1(\lambda)\lambda b(P_1)(1-\alpha) = (1-\lambda)a(P_1)\alpha$$

and $a(P_1)$, $b(P_1)$, and P_1 are the same for the states with the same λ. Then, a late consumer chooses a in period 1 to maximize the expected utility of his period-two portfolio

$$\sum_R u\left[\left(\left(\frac{a(P_1)\alpha}{P_1}+1-\alpha\right)R+(1-a(P_1))\alpha\right)\omega\right]g(R)$$

subject to

$$0 \le a(P_1) \le 1.$$

We now state and prove (in the appendix) the following useful Lemma.

Lemma 3 *There exists a strictly positive price, which we denote by $\overline{P_1}$, such that for all $P_1 \ge \overline{P_1}$, late types prefer to completely diversify their portfolios, i.e. their demand for the long-term asset for a given λ is zero. Similarly, there exists a strictly positive price, which we denote by $\underline{P_1}$, such that for all $P_1 \le \underline{P_1}$, late*

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consumers want to hold long-term asset only and thus sell their entire holdings of the short-term asset in period 1.

Also, we show that $P_1 \geq L$ always holds.

Lemma 4 *Price P_1 is always greater or equal to the liquidation return L and early consumers supply a strictly positive amount of the long-term asset to the market.*

If in an equilibrium price $P_1(\lambda)$ is equal to the liquidation return L, then we know from lemma 3 that late consumers prefer to hold the long-term asset only. This is because $L < \underline{P_1}$ and the long-term asset offers a strictly higher net return that the short-term asset. Thus,

$$D = (1 - \lambda) \alpha \omega$$

in this case. Early consumers are indifferent between private liquidation of the long-term asset and its sale. They will thus supply an amount b such that

$$L = \frac{(1 - \lambda) \alpha}{\lambda b (1 - \alpha)}.$$

If in an equilibrium price $P_1(\lambda) > L$, then early consumers prefer to sell their entire long-term asset holdings in the market, i.e. $b(P_1) = 1$. Thus, the supply of the long-term asset is given by $\lambda (1 - \alpha) \omega$ and is fixed for a given λ. This implies that the price in the market cannot exceed $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$ since $(1 - \lambda) \alpha \omega$ is the maximum possible level of the demand for the long-term asset, D. If for a given λ, we have

$$\frac{(1 - \lambda) \alpha}{\lambda (1 - \alpha)} < \underline{P_1},$$

then late types always want to hold the long-term asset only and thus sell their entire short-term asset holdings, $a = 1$. The equilibrium price is equal to $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$.

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If, on the other hand,

$$\frac{(1-\lambda)\,\alpha}{\lambda\,(1-\alpha)} \geq \underline{P_1}$$

holds, then late consumers prefer to diversify their portfolio and hold both the short-term and the long-term asset. They choose an optimal amount of the short-term asset to sell, $0 < a \leq 1$, with the resulting equilibrium price being $P_1 \in [\underline{P_1}, \overline{P_1})$. We summarize our findings in the following proposition.

Proposition 5 *For a given λ, there are three possible cases. 1) The equilibrium price is equal to L. Late consumers prefer to hold the long-term asset only and early consumers sell the fraction $\frac{\alpha(1-\lambda)}{L\lambda(1-\alpha)}$ of their long-term asset holdings. 2) The price in the market that would arise if late consumers sold their entire short-term asset holdings, $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$, is smaller than $\underline{P_1}$. Then, late consumers prefer to hold the long-term asset only and the resulting equilibrium price is equal to $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$. 3) The price $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$ is greater or equal to $\underline{P_1}$. Then, late consumers prefer to diversify their portfolio holdings. They choose $0 < a \leq 1$ optimally and the resulting equilibrium price is given by $P_1 = \frac{(1-\lambda)a\alpha}{\lambda(1-\alpha)}$ with $P_1 \in [\underline{P_1}, \overline{P_1})$.*

Let us look at the decisions made in period 0. Consider a consumer who chooses portfolio $(\alpha, 1 - \alpha)$. At date 1, if a consumer turns out to be an early type, he either sells all of his long-term asset holdings at price $P_1(\lambda) > L$, or sells a fraction $b(P_1)$ at price $P_1 = L$ and liquidates the remaining part of the long-term holdings privately. His period-one wealth, which we denote by c_1, can be written as

$$c_1(\lambda) = [\alpha + (1-\alpha)\,P_1(\lambda)]\,\omega.$$

If a consumer turns out to be a late type, he adjusts his portfolio holdings in period 1, gathers realized return on the long-term asset in period 2, and

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consumes his wealth. Late consumer's period-two wealth is equal to

$$c_2(\theta) = \left(\left(\frac{a(P_1)\alpha}{P_1(\lambda)} + 1 - \alpha\right)R + (1 - a(P_1))\alpha\right)\omega.$$

A consumer chooses α to maximize the expected utility that can be achieved in the asset market, $EU^M(\alpha; a, P_1)$:

$$\sum_R \sum_\lambda [\lambda u(c_1(\lambda)) + (1 - \lambda) u(c_2(\theta))] f(\lambda) g(R) \qquad (MP)$$

subject to

$$0 \leq \alpha \leq 1.$$

Let μ_2 denote the Lagrangian multiplier on the constraint $\alpha \geq 0$. Let μ_3 denote the Lagrangian multiplier on the constraint $1 - \alpha \geq 0$. Then, optimal α must satisfy the following first-order condition:

$$\sum_R \sum_\lambda \left[\lambda u'(c_1(\lambda))[1 - P_1(\lambda)] + (1 - \lambda) u'(c_2(\theta))\left(\left(\frac{a(P_1)}{P_1(\lambda)} - 1\right) \times\right.\right.$$
$$R + 1 - a(P_1))]\omega f(\lambda) g(R) + \mu_2 - \mu_3 = 0.$$

We can prove the following proposition.

Proposition 6 *In equilibrium, consumers prefer to diversify their period-zero portfolios by investing a strictly positive fraction of the endowment in the long-term asset and thus a positive fraction of the endowment in the short-term asset.*

Using the standard fixed-point argument we can show that there exists an asset market equilibrium for all parameter values.

Proposition 7 *There exists an asset market equilibrium for all parameter values.*

We now turn our attention to analyzing the behavior of banks in the economy, in which banks are the only alternative to the autarky (asset market is non-existent).

5   Banking Sector

Banks operate in a perfectly competitive environment and thus their objective is to maximize the expected utility of depositors.

To retain tractability, we restrict banks to offer demand deposit contracts, i.e. contracts of the form $(c_1, c_2(\theta))$. A return c_1 is fixed and is promised to a depositor withdrawing in period 1^{14}. A bank stands up to this obligation unless it has run out of funds. In period 2, whatever is left in the bank is divided equally among the remaining depositors. Due to the variation in the aggregate demand for liquidity and the return on the long-term asset, this amount, $c_2(\theta)$, varies across states. In period 0, banks decide how much to invest in the short-term asset. Let β denote the fraction of bank's resources put into the short-term asset. This variable is also specified in the banking contract. The remaining resources are invested in the long-term asset. If the banks need to liquidate some or all of their long-term asset holdings in period 1, they use the liquidation technology and gather return equal to L per unit of the long-term asset held.

In period 0, an individual decides whether or not to join a bank. If he does not deposit his endowment in the bank, he can store it and consume later (in period 1 if early and in period 2 if late). This would yield utility level $\overline{u}$. Banks can always provide this same utility level by investing all resources in the short-term asset and offering the contract $(c_1 = 1, c_2 = 1)$. Given that banks can provide a better consumption profile, all consumers choose to participate in

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[14] Wallace [27] argues that in the economy with aggregate uncertainty about λ, the best feasible contract will in general have c_1 depend on the order in which consumers arrive to the bank. Incorporating this feature into our analysis would not, qualitatively, change our conclusions and would make the analysis harder.

the banking sector.

After banks have set the banking contract and consumers have made their deposits, the so-called "post-deposit game" begins. Each individual learns his type at the beginning of period 1 and decides whether to withdraw from the bank in period 1 or in period 2. We assume that banks serve their customers sequentially and continue paying off c_1 until they runs out of funds. There is no suspension of convertibility. Early types always withdraw in period 1. Late consumers have a choice to withdraw in period 1 or in period 2.

The bank always offers a contract which is incentive-compatible, i.e. a late consumer's expected payoff from withdrawing in period 2 is higher than his payoff from withdrawing early given that all other late consumers wait until period 2 to withdraw. Following the literature, we look at symmetric, pure-strategy equilibria to the game played by late consumers. Two types of equilibria are possible. In the "no run" Nash equilibrium, early consumers withdraw in period 1 and late consumers wait until period 2 to make their withdrawals. However, whenever the bank chooses an incentive-compatible contract such that if all depositors showed up at the bank in period 1, it would not have enough resources to pay off c_1 to everyone, there exists another, bank run, Nash equilibrium to the game. The reason is that if a late consumer believes that all other late consumers will try to withdraw in period 1, it is in his best interest to withdraw, too (and store the good to consume it in period 2). If he waited until period 2, his return on the bank withdrawal would be zero (the bank runs out of funds by the end of period 1). When a late consumer chooses to withdraw from the bank in period 1, we say that he "runs". We define a bank run equilibrium below.

Definition 8 (Bank Run) *Panic-based bank run equilibrium is a Nash equilibrium to the post-deposit game such that each late consumer finds it optimal to withdraw in period* 1 *because he believes that all other late consumers will*

withdraw in period 1 *and hence the bank will run out of funds before the end of period* 1. *Thus,* **both** *early and late consumers withdraw from the bank at* $t = 1$.

On the other hand, if a bank has enough resources to pay off c_1 to everyone in period 1, it is a dominant strategy for late consumers to withdraw in period 2 and there is only one (no run) equilibrium to the post-deposit game. The contract satisfying this property is said to be run proof. A bank is always able to offer a run proof contract. This also implies that for every banking contract, at least one symmetric, pure-strategy equilibrium exists: the no-run outcome is an equilibrium if the banking contract is run-proof.

If the aggregate demand for liquidity in some state of the world turns out be unexpectedly high, the bank can "run-out" of funds. We define the concept below.

Definition 9 (Running-out) *Running-out occurs when a bank does not have enough resources to pay off c_1 to all* **early** *consumers even after liquidating its long-term asset holdings. Only a proportion of the early consumers is served by the bank (those first in line).*

It is important to note that in the case of running-out only early consumers withdraw from the bank in period 1. Late types wait until period 2 to withdraw (they don't observe the realization of the aggregate liquidity shock and thus don't know that the bank will run out of funds in period 1). Thus, the payoff of all late consumers and the proportion of the early consumers that was not served by the bank in period 1 is equal to zero.

To analyze a bank's problem, we specify some useful notation[15]. Let $\underline{\lambda}$ denote the fraction of consumers withdrawing in period 1 such that this level of withdrawals just exhausts bank's investment in the short-term asset, β. (A bank

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[15] Bank's optimization problem in this section is similar to the bank's problem in Ennis and Keister [16].

will first use the short-term asset to pay off first-period withdrawals; only after this source is depleted, it will start liquidating its long-term investment.) If the realization of λ is such that $\lambda < \underline{\lambda}$, a part of the investment in the short-term asset will not be used to pay off consumers in period 1 and will be reinvested for one more period. Another variable of interest is $\underline{\underline{\lambda}}$, which denotes a proportion of consumers who get c_1 before the bank completely runs out of funds. For λ such that $\underline{\lambda} < \lambda < \underline{\underline{\lambda}}$, the bank needs to liquidate a part of its long-term investment to satisfy liquidity demand in period 1. We let $\Lambda_{\underline{\lambda}}$ be the subset of the set of possible values of λ such that for all $\lambda \in \Lambda_{\underline{\lambda}}$, $\lambda \leq \underline{\lambda}$ holds. Similarly, let $\Lambda_{\underline{\underline{\lambda}}}$ be the subset of the set of possible values of λ such that for all $\lambda \in \Lambda_{\underline{\underline{\lambda}}}$, $\underline{\lambda} < \lambda < \underline{\underline{\lambda}}$ holds. Finally, let Λ_λ be a shortcut notation for $\Lambda_{\underline{\lambda}} \cup \Lambda_{\underline{\underline{\lambda}}}$.

We first briefly discuss a bank's problem and properties of the optimal contract when the *ex post* types (early/late) are observable. Then, we analyze the main case of our interest: the case when individual types are unobservable to the bank.

5.1   First Best Contract

If *ex post* types (early/late) were observable, a bank would choose $(\beta, c_1, c_2(\theta))$ to maximize

$$\sum_R \sum_{\lambda \in \Lambda_\lambda} [\lambda u(c_1) + (1-\lambda) u(c_2(\theta))] f(\lambda) g(R) + \sum_{\lambda \in \Lambda \backslash \Lambda_\lambda} \left[\underline{\underline{\lambda}} u(c_1) \right] f(\lambda)$$

subject to

$$\underline{\lambda} = \min \left\{ \frac{\beta}{c_1}, 1 \right\}$$

$$\underline{\underline{\lambda}} = \min \left\{ \frac{\beta + L(1-\beta)}{c_1}, 1 \right\} \qquad (FB)$$

$$0 < \beta \leq 1, \ c_1 \geq 0$$

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$$c_2\left(\theta\right) = \begin{cases} \frac{\beta+R(1-\beta)-\lambda c_1}{1-\lambda} & \text{if } \lambda \in \Lambda_{\underline{\lambda}} \\ \frac{R\left[1-\beta+\frac{\beta-\lambda c_1}{L}\right]}{1-\lambda} & \text{if } \lambda \in \Lambda_{\underline{\underline{\lambda}}} \end{cases}.$$

Banks serve early consumers sequentially and continue paying off c_1 in period 1 until they run out of funds. All late consumers arrive to the bank in period 2. If the liquidity demand in some state of the world turns out to be high, i.e. $\lambda \geq \underline{\underline{\lambda}}$, banks serve first $\underline{\underline{\lambda}}$ early consumers and the return on the bank withdrawal for all the remaining consumers, namely $(\lambda - \underline{\underline{\lambda}})$ early types and $(1 - \lambda)$ late types, is equal to zero.

In the next Lemma, we state a result that is well-known in the bank runs literature.

Lemma 10 *Suppose λ is non-stochastic. Then, the first best contract satisfies $c_1^* > 1$ and $c_2^*\left(\theta\right) < R$. This implies the preference for liquidity (and cross-subsidy) between early and late consumers.*

Risk-averse consumers want *ex ante* insurance against the "bad luck" of becoming early consumers in period 1. They prefer a higher period-one consumption at the expense of a somewhat lower period-two consumption.

5.2 Asymmetric Information in period 1

We now analyze the case when *ex post* types are unobservable to the bank. At the beginning of period 1, a consumer learns his type and it is private information. How does this change the optimization problem banks face?

We know that a bank always offers an incentive-compatible contract (a contract that admits a no run equilibrium to the post-deposit game). Within the class of incentive compatible contracts we can distinguish two types of contracts: 1) run proof contracts and 2) run admitting contracts.

Run proof contracts are contracts such that it is a *dominant* strategy for a

late consumer to wait until period 2 to withdraw. Run proof contracts must satisfy the following condition

$$c_1 \leq \beta + L(1 - \beta).$$

The condition states that the bank has always enough resources to pay off c_1 to everyone in period 1. This eliminates panic-based bank runs since a late consumer's payoff from withdrawing in period 2 is at least as high as his payoff from withdrawing in period 1 regardless of when other late consumers make their withdrawals. Thus, there is only one, no run, equilibrium to the post-deposit game.

If the bank finds it optimal to offer a run proof contract, it will choose the best contract satisfying the above "run proof" condition, i.e. the contract solving:

$$\max_{\beta, c_1, c_2(\theta)} \sum_R \sum_\lambda \left[\lambda u(c_1) + (1 - \lambda) u(c_2(\theta)) \right] f(\lambda) g(R) \text{ subject to}$$

$$\underline{\lambda} = \min\left\{ \frac{\beta}{c_1}, 1 \right\}$$

$$0 < \beta \leq 1, \ 0 \leq c_1 \leq \beta + L(1 - \beta) \qquad\qquad (RP)$$

$$c_2(\theta) = \begin{cases} \frac{\beta + R(1-\beta) - \lambda c_1}{1 - \lambda} & \text{if } \lambda \leq \underline{\lambda} \\ \frac{R\left[1 - \beta + \frac{\beta - \lambda c_1}{L}\right]}{1 - \lambda} & \text{if } \underline{\lambda} < \lambda < 1 \end{cases}.$$

The run-proof condition implies that $\underline{\lambda} = 1$.

Let us denote the solution to the problem (RP) by $\left(\beta^{RP}, c_1^{RP}, c_2^{RP}(\theta) \right)$ and the corresponding maximized level of utility by EU^{RP}. Banks are *always* able to offer a run proof contract. However, there is a cost to making the contract run proof. A run proof contract cannot provide liquidity insurance since $c_1 \leq \beta + L(1 - \beta) \leq 1$ for all $0 \leq \beta \leq 1$. This lowers the welfare of

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risk-averse consumers.

A run admitting contract admits *two* equilibria to the post-deposit game, a no run equilibrium and a run equilibrium. It satisfies the following condition

$$c_1 > \beta + L(1 - \beta).$$

The condition states that the bank would for sure run out of funds in period 1 if all depositors arrived to the bank in period 1. Hence, if a late consumer believes that all other late consumers will run in period 1, it is in his best interest to run, too since waiting until period 2 would yield zero payoff.

When there are two equilibria to the post-deposit game, a device that will coordinate consumers' actions is needed. A common approach in the literature is to assume an existence of a publicly observed sunspot signal. After deposit decisions and portfolio choices are made, a number σ is drawn from a uniform distribution on $[0, 1]$. The draw itself is unrelated to any other variable in the economy. Given σ, late consumers behave according to the following decision rule:

$$\text{run if } \sigma \leq \pi,$$

$$\text{don't run otherwise}$$

for some number $\pi \in [0, 1]$. Variable π is the probability of a run outcome[16]. Naturally, a run proof contract induces $\pi = 0$ and the sunspot realization is ignored by individuals.

Let us start by assuming that the probability of a run π is a fixed, exogenous parameter of the economy. Banks take π as given when choosing the optimal contract and consumers have rational expectations of the following form: They

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[16]Peck and Shell [24], whose approach we follow here, use the term "propensity to run".

expect a bank run with exogenous probability π whenever the banking contract admits a run equilibrium.

We can now formulate a bank's maximization problem. The bank

$$
\max_{\beta, c_1, c_2(\theta)} (1 - \pi) \left\{ \sum_R \sum_{\lambda \in \Lambda_\lambda} \left[\lambda u(c_1) + (1 - \lambda) u(c_2(\theta)) \right] f(\lambda) g(R) \right. \qquad (BP)
$$
$$
\left. + \sum_{\lambda \in \Lambda \backslash \Lambda_\lambda} \left[\underline{\lambda} u(c_1) \right] f(\lambda) \right\} + \pi \underline{\lambda} u(c_1) \text{ subject to}
$$

the same set of constraints as in the problem (FB). The first term in the objective function (multiplied by $(1 - \pi)$) gives the expected utility of the contract in case there is no run. The term multiplied by π reflects that in the event of a run, the bank's resources will for sure be depleted by the end of period 1 and thus only proportion $\underline{\lambda}$ of consumers will be served (those first in line). Optimal contract must be incentive-compatible, i.e. a late consumer's expected payoff from withdrawing early must be smaller than his expected payoff from waiting until period 2 conditional on other late consumers waiting until period 2:

$$
\sum_{\lambda \in \Lambda_\lambda} u(c_1) f_p(\lambda) + \sum_{\lambda \in \Lambda \backslash \Lambda_\lambda} \left[\underline{\lambda} \frac{1}{\lambda} u(c_1) \right] f_p(\lambda) \leq \sum_R \sum_{\lambda \in \Lambda_\lambda} u(c_2(\theta)) f_p(\lambda) g(R).
$$

As is standard in the literature, we will assume throughout the paper that individuals truthfully reveal their type if they are indifferent between doing so and not doing so.

We know from Peck and Shell [24] that if π is sufficiently small, banks find it optimal to offer a run admitting contract. In the banks-only economy however, banks face a trade-off between the liquidity provision and the possibility of a run. For low values of π, consumers are willing to tolerate the possibility of a bank run since a run admitting contract insures then against private preference shocks. As π increases, consumers' welfare goes down because if there is a run

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and an individual is not served, his consumption is equal to zero. There exists a threshold level of probability of a run, $\widehat{\pi}$, such that for any $\pi \geq \widehat{\pi}$ banks find it optimal to offer a run proof contract (it is strictly preferred to the run admitting contract).

Suppose for a moment that a bank would hold a portfolio that is close to a run proof portfolio. Would consumers believe that the probability of a run this bank faces is relatively small? Put more generally, should not bank's portfolio choice influence consumers' beliefs about the likelihood of a run π? A way to capture this influence is to consider a modified version of rational expectations beliefs that are linked to the parameters of the banking contract. Work by Ennis and Keister [16] is an example of such an approach and we adopt it here[17].

The underlying intuition is that a mere possibility of a run alters banks' behavior by affecting their choice of the optimal contract (even though a run may not occur eventually). Thus, the sunspot equilibrium allocation is not a mere randomization over the equilibrium allocations of the economy without sunspots. We therefore retain Diamond and Dybvig [15] spirit that bank runs are to some extent chance events (this is represented by the randomness of the sunspot variable σ). At the same time, portfolio decisions of banks, asset returns, and liquidity demand uncertainty affect determination of the *ex ante* probability of a run π. This framework allows to model bank runs as chance events after taking fundamentals into account.

A technical device used to formalize a relation between π and parameters of the contract is the so-called risk-factor of the run equilibrium, denoted by ϕ. The risk-factor represents how "risky" is running on the bank from the point of view of the late consumer. The riskiness stems from the fact that he does not know what other late consumers will do. The risk-factor ϕ reflects the strength

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[17]We refer an interested reader to their paper for the extensive discussion of the idea and technical details.

of incentives of individuals to run, which in turn determines the probability of a run, π.

Formally, $\phi(\theta)$ is determined as follows. Let $(\beta, c_1, c_2(\theta))$ be the contract offered by the bank in period 0. Let $\xi(\theta)$ denote a late consumer's prior belief about the probability that all other late consumers will run in state θ and period 1. For $\xi(\theta) = \phi(\theta)$, a late consumer is indifferent between withdrawing in period 1 and period 2, i.e. the risk-factor is a cutoff level of $\xi(\theta)$ such that for all $\xi(\theta) > \phi(\theta)$ running is the unique optimal action in state θ. For $\lambda \in \Lambda_\lambda$, $\phi(\theta)$ is given by the following equation

$$\phi(\theta) \underline{\underline{\lambda}} u(c_1) + (1 - \phi(\theta)) u(c_1) = (1 - \phi(\theta)) u(c_2(\theta)).$$

The left hand side is a late consumer's expected payoff of running when he believes that with probability $\phi(\theta)$ all other late consumers will run. The right hand side gives the expected payoff of waiting until period 2 given the same belief. Rearranging yields

$$\phi(\theta) = \frac{u(c_2(\theta)) - u(c_1)}{u(c_2(\theta)) - u(c_1) + \underline{\underline{\lambda}} u(c_1)}.$$

We can see that $\phi(\theta) \in [0, 1]$. If the risk-factor is low, probability that $\xi(\theta) > \phi(\theta)$ increases and a late consumer chooses to run for a wider range of beliefs about the actions of other late consumers.

Remark 11 *It is only sensible to consider how risky is running on the bank in state θ if the state-θ banking allocation $(c_1, c_2(\theta))$ admits two equilibria to the post-deposit game. If it is always optimal for a late consumer to run in state θ, then $\phi(\theta) = 0$. This is the case, for example, for all $\lambda \in \Lambda \setminus \Lambda_\lambda$ since here a bank will run out of funds by the end of period 1 and it is thus optimal for a late consumer to withdraw in period 1. Similarly, $\phi(\theta) = 0$ if $c_1 > c_2(\theta)$ holds*

for some θ (yet the incentive-compatibility constraint is still satisfied).

Remark 12 *What is the relation between the approach of Peck and Shell [24] and the risk-factor of the run equilibrium? We know that there are four kinds of payoffs a late consumer can potentially consider: payoffs from withdrawing in period 1 versus period 2 given that other late consumers withdraw in period 2 (incentive-compatibility) and payoffs from withdrawing in period 1 versus period 2 given that others run. In Peck and Shell[24], the probability of a run is fixed and all that matters is incentive-compatibility. In the risk-factor approach, all four payoffs play a role in the analysis. They are combined in the way described above.*

The *ex ante* (here, beginning of period 1) risk-factor ϕ is defined as

$$\phi = \sum_R \sum_\lambda \phi\left(\theta\right) f_p\left(\lambda\right) g\left(R\right).$$

The lower ϕ the higher a late consumer's *ex ante* incentive to run. Hence, we assume that probability of a run π is a decreasing function of the risk-factor ϕ. We do so to be able to use the established terminology: the lower probability of a run, the better for the bank (as opposed to the bank aiming at increasing the risk-factor of the run equilibrium). For simplicity, we posit a linear relation:

$$\pi\left(\phi\right) = m - h\phi.$$

Essentially, we are re-scaling ϕ (which is determined endogenously). We are not interested in the value of π per se. We want to compare the probability of a run banks face in the banks-only economy with the probability of a run in the economy with banks and markets. Note that for $h = 0$ we are back to an exogenously given π considered initially.

We are interested in systemic runs, i.e. the situation when the probability

of a run on a particular bank is determined by the economy-wide (average) contract. When choosing the optimal contract, banks know and take as given the relation between their contract and the risk-factor ϕ and thus probability of a run π. The probability of a run is determined by the rational expectations condition that requires the probability of a run, which is taken as given by banks, to be the same as the probability implied by the contract all banks choose[18].

6   Mixed Financial System

We now analyze an economy in which consumers can choose whether to invest their endowments in the asset market, deposit contracts, or both. As in the previous section, banks' objective is to maximize the welfare of individuals.

In our setup, banks and markets provide different services to consumers. Banks provide an *ex ante* insurance against private liquidity shocks. They guarantee (unless there is a run) a certain level of consumption in period 1. Moreover, they are not exposed to asset price fluctuations unless in distress (in which case they might choose to liquidate their long-term asset holdings in the market; more details shortly). However, banks may be subject to runs. We know that if there is a run and a consumer is not served, the return on his bank withdrawal is zero regardless of his type. Having (part of) the endowment invested in the asset market guarantees positive consumption levels for both early and late types in *all* states of the world. On the other hand, the asset market cannot provide insurance against private preference shocks and consumers trading in the market are exposed to the market risk.

We might therefore expect that risk-averse individuals will find it optimal to

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[18]It is possible to consider the environment where the probability of a run on an individual bank is given by that bank's deposit contract, i.e. runs are idiosyncratic. The equilibrium would be indistinguishable from the systemic case: either all banks would experience a run or none would since all banks would choose the same contract. However, each bank would internalize the effect of its contract on π. This would lower equilibrium π. See Ennis and Keister [16].

accept the banking-financial contract in period 0. By investing (part of) their endowment in the market, consumers can insure themselves against bank runs. Even in the case of a bank run occurring and a consumer not being served, a consumer still has his market investment so he will never end up with zero consumption level.

6.1 Banking-Financial Contract

The banking-financial contract specifies what part of a consumer's endowment is invested in the asset market. We denote this amount by W, $0 \leq W \leq 1$. The contract further specifies what fraction of W is invested in the short-term asset. This fraction is denoted by α. Fraction $(1 - \alpha)$ is invested in the long-term asset.

The remaining part of a consumer's endowment, $(1 - W)$, is exchanged for the deposit contract. As in the previous section, period-one return on the demand deposit investment is a fixed payoff multiple c_1^B. Period-two payoff multiple $c_2^B(\theta)$ depends on the realization of λ and R. More precisely, period-one return on banking withdrawals is $(1 - W) c_1^B$ and, similarly, period-two return on withdrawals in state θ is equal to $(1 - W) c_2^B(\theta)$. Banks choose in period 0 what fraction of $(1 - W)$ is invested in the short-term asset (β). The remaining resources are invested in the long-term asset (fraction $(1 - \beta)$).

If banks' short-term asset holdings are not enough to cover withdrawals in period 1, they can acquire the short-term asset in the market by selling some or all of their long-term asset holdings. Of course, banks can still privately liquidate the long-term asset and gather a return L per unit[19]. A bank finds out how high is the aggregate demand for liquidity in period 1 by paying off c_1^B using the short-term asset holdings first and giving out "slips", i.e. promises of c_1^B,

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[19]We know that banks will need to liquidate (either in the market or privately) their entire long-term asset holdings in case there is a run. They will need to liquidate a part of the long-term asset holdings if $\underline{\lambda} < \lambda < \underline{\underline{\lambda}}$.

once it runs out of the short-term asset. Given this information, a bank decides what fraction of the long-term asset holdings will be sold given its expectations about the price. We assume that banks know and take as given price function P_1. The reason we assume banks are price-takers is to keep symmetry with individuals who are also price-takers. Let $\widehat{\gamma}$ denote a fraction of the long-term asset holdings banks need to liquidate in period 1. Then, the equilibrium price of the long-term asset when banks trade in the market is given by

$$P_1^L(\lambda) = \frac{(1-\lambda)\,a\alpha W}{\lambda b\,(1-\alpha)\,W + \widehat{\gamma}\,(1-\beta)\,(1-W)},$$

where superscript L stands for "liquidation". We know from Lemma 2 that the equilibrium price when banks do not trade in the market fully reveals λ and from Lemma 4 that $P_1(\lambda) \geq L$ for all λ. We can show that the same holds true for $P_1^L(\lambda)$.

Lemma 13 *Equilibrium price of the long-term asset when banks trade in the market fully reveals λ and is always greater or equal to the liquidation return L.*

After a consumer learns his type, he decides when to withdraw from the bank and orders a portfolio rebalancing transaction. An early consumer decides what fraction of his long-term asset holdings to sell and a late consumer chooses an optimal amount of the long-term asset to acquire, given the expectations about the price P_1. The consumer is only allowed to adjust his period 0 portfolio, e.g. if he sells the short-term asset, he cannot sell a larger amount of the short-term asset than the amount he invested in period 0.

Banks choose α, W, β, and demand deposit multiples $\left(c_1^B, c_2^B\,(\theta)\right)$ to maximize the expected utility of consumers. Bank's optimization problem (BMP) is formulated below[20]:

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[20] We present the first-order conditions for this problem in the Appendix, p. 54.

$$\max_{\alpha,W,\beta,c_1^B,c_2^B(\theta)} (1-\pi)\left\{\sum_R \sum_{\lambda\in\Lambda_\lambda} \left[\lambda u\left(c_1\right) + (1-\lambda) u\left(c_2\left(\theta\right)\right)\right] f\left(\lambda\right) g\left(R\right) + \right.$$

$$\left.\sum_R \sum_{\lambda\in\Lambda\backslash\Lambda_\lambda} \left[\underline{\lambda} u\left(c_1^S\right) + \left(\lambda-\underline{\lambda}\right) u\left(c_1^N\right) + (1-\lambda) u\left(c_2^N\right)\right] f\left(\lambda\right) g\left(R\right)\right\} +$$

$$\pi\left\{\sum_R \sum_{\lambda\in\Lambda} \underline{\lambda}\left[\lambda u\left(c_1^S\right) + (1-\lambda) u\left(c_2^S\left(\theta\right)\right)\right] + \left(1-\underline{\lambda}\right)\left[\lambda u\left(c_1^N\right) + \right.\right.$$

$$\left.\left.(1-\lambda) u\left(c_2^N\right)\right] f\left(\lambda\right) g\left(R\right)\right\} \text{ subject to}$$

$$\underline{\lambda} = \min\left\{\frac{\beta}{c_1^B},1\right\}, \; \underline{\underline{\lambda}}\left(\lambda\right) = \min\left\{\frac{\beta + \max\left\{P_1^L\left(\lambda\right),L\right\}\left(1-\beta\right)}{c_1^B},1\right\}, \;\; (BMP)$$

$$0 < \beta \le 1, \; 0 \le \gamma \le 1, \; c_1^B \ge 0, \; 0 \le W \le 1,$$

$$c_2^B\left(\theta\right) = \begin{cases} \frac{\beta + R(1-\beta) - \lambda c_1^B}{1-\lambda} & \text{if } \lambda \le \underline{\lambda} \\ \frac{R\left[1-\beta+\frac{\beta-\lambda c_1^B}{\max\left\{P_1^L(\lambda),L\right\}}\right]}{1-\lambda} & \text{if } \underline{\lambda} < \lambda < \underline{\underline{\lambda}} \end{cases},$$

$$\sum_R \sum_{\lambda\in\Lambda_\lambda} u\left(c_2^W\left(\theta\right)\right) f_p\left(\lambda\right) g\left(R\right) + \sum_R \sum_{\lambda\in\Lambda\backslash\Lambda_\lambda} \left[\underline{\underline{\lambda}}\frac{1}{\lambda} u\left(c_2^W\left(\theta\right)\right) + \right.$$

$$\left.\left(1 - \underline{\underline{\lambda}}\frac{1}{\lambda}\right) u\left(c_2^{WN}\left(\theta\right)\right)\right] f_p\left(\lambda\right) g\left(R\right) \le \sum_R \sum_{\lambda\in\Lambda_\lambda} u\left(c_2\left(\theta\right)\right) f_p\left(\lambda\right) g\left(R\right),$$

where

$$c_1\left(\lambda\right) = \begin{cases} \left[\alpha + (1-\alpha) P_1\left(\lambda\right)\right] W + (1-W) c_1^B & \text{if } \lambda \le \underline{\lambda} \\ \left[\alpha + (1-\alpha) P_1^L\left(\lambda\right)\right] W + (1-W) c_1^B & \text{if } \underline{\lambda} < \lambda < \underline{\underline{\lambda}} \end{cases},$$

$$c_2\left(\theta\right) = \begin{cases} \left[\left(\frac{a\alpha}{P_1(\lambda)} + 1 - \alpha\right) R + (1-a)\alpha\right] W + (1-W) c_2^B\left(\theta\right) & \text{if } \lambda \le \underline{\lambda} \\ \left[\left(\frac{a\alpha}{P_1^L(\lambda)} + 1 - \alpha\right) R + (1-a)\alpha\right] W + (1-W) c_2^B\left(\theta\right) & \text{if } \underline{\lambda} < \lambda < \underline{\underline{\lambda}} \end{cases},$$

$$c_2^W(\theta) = \left[\left(\frac{a\alpha}{P_1(\lambda)} + 1 - \alpha\right)R + (1-a)\alpha\right]W + (1-W)c_1^B,$$

$$c_1^S(\lambda) = \left[\alpha + (1-\alpha)P_1^L(\lambda)\right]W + (1-W)c_1^B,$$

$$c_2^S(\theta) = \left[\left(\frac{a\alpha}{P_1^L(\lambda)} + 1 - \alpha\right)R + (1-a)\alpha\right]W + (1-W)c_1^B,$$

$$c_1^N(\lambda) = \left[\alpha + (1-\alpha)P_1^L(\lambda)\right]W,$$

$$c_2^{WN}(\theta) = \left[\left(\frac{a\alpha}{P_1(\lambda)} + 1 - \alpha\right)R + (1-a)\alpha\right]W,$$

$$c_2^N(\theta) = \left[\left(\frac{a\alpha}{P_1^L(\lambda)} + 1 - \alpha\right)R + (1-a)\alpha\right]W.$$

Variable $c_1(\lambda)$ represents the total consumption of an early consumer if there is no run on the bank. Variable $c_2(\theta)$ is the total consumption of a late consumer if there is no run. If a bank run occurs in period 1 and a consumer is served, his consumption is equal to $c_1^S(\lambda)$ if he is an early type and $c_2^S(\theta)$ if he is a late type. In case there is a run or running-out and a consumer is not served, he is left with his market investment only. We let $c_1^N(\lambda)$ and $c_2^N(\theta)$ denote consumption bundles of an early and late type, respectively, for this case. Finally, $c_2^W(\theta)$ is the total consumption of a late consumer who decides to withdraw in period 1 when other late consumers withdraw in period 2. In case of running out, a late consumer would get $c_2^{WN}(\theta)$. It is important to realize that price P_1 varies depending on whether or not banks trade in the market and how much long-term asset banks sell.

Incentive-compatibility is now defined in terms of *total* consumptions of a late consumer in relevant states (it includes the corresponding return on the market portfolio).

If $W = 1$ in the optimum, the problem (BMP) reduces to the problem (MP) from section 4. If $W = 0$ in the optimum, the problem (BMP) reduces to the problem (BP) from section 5.

We denote the risk-factor of the run equilibrium in state θ in the economy with banks and markets by $\phi^M(\theta)$. For $\lambda \in \Lambda_\lambda$ it is determined by the following equation

$$\phi^M(\theta)\left[\underline{\lambda}u\left(c_2^S(\theta)\right) + \left(1-\underline{\lambda}\right)u\left(c_2^N(\theta)\right)\right] + \left(1-\phi^M(\theta)\right)u\left(c_2^W(\theta)\right) =$$
$$\phi^M(\theta)u\left(c_2^N(\theta)\right) + \left(1-\phi^M(\theta)\right)u\left(c_2(\theta)\right).$$

Rearranging yields

$$\phi^M(\theta) = \frac{u\left(c_2(\theta)\right) - u\left(c_2^W(\theta)\right)}{u\left(c_2(\theta)\right) - u\left(c_2^W(\theta)\right) + \underline{\lambda}\left(u\left(c_2^S(\theta)\right) - u\left(c_2^N(\theta)\right)\right)}.$$

As in section 5, the *ex ante* incentive to run ϕ^M is given by

$$\phi^M = \sum_R \sum_{\lambda \in \Lambda} \phi^M(\theta)f_p(\lambda)g(R)$$

and the probability of a run, denoted by π^M, is defined as

$$\pi^M = m - h\phi^M,$$

where m and h are some positive constants.

We can now see how the market investment can lower the probability of a run banks face. We know that in the banks-only economy, if there is a run and a consumer is not served, his consumption is equal to zero. Using our notation, $c_1^N = 0$ and, more importantly, $c_2^N = 0$. In the economy with banks and markets, as long as consumer diversifies by investing in both the asset market and deposit contracts, $c_1^N(\lambda)$ and $c_2^N(\theta)$ are positive in all states of the world. The higher $c_2^N(\theta)$, *ceteris paribus*, the higher $\phi^M(\theta)$, and the lower π^M. There is an additional channel through which the market investment lowers a

late consumer's incentive to run. In case there is a run, banks need to liquidate their entire long-term asset holdings. When banks sell the long-term asset in the market, its price decreases[21]. This creates capital gains for late consumers who are the buyers of the long-term asset. Thus, $c_2^N(\theta)$ increases and the probability of a run on the banking sector further decreases.

7   Examples and Discussion

Due to the complexity of the banks' problem we proceed by computing a series of numerical examples which illustrate properties of the optimal contract[22].

The functional form for the utility function is $u(c) = \frac{4c}{1+4c}$[23]. Random variable λ can take on two values, $\lambda_1 = 0.3$ and $\lambda_2 = 0.35$, each with equal probability. We only consider a small variability in λ since we are primarily concerned with the phenomenon of bank runs (as opposed to running-out). The random variable R can also take on two values, namely $R_1 = 1.05$ and $R_2 = 1.15$, each with equal probability. Thus, there are four states of the world $\theta = \{1, 2, 3, 4\}$, each occurring with probability $1/4$.

θ	1	2	3	4
λ	λ_1	λ_1	λ_2	λ_2
R	R_1	R_2	R_1	R_2

When solving for the optimal banking allocation in the banks-only economy, we set $L = P_1^L$, where P_1^L is the price that clears the market in the banking-financial economy when banks liquidate their entire long-term asset holdings in the asset market (keeping all other parameter values the same). This facilitates

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[21] The asset market is "down". Historically, banking crises were often accompanied by the stock market crashes.

[22] The computations were performed using Maple 9.51. Details are available from the author.

[23] Thus, the autarky level of utility is equal to $\bar{u} = 0.8$.

the direct comparison of the properties of the banking contract in the economy with banks only and the banking-financial contract.

7.1 Fixed Probability of a Run

We start by assuming that π is a fixed, exogenous parameter of the economy. We set $\pi = 0.5\%$. We compute the optimal contract in the economy with banks only (Example 1) and in the economy with banks and markets (Example 2) and compare banks' behavior in the two cases.

Example 1 (Banking Allocation I) We present the results in Table 1.

β	c_1	$c_2\left(1\right), c_2\left(2\right), c_2\left(3\right), c_2\left(4\right)$	$\underline{\underline{\lambda}}_1, \underline{\underline{\lambda}}_2$	$\widehat{\pi}\left(\%\right)$	EU^B
0.35	1.0	$1.046, 1.139, 1.049, 1.149$	$0.85, 0.77$	2.44	0.80868

Table 1: Optimal Banking Allocation, $\pi = 0.5\%$

Consumers exchange their entire endowment for the demand deposit contract. Banks invest a fraction β equal to 0.35 in the short-term asset. Those consumers who turn out to be early in period 1 can withdraw $c_1 = 1.0$ from the bank. These two variables together with the liquidation value of banks' long-term asset holdings determine what fraction of consumers will be served by the bank in the event of a run ($\underline{\lambda}$). We have that banks will serve a fraction equal to 0.85 in states 1 and 2 and a fraction equal to 0.77 in states 3 and 4. If the realization of R is low, i.e. if $\theta = 1$ or $\theta = 3$, late consumers get a relatively low period-two consumption, $c_2^*\left(1\right) = 1.046$ and $c_2^*\left(3\right) = 1.049$.

As the probability of a run increases, the expected utility from the run-admitting contract decreases. We can compute the cutoff level of the probability of a run beyond which the run proof contract becomes optimal. Banks start offering the run proof contract when its benefit (immunity to runs) exactly

outweighs its cost (no provision of the liquidity insurance). For the parameter values considered, this occurs for π equal to 2.44%. We denote this value by $\widehat{\pi}$. We list the solution to the run proof problem in Table 2.

β^{RP}	c_1^{RP}	$c_2^{RP}(1), c_2^{RP}(2), c_2^{RP}(3), c_2^{RP}(4)$	EU^{RP}
0.271	0.784	$1.145, 1.249, 1.172, 1.284$	0.80585

Table 2: Run Proof Allocation

The best run proof allocation has $\beta^{RP} = 0.271$ and $c_1^{RP} = 0.784$. In this case, banks significantly reduce the period-one return on withdrawals to make sure that they have enough resources to pay off c_1^{RP} to everyone in period 1. The elimination of the liquidity insurance provision reduces the expected utility to 0.80585.

Example 2 (Banking-Financial Allocation I) We list the solution to the problem (BMP) in Tables 3, 4, and 5.

α^*	$P_1(\lambda_1), P_1(\lambda_2)$	$a(P_1), a(P_2)$	W^*	$P_1^L(\lambda_1), P_1^L(\lambda_2)$
0.331	$1.096, 0.918$	$0.95, 1$	0.862	$0.768, 0.642$

Table 3: Banking-Financial Allocation I, $\pi = 0.5\%$

In the optimum, individuals invest $W^* = 0.862$ in the asset market out of which about one third, $\alpha^* = 0.331$, is invested in the short-term asset. For completeness, $a^*(P_1(\lambda_1)) = 0.950$ and $a^*(P_2(\lambda_2)) = 1$. Table 3 also lists equilibrium asset prices which reveal the aggregate demand for liquidity, λ. We present two sets of prices: one arises if banks do not trade in the market and the other, with superscript L, arises if banks need to liquidate their entire long-term asset holdings. We see that $P_1^L(1)$ and $P_1^L(2)$ are lower than $P_1(1)$ and $P_1(2)$,

respectively.

β^*	c_1^B	$c_2^B\left(1\right), c_2^B\left(2\right), c_2^B\left(3\right), c_2^B\left(4\right)$	$\underline{\underline{\lambda}}_1, \underline{\underline{\lambda}}_2$	$\widehat{\pi}^*\left(\%\right)$
0.371	1.061	$1.019, 1.109, 1.016, 1.112$	$0.81, 0.73$	6.3

Table 4: Banking-Financial Allocation II, $\pi = 0.5\%$

Table 4 presents banks decisions in the banking-financial economy. Banks invest much more in the short-term asset (optimal β^* has increased as compared to the banks-only economy), which enables them to provide *more* liquidity insurance (higher c_1^B at the expense of lower $c_2^B\left(\theta\right)$). This also implies that banks are able to serve a smaller number of consumers in the event of a run. Banks are less concerned about the proportion of people served because those not served will still consume a positive amount: c_1^N and c_2^N are greater than zero. On the contrary, in the banks-only economy, those not served end up with zero consumption and thus banks aim at serving more consumers in a run. Hence, the market investment acts as insurance against runs and the existence of this insurance gives banks more flexibility in their portfolio choice. In the banking-financial economy, banks "specialize" in liquidity provision.

We find that the run proof contract is not offered in the banking-financial economy. Intuitively, this is because the provision of the liquidity insurance constitutes the comparative advantage of banks in this model. If banks do not provide the liquidity insurance in the economy with markets, consumers have no reason to invest in deposit contracts. Instead, as the probability of a run increases, consumers invest increasingly more in the asset market since the demand deposit contract becomes more risky. We can compute the cutoff level of the probability of a run beyond which investing in the asset market only becomes optimal. This is also the point beyond which consumers no longer get the *ex ante* liquidity insurance since it can only be provided by banks. For the

parameter values considered, this occurs for π equal to 6.3%. We denote this value by $\widehat{\pi}^*$ (see Table 4). We list the optimal asset market allocation in Table 6.

It is interesting to note that the cutoff value of π in the banking-financial economy above which runs are no longer a concern (and no *ex ante* liquidity insurance is provided) is much higher than the similar cutoff in the banks-only economy, $\widehat{\pi}^* > \widehat{\pi}$.

Table 5 summarizes consumer's final consumption allocations (consisting of the return on the market investment and bank withdrawals) in each of the four aggregate states of the world. If there is no run, early and late types receive $c_1^* (\theta)$ and $c_2^* (\theta)$, respectively. In the event of a run, early consumers get $c_1^S (\theta)$ if served and $c_1^N (\theta)$ if not, and late consumers get $c_2^S (\theta)$ if served and $c_2^N (\theta)$ if not.

θ	$(c_1^* (\theta), c_2^* (\theta))$	$(c_1^S (\theta), c_2^S (\theta))$	$(c_1^N (\theta), c_2^N (\theta))$	EU^{BM}
1	$(1.064, 1.02)$	$(0.875, 1.142)$	$(0.728, 0.995)$	0.80955
2	$(1.064, 1.115)$	$(0.875, 1.237)$	$(0.728, 1.090)$	0.80955
3	$(0.961, 1.072)$	$(0.802, 1.219)$	$(0.655, 1.072)$	0.80955
4	$(0.961, 1.174)$	$(0.802, 1.321)$	$(0.655, 1.174)$	0.80955

Table 5: Banking-Financial Allocation III, $\pi = 0.5\%$

Example 3 (Asset Market Allocation) Consumers invest 0.330 fraction of their initial endowment in the short-term asset in period 0. In period 1 they adjust their portfolio holdings. Optimal consumption allocations for the two types are listed below. The resulting expected utility is equal to 0.809528.

θ	α^*	$P_1(\lambda)$	$a^*(P_1(\lambda))$	$(c_1(\theta), c_2(\theta))$	EU^M
1	0.330	1.096	0.954	$(1.065, 1.020)$	0.809528
2	0.330	1.096	0.954	$(1.065, 1.116)$	0.809528
3	0.330	0.915	1	$(0.943, 1.082)$	0.809528
4	0.330	0.915	1	$(0.943, 1.185)$	0.809528

Table 6: Asset Market Allocation

In summary, as long as the probability of a run remains sufficiently low, i.e. below $\widehat{\pi}^*$, the expected utility from the banking-financial allocation is higher when compared to both the banking allocation and the asset market allocation, $EU^{BM} > EU^M > EU^B$. This is because banks and markets provide different services to consumers.

7.2 The Risk-Factor of the Run Equilibrium

We now let π depend on the properties of the contract offered. We model the probability of a run banks face using the concept of the risk-factor of the run equilibrium described in section 5.2. Constant m is set equal to 0.01 and constant h is equal to 0.002. (We choose m and h such that re-scaling ϕ yields a reasonable value of the probability of a run (between 0 and 1)).

Example 4 (Banking Allocation II) We present the optimal banking allocation in Table 7.

β^*	c_1^*	$c_2^* : (1), (2), (3), (4)$	$\underline{\underline{\lambda}}_1, \underline{\underline{\lambda}}_2$	ϕ^*	$\pi(\%)$	EU^B
0.34	0.976	$1.06, 1.15, 1.06, 1.17$	$0.962, 0.852$	0.0534	0.989	0.8085

Table 7: Optimal Banking Allocation II

Qualitatively, the results are similar to those in Table 1. Two variables are

new: the risk-factor of the run equilibrium ϕ^* is equal to 0.0534 and the implied probability of a run is given by 0.989%. We want to know what are the effects of the presence of the asset market on the probability of a run banks face.

Example 5 (Banking-Financial Allocation II) We can see (Table 9) that the riskiness of running in the banking-financial economy increases, $\phi^M = 0.0556$. Hence, consumers' incentives to run decrease and the probability of a run banks face is reduced in our example, $\pi^M = 0.98887$. Two mechanisms play a role here. First, the market investment acts as insurance in the event of a run, i.e. $c_1^N > 0$ and $c_2^N > 0$ whereas in the banks-only economy $c_1^N = c_2^N = 0$. In the banking-financial economy, the higher $c_2^N(\theta)$, *ceteris paribus*, the higher $\phi^M(\theta)$, and the lower π^M. Second, when banks sell the long-term asset in the market, its price decreases. This creates capital gains for late consumers who are the buyers of the long-term asset. Thus, $c_2^N(\theta)$ increases and the probability of a run on the banking sector further decreases.

α^*	$(P_1(\lambda_1), P_1(\lambda_2))$	W^*	$(P_1^L(\lambda_1), P_1^L(\lambda_2))$
0.331	$(1.096, 0.918)$	0.92	$(0.906, 0.744)$

Table 8: Banking-Financial Allocation I

β^*	c_1^B	$c_2^B(1), c_2^B(2), c_2^B(3), c_2^B(4)$	$\underline{\underline{\lambda}}_1, \underline{\underline{\lambda}}_2$	ϕ^M	π^M (%)
0.371	1.061	$1.019, 1.109, 1.016, 1.112$	$0.887, 0.791$	0.0556	0.98887

Table 9: Banking-Financial Allocation II

In general, whether the probability of a run banks face in the banking-financial economy is lower than in the banks-only economy depends on how dramatic the relative reduction in the period-one return on withdrawals in the banks-only economy is. We know that banks offer a considerably smaller degree of the liquidity provision in the banks-only economy, i.e. c_1^* is relatively small

and $c_2^*(\theta)$ is relatively high. On the contrary, the banking-financial contract offers more liquidity insurance which reduces the difference between the period-one and the period-two consumption allocations and, *ceteris paribus*, decreases the riskiness of running. However, as long as the reduction in the period-one return on withdrawals in the banks-only economy is relatively small, the two mechanisms we discussed above will play a major role. Hence, we can expect that the probability of a run banks face in the banking-financial economy is smaller than the probability of a run in the banks-only economy.

Returning to our example, banks face a *smaller* probability of a run while providing *more* liquidity insurance when compared to the optimal banking allocation. In the banks-only economy, banks face a trade-off between the provision of the liquidity insurance and stability. In the banking-financial economy, the market investment weakens this trade-off since it provides insurance in the event of a bank run.

Table 10 summarizes consumer's final consumption allocations (consisting of the return on the market investment and bank withdrawals) in each of the four aggregate states of the world. Again, consumers are *ex ante* better off in the economy with banks and markets, i.e. $EU^{BM} > EU^M > EU^B$.

θ	$(c_1^*(\theta), c_2^*(\theta))$	$(c_1^S(\theta), c_2^S(\theta))$	$(c_1^N(\theta), c_2^N(\theta))$	EU^{BM}
1	$(1.064, 1.020)$	$(0.947, 1.084)$	$(0.862, 0.999)$	0.80954
2	$(1.064, 1.115)$	$(0.947, 1.179)$	$(0.862, 1.094)$	0.80954
3	$(0.955, 1.076)$	$(0.847, 1.161)$	$(0.762, 1.076)$	0.80954
4	$(0.955, 1.178)$	$(0.847, 1.263)$	$(0.762, 1.178)$	0.80954

Table 10: Banking-Financial Allocation III

8 Conclusion

In this paper, we analyze an incomplete markets economy in which consumers can choose whether to invest their endowment in the asset market (indirectly through banks), banks offering demand deposit contracts, or both. Banks and markets provide different services to consumers.

In a nonstochastic economy, the first-best (full information) banking contract is strictly preferred to the market allocation. This is because banks create an *ex ante* cross-subsidy between the early and late consumers. This liquidity insurance is valuable for risk-averse consumers and is impossible to achieve in the competitive market. However, the cross-subsidy makes banks vulnerable to runs. A banking-financial contract that allows consumers to invest part of their endowment in the market indirectly, through banks, is strictly preferred to the optimal banks-only contract since the market investment acts as insurance in the event of a bank run.

In a stochastic economy, the *ex ante* insurance against private preference shocks can still be provided by the banking sector only. The stochastic nature of the economy introduces an additional mechanism. Aggregate liquidity shocks cause fluctuations of the asset prices in the market. Thus, individual market portfolios are subject to market risk. Changing prices redistribute wealth between the early and late consumers and can smooth consumption in *some* states of the world. Of course, this market redistribution occurs *ex post*. Nevertheless, this mechanism increases the expected utility from markets-only allocation as compared to the nonstochastic case.

Within this framework, we study the implications of financial deepening for banks' role in liquidity provision and for the probability of a run on the banking sector.

Consumers insure themselves against bank runs by investing in the market

portfolio. This insurance reduces their incentives to run and, in our examples, probability of a run *decreases*. However, it also means giving up an *ex ante* cross-subsidy on the fraction of wealth invested in the market. Banks reward consumers for getting insurance that gives them more flexibility in their portfolio choice by offering a *higher* cross-subsidy on the portion of wealth invested in the demand deposit contract (as compared to the economy with banks only). The trade-off between the provision of the liquidity insurance and stability is weakened. The *ex ante* welfare increases even though markets remain incomplete.

In this work, we do not consider other services provided by banks, such as payment services. Incorporating these aspects of banking into our model would allow for a richer set of interactions between banks and markets. This is left for future research.

An important implication of our model is that emerging economies can reduce occurrence of panic-based runs and thus increase stability of their banking sector by supporting the development of asset markets.

Appendix

LEMMA 2 *Price P_1 fully reveals λ in period* 1.

Proof. We first show that price P_1 cannot be non-revealing. Suppose not. Then, $P_1 = const$ for all states. Then, all consumers know is their own type and they choose the same a and b for all states. But then, $P_1 = \frac{(1-\lambda)a\alpha}{\lambda b(1-\alpha)}$ must be constant across states. This can only be the case if $a = 0 \; \forall \theta$ and the resulting $P_1 = 0$. But then, $a = 0$ is not optimal. This is not an equilibrium. Contradiction.

We proceed by showing that P_1 cannot reveal λ only partly. Suppose not. Then, there exist two states of the world with distinct realizations of λ, say λ_1 and λ_2, $\lambda_1 \neq \lambda_2$, and at the same time $P_1(\lambda_1) = P_1(\lambda_2)$. Then, consumers must choose the same a and b in these two states. By the same argument as above, this cannot be the case.

It follows that P_1 always fully reveals λ in period 1. ∎

LEMMA 3 *There exists a strictly positive price, which we denote by $\overline{P_1}$, such that for all $P_1 \geq \overline{P_1}$, late types prefer to completely diversify their portfolios, i.e. their demand for the long-term asset for a given λ is zero. Similarly, there exists a strictly positive price, which we denote by $\underline{P_1}$, such that for all $P_1 \leq \underline{P_1}$, late consumers want to hold long-term asset only and thus sell their entire holdings of the short-term asset in period* 1.

Proof. A late consumer chooses a to maximize the expected utility of his portfolio in period 2

$$\sum_R u\left[\left(\left(\frac{a(P_1)\alpha}{P_1} + 1 - \alpha\right)R + (1 - a(P_1))\alpha\right)\omega\right]g(R)$$

subject to

$$0 \leq a(P_1) \leq 1.$$

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Let μ_0 and μ_1 be the Lagrangian multipliers on the two constraints. The first-order condition is

$$\sum_R u'\left[\left(\left(\frac{a\left(P_1\right)\alpha}{P_1}+1-\alpha\right)R+\left(1-a\left(P_1\right)\right)\alpha\right)\omega\right]\times$$
$$\left[\frac{R}{P_1}-1\right]\alpha g\left(R\right)+\mu_0-\mu_1=0$$

and $\mu_0 \geq 0$, $\mu_1 \geq 0$.

First, consider case $a = 0$, $\mu_0 \geq 0$ and $\mu_1 = 0$. Then,

$$\sum_R u'\left[\left(\left(1-\alpha\right)R+\alpha\right)\omega\right]\left[\frac{R}{P_1}-1\right]\alpha\omega g\left(R\right)+\mu_0=0$$

Thus, it must be the case that

$$\sum_R u'\left[\left(\left(1-\alpha\right)R+\alpha\right)\omega\right]\left[\frac{R}{P_1}-1\right]\alpha\omega g\left(R\right)\leq 0 \qquad (C1)$$

with equality for $\mu_0 = 0$. Note that it cannot be the case that $P_1 < R$ for all realizations of R. Also, if $P_1 = R$ always holds, then the price reveals R, contradiction with price revealing λ only. Then, we must have either that $P_1 > R$ always holds[24] or that $P_1 < R$ and $P_1 > R$ for some realizations of R.

Second, consider case $a = 1$, $\mu_0 = 0$ and $\mu_1 \geq 0$. Then, the first-order condition is as follows

$$\sum_R u'\left[\left(\frac{\alpha}{P_1}+1-\alpha\right)R\omega\right]\left[\frac{R}{P_1}-1\right]\alpha\omega g\left(R\right)-\mu_1=0.$$

Then,
$$\sum_R u'\left[\left(\frac{\alpha}{P_1}+1-\alpha\right)R\omega\right]\left[\frac{R}{P_1}-1\right]\alpha\omega g\left(R\right)\geq 0 \qquad (C2)$$

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[24] We shall see later (in the Proof of Proposition 6.) that this cannot be the case because the short-term asset would be dominated by the long-term asset in this case.

with equality for $\mu_1 = 0$. Here, it cannot be the case that $P_1 \geq R$ holds for all realizations of R.

Finally, consider case $0 < a < 1$ and $\mu_0 = \mu_1 = 0$. Then,

$$\sum_R u' \left[\left(\left(\frac{a\,(P_1)\,\alpha}{P_1} + 1 - \alpha \right) R + (1 - a\,(P_1))\,\alpha \right) \omega \right] \left[\frac{R}{P_1} - 1 \right] \alpha g\,(R) = 0.$$

We can see that $P_1 \leq R$ $(P_1 \geq R)$ cannot hold for all realizations of R since that would require (for the first-order condition to hold) $P_1 = R$ and thus the price would reveal R, contradiction with price revealing λ only. Thus, we must have $P_1 > R$ and $P_1 < R$ for some realizations of R.

Now, there exists a price, which we denote by $\overline{P_1}$, such that for all $P_1 \geq \overline{P_1}$, late types' demand for the long-term asset for a given λ is zero: $a = 0$. This price is the minimum price that satisfies $(C1)$. Note that the left hand side of $(C1)$ is decreasing in P_1:

$$- \sum_R u'[((1 - \alpha)\,R + \alpha)\,\omega] \frac{R}{P_1^2} \alpha \omega g\,(R) < 0$$

and so all $P_1 \geq \overline{P_1}$ will satisfy $(C1)$ and thus $a = 0$.

Similarly, there exists a price, which we denote by $\underline{P_1}$, such that for all $P_1 \leq \underline{P_1}$, late consumers want to sell their entire holdings of the short-term asset in exchange for the long-term asset. For example, for any $P_1 \leq \min_\theta \{R\}$, condition $(C2)$ is satisfied and $a = 1$. ∎

LEMMA 4 *Price P_1 is always greater or equal to the liquidation return L and early consumers supply a strictly positive amount of the long-term asset to the market.*

Proof. Suppose not, i.e. there exists λ such that $P_1\,(\lambda) < L$. Then, we have $b\,(P_1) = 0$ and $S = 0$. For the market to clear, it must be the case that $D = 0$ implying $a\,(P_1) = 0$. We know from the previous lemma that $a = 0$ if and only

if $P_1 \geq \overline{P_1} > 1 > L$. Contradiction with $P_1 < L$. The claim follows. ∎

PROPOSITION 5 *For a given λ, there are three possible cases. 1) The equilibrium price is equal to L. Late consumers prefer to hold the long-term asset only and early consumers sell the fraction $\frac{\alpha(1-\lambda)}{L\lambda(1-\alpha)}$ of their long-term asset holdings. 2) The price in the market that would arise if late consumers sold their entire short-term asset holdings, $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$, is smaller than $\underline{P_1}$. Then, late consumers prefer to hold the long-term asset only and the resulting equilibrium price is equal to $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$. 3) The price $\frac{(1-\lambda)\alpha}{\lambda(1-\alpha)}$ is greater or equal to $\underline{P_1}$. Then, late consumers prefer to diversify their portfolio holdings. They choose $0 < a \leq 1$ optimally and the resulting equilibrium price is given by $P_1 = \frac{(1-\lambda)a\alpha}{\lambda(1-\alpha)}$ with $P_1 \in [\underline{P_1}, \overline{P_1})$.*

Proof. We want to show that $P_1 < \overline{P_1}$ always holds in equilibrium. Suppose not and we have $P_1 \geq \overline{P_1} > 0$ for some λ. We know that $P_1 = \frac{(1-\lambda)a\alpha}{\lambda(1-\alpha)}$. The fact that $P_1 \geq \overline{P_1}$ implies (by definition of $\overline{P_1}$) that $a = 0$ and hence we get $P_1 = 0$. Contradiction with $P_1 \geq \overline{P_1} > 0$. ∎

PROPOSITION 6 *In equilibrium, consumers prefer to diversify their period-zero portfolios by investing a strictly positive fraction of the endowment in the long-term asset and thus a positive fraction of the endowment in the short-term asset.*

Proof. We prove each property in turn by contradiction.

First, suppose an individual finds it optimal to hold the short-term asset only, i.e. $\alpha = 1$, $\mu_2 = 0$ and $\mu_3 \geq 0$. At $t = 1$, then, price P_1 must be high enough so as to induce zero demand for the long-term asset, i.e. $a(P_1) = 0$. Sufficient condition would be $P_1 > \max\{R\}$ (see condition $(C1)$). The necessary condition is $P_1 > 1$ in all states (since R is always greater than 1). In this case,

however, the first-order condition

$$\sum_R \sum_\lambda \left[\lambda u'(c_1(\theta)) \left[1 - P_1(\lambda) \right] + (1 - \lambda) u'(c_2(\theta)) \right.$$

$$\left. (1 - R) \right] \omega f(\lambda) g(R) - \mu_3 = 0.$$

cannot hold (short-term asset dominates the long-term asset). Contradiction. We must have and $\alpha < 1$ and $\mu_2 = 0$ in the optimum.

Now suppose that optimal α is equal to zero, i.e. an individual only invests in the long-term asset. Then, $\mu_2 \geq 0$ and $\mu_3 = 0$. Since λ is always greater than 0 (by assumption), early consumers will want to offer their long-term asset holdings for sale. Since late types have nothing to offer in exchange, the price in the market P_1 must be equal to zero for all states. In this case, $c_1 = 0$ and $c_2 = R\omega$. However, given that the price of the long-term asset is zero, short-term asset in fact dominates the long-term asset. Thus, $\alpha^* = 0$ cannot be optimal. Contradiction.

Thus, we have $0 < \alpha < 1$ in the optimum and α^* satisfies:

$$\sum_R \sum_\lambda \left[\lambda u'(c_1(\theta)) \left[1 - P_1(\lambda) \right] + (1 - \lambda) u'(c_2(\theta)) \right.$$

$$\left. \left(\left(\frac{a(P_1)}{P_1(\lambda)} - 1 \right) R + 1 - a(P_1) \right) \right] f(\lambda) g(R) = 0.$$

∎

LEMMA 10 *Suppose λ is non-stochastic. Then, the first best contract satisfies $c_1^* > 1$ and $c_2^*(\theta) < R$. This implies the preference for liquidity (and cross-subsidization) between early and late consumers.*

Proof. We first show that $c_1^* > 0$ and $\beta^* < 1$ holds for the solution to the problem (FB). We then consider the case of non-random λ and sketch the proof of the Lemma.

Lagrangian L for the problem (FB) is as follows:

$$\sum_R \sum_{\lambda \in \Lambda_\lambda} \left[\lambda u\left(c_1\right) + (1-\lambda) u\left(c_2\right)\right] f\left(\lambda\right) g\left(R\right) + \sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \underline{\lambda} u\left(c_1\right) \times \tag{1}$$

$$f\left(\lambda\right) g\left(R\right) + \mu_2 \left(1 - \beta\right) + \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3\left(\theta\right) \left[\beta + R\left(1 - \beta\right) - \lambda c_1 - \left(1 - \lambda\right) c_2\left(\theta\right)\right] \times$$

$$f\left(\lambda\right) g\left(R\right) + \mu_1 c_1 + \sum_R \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \mu_3\left(\theta\right) \left[R\left(1 - \beta + \frac{\beta - \lambda c_1}{L}\right) - \left(1 - \lambda\right) c_2\left(\theta\right)\right] \times$$

$$f\left(\lambda\right) g\left(R\right) + \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \varphi\left(\theta\right) \left[\beta - \lambda c_1\right] f\left(\lambda\right) + \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \varphi\left(\theta\right) \left[\beta + L\left(1 - \beta\right) - \lambda c_1\right] f\left(\lambda\right).$$

First order conditions (FOCs) with respect to c_1, $c_2\left(\theta\right)$, and β are:

$$\sum_R \sum_{\lambda \in \Lambda_\lambda} \lambda u'\left(c_1\right) f\left(\lambda\right) g\left(R\right) + \sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \left(\beta + L\left(1 - \beta\right)\right) \times \tag{2}$$

$$\frac{u'\left(c_1\right) c_1 - u\left(c_1\right)}{c_1^2} f\left(\lambda\right) g\left(R\right) + \mu_1 - \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3\left(\theta\right) \lambda f\left(\lambda\right) g\left(R\right) -$$

$$- \sum_R \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \mu_3\left(\theta\right) \lambda \frac{R}{L} f\left(\lambda\right) g\left(R\right) - \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \varphi\left(\theta\right) \lambda f\left(\lambda\right) - \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \varphi\left(\theta\right) \times$$

$$\lambda f\left(\lambda\right) = 0, \ \mu_1 \geq 0.$$

$$u'\left(c_2\left(\theta\right)\right) f\left(\lambda\right) g\left(R\right) - \mu_3\left(\theta\right) f\left(\lambda\right) g\left(R\right) = 0, \ \mu_3\left(\theta\right) \geq 0 \text{ for each } \theta \tag{3}$$

$$\sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \frac{u\left(c_1\right)}{c_1} \left(1 - L\right) f\left(\lambda\right) g\left(R\right) - \mu_2 + \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3\left(\theta\right) \left[1 - R\right] \times \tag{4}$$

$$f\left(\lambda\right) g\left(R\right) + \sum_R \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \mu_3\left(\theta\right) R\left(\frac{1}{L} - 1\right) f\left(\lambda\right) g\left(R\right) + \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \varphi\left(\theta\right) f\left(\lambda\right) +$$

$$\sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \varphi\left(\theta\right) \left(1 - L\right) f\left(\lambda\right) = 0, \ \mu_2 \geq 0.$$

Complementary slackness conditions are:

$$\mu_1 c_1 = 0$$

$$\mu_2 \left(1 - \beta\right) = 0$$

$$\mu_3\left(\theta\right)\left[\beta + R\left(1 - \beta\right) - \lambda c_1 - \left(1 - \lambda\right)c_2\left(\theta\right)\right] = 0 \text{ for } \lambda \in \Lambda_{\underline{\lambda}}$$

$$\mu_3\left(\theta\right)\left[R\left(1 - \beta + \frac{\beta - \lambda c_1}{L}\right) - \left(1 - \lambda\right)c_2\left(\theta\right)\right] = 0 \text{ for } \lambda \in \Lambda_{\underline{\underline{\lambda}}}$$

$$\varphi\left(\theta\right)\left(\beta - \lambda c_1\right) = 0 \text{ for } \lambda \in \Lambda_{\underline{\lambda}}$$

$$\varphi\left(\theta\right)\left(\beta + L\left(1 - \beta\right) - \lambda c_1\right) = 0 \text{ for } \lambda \in \Lambda_{\underline{\underline{\lambda}}}$$

Note that equation 3 implies $u'(c_2\left(\theta\right)) = \mu_3\left(\theta\right)$ and thus $\mu_3\left(\theta\right) > 0$. We now claim that $c_1^* > 0$ must hold. Suppose not and $c_1^* = 0$. Then, $\underline{\lambda} = \underline{\underline{\lambda}} = 1$ and $\varphi\left(\theta\right) = 0$ for all θ. equation 2 reduces to

$$\sum_R \sum_{\lambda \in \Lambda} \lambda\left[u'(c_1) - u'(c_2\left(\theta\right))\right] f\left(\lambda\right) g\left(R\right) + \mu_1 = 0.$$

However, $c_1 = 0$ together with the strict concavity of u give $u'(c_1) - u'(c_2\left(\theta\right)) > 0$ and thus equation 2 cannot hold. Contradiction. Thus, we have $c_1^* > 0$ and $\mu_1^* = 0$.

Furthermore, $\beta^* < 1$ and $\mu_2^* = 0$. To see this, suppose otherwise. Then, $\mu_3\left(\theta\right) = 0$ for $\lambda \in \Lambda_{\underline{\underline{\lambda}}}$ for the complementarity slackness condition to hold. But this is not possible since $\mu_3\left(\theta\right) > 0$ for all θ. Then, banks need to ensure that all $\lambda \leq \underline{\lambda} = \underline{\underline{\lambda}}$. It follows from $\mu_3\left(\theta\right) > 0$ that $c_2\left(\theta\right) = \frac{1 - \lambda c_1}{1 - \lambda}$. incentive-compatibility requires that

$$\sum_{\lambda \in \Lambda_{\underline{\lambda}}} u\left(c_1\right) f_p\left(\lambda\right) + \sum_{\lambda \in \Lambda \backslash \Lambda_{\underline{\lambda}}} \left[\underline{\underline{\lambda}}\frac{1}{\lambda} u\left(c_1\right)\right] f_p\left(\lambda\right) \leq \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} u\left(c_2\left(\theta\right)\right) f_p\left(\lambda\right) g\left(R\right).$$

For this condition to hold there must be at least one λ such that $u\left(c_1\right) \leq u\left(c_2\left(\theta\right)\right)$, which is equivalent to $c_1 \leq \frac{1 - \lambda c_1}{1 - \lambda}$ for this λ. It follows that $c_1 \leq 1$ must hold. This implies that $\underline{\lambda} = \underline{\underline{\lambda}} = 1$ and $\varphi\left(\theta\right) = 0$. But then, equation 4

cannot hold. Contradiction.

Now, if λ is non-random, the bank knows exactly how many consumers will arrive in period 1. Intuitively, the bank will invest such that no inefficient liquidation of the long-term asset takes place in period 1. It is easy to show that $c_1 = \frac{\beta}{\lambda}$ and $c_2 = \frac{(1-\beta)R}{1-\lambda}$ in the optimum. FOC yields:

$$\sum_R u'(c_1) g(R) = \sum_R R u'(c_2) g(R).$$

We want to show that $c_1 > 1$ and $c_2 < R$ in the optimum. Suppose that $c_1^* = 1$ and thus $c_2^* = R$ instead. We know that $c u'(c)$ is decreasing in c for $[1, \infty)$ since $-\frac{c u'(c)}{u'(c)} > 1$ on this interval. Given that R is always greater than 1, we get $u'(1) > R u'(R)$ for all R. But then, we have $\sum_R u'(1) g(R) > \sum_R R u'(R) g(R)$ and thus the pair $(1, R)$ does not satisfy the FOC. Strict concavity of u implies that $c_1^* > 1$ and $c_2^* < R$ must hold. ∎

LEMMA 13 *Equilibrium price of the long-term asset when banks trade in the market fully reveals λ and is always greater or equal to the liquidation return L.*

Proof. Suppose not, i.e. there exist two distinct values of λ, say λ_1 and λ_2, $\lambda_1 \neq \lambda_2$, such that $P_1^L(\lambda_1) = P_1^L(\lambda_2)$. Then, it must be the case that $a\left(P_1^L(\lambda_1)\right) = a\left(P_1^L(\lambda_2)\right)$, $b\left(P_1^L(\lambda_1)\right) = b\left(P_1^L(\lambda_2)\right)$, and $\widehat{\gamma}\left(P_1^L(\lambda_1)\right) = \widehat{\gamma}\left(P_1^L(\lambda_2)\right)$. This implies that $a\left(P_1^L(\lambda_1)\right) = a\left(P_1^L(\lambda_2)\right) = 0$ must hold and thus $P_1 = 0$. But then, $a = 0$ is not optimal. Contradiction. ∎

FOCs of the problem (BMP) with respect to c_1^B, $c_2^B(\theta)$, β, W, and α are

$$
(1-\pi)\left\{\sum_R \sum_{\lambda \in \Lambda_\lambda} \lambda u'(c_1)(1-W) f(\lambda) g(R) + \sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \underline{\underline{\lambda}} \times \right. \tag{5}
$$

$$
\frac{u'(c_1)(1-W) c_1^B - u(c_1) + u(c_1^N)}{c_1^B} f(\lambda) g(R) \Bigg\} + \pi \left\{ \frac{1}{c_1^B} \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \underline{\lambda} \times \right.
$$

$$
\left[\lambda \left(u'(c_1)(1-W) c_1^B - u(c_1) \right) + (1-\lambda)\left(u'(c_2^S)(1-W) c_1^B - u(c_2^S) \right) + \right.
$$

$$
\left. \lambda u(c_1^N) + (1-\lambda) u(c_2^N) \right] f(\lambda) g(R) \Bigg\} + \mu_1 - \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3(\theta) \lambda f(\lambda) g(R) -
$$

$$
\sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3(\theta) \lambda \frac{R}{L} f(\lambda) g(R) - \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \varphi(\theta) \lambda f(\lambda) - \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \varphi(\theta) \lambda f(\lambda) - \mu_6 \times
$$

$$
\left(\sum_R \sum_{\lambda \in \Lambda_\lambda} u'(c_1^W)(1-W) f_p(\lambda) g(R) + \sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \frac{u'(c_1^W)(1-W) c_1^B - u(c_1^W)}{c_1^B} \right.
$$

$$
\left. \frac{\underline{\underline{\lambda}}}{\lambda} f_p(\lambda) g(R) \right) = 0, \ \mu_1 \geq 0
$$

$$
(1-\pi)(1-\lambda) u'(c_2)(1-W) f(\lambda) g(R) - \mu_3(1-\lambda) f(\lambda) g(R) + \mu_6 \times \tag{6}
$$

$$
u'(c_2)(1-W) f_p(\lambda) g(R) = 0 \text{ and } \mu_3 \geq 0
$$

$$
(1-\pi) \sum_R \sum_{\lambda \in \Lambda \setminus \Lambda_\lambda} \frac{u(c_1) - u(c_1^N)}{c_1^B} \left(1 - \max\left\{ P_1^L(\lambda), L \right\} \right) f(\lambda) g(R) + \tag{7}
$$

$$
\pi \left\{ \frac{1}{c_1^B} \sum_R \sum_{\lambda \in \Lambda} \left(1 - \max\left\{ P_1^L(\lambda), L \right\} \right) \left[\lambda u(c_1) + (1-\lambda) u(c_2^S) - \lambda u(c_1^N) - \mu_2 - \right.
$$

$$
\left. (1-\lambda) u(c_2^N) \right] f(\lambda) g(R) \Bigg\} + \sum_R \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \mu_3(\theta)(1-R) f(\lambda) g(R) + \sum_R \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \mu_3(\theta)
$$

$$
R\left(\frac{1}{\max\left\{ P_1^L(\lambda), L \right\}} - 1 \right) f(\lambda) g(R) + \sum_{\lambda \in \Lambda_{\underline{\underline{\lambda}}}} \varphi(\theta)\left(1 - \max\left\{ P_1^L(\lambda), L \right\} \right) f(\lambda) +
$$

$$+ \sum_{\lambda \in \Lambda_{\underline{\lambda}}} \varphi(\theta) f(\lambda) - \mu_6 \sum_R \sum_{\lambda \in \Lambda \smallsetminus \Lambda_\lambda} \frac{1 - \max\left\{P_1^L(\lambda), L\right\}}{\lambda c_1^B} u\left(c_2^W\right) f_p(\lambda) g(R) = 0,$$

$$\mu_2 \geq 0$$

$$(1-\pi)\left\{\sum_R \sum_{\lambda \in \Lambda_\lambda} \left[\lambda u'(c_1)\left(\alpha + (1-\alpha)P_1 - c_1^B\right) + (1-\lambda)u'(c_2) \times \right.\right. \tag{8}$$

$$\left.\left.\left(\left(\frac{a\alpha}{P_1} + 1 - \alpha\right)R + (1-a)\alpha - c_2^B\right)\right] f(\lambda)g(R) + \sum_R \sum_{\lambda \in \Lambda \smallsetminus \Lambda_\lambda} \left[\underline{\lambda} u'(c_1) \times \right.\right.$$

$$\left(\alpha + (1-\alpha)P_1 - c_1^B\right) + \left(\lambda - \underline{\lambda}\right)u'\left(c_1^N\right)\left(\alpha + (1-\alpha)P_1\right) + (1-\lambda)u'\left(c_2^N\right) \times$$

$$\left.\left(\left(\frac{a\alpha}{P_1} + 1 - \alpha\right)R + (1-a)\alpha\right)\right] f(\lambda)g(R)\right\} + \pi\left\{\sum_R \sum_{\lambda \in \Lambda} \underline{\underline{\lambda}}\left[\lambda u'(c_1) \times \right.\right.$$

$$\left.\left(\alpha + (1-\alpha)P_1^L - c_1^B\right) + (1-\lambda)u'\left(c_2^S\right)\left(\left(\frac{a\alpha}{P_1^L} + 1 - \alpha\right)R + (1-a)\alpha - c_1^B\right)\right]$$

$$+ \left(1 - \underline{\underline{\lambda}}\right)\left[\lambda u'\left(c_1^N\right)\left(\alpha + (1-\alpha)P_1^L\right) + (1-\lambda)u'\left(c_2^N\right)\left(\left(\frac{a\alpha}{P_1^L} + 1 - \alpha\right)R + \right.\right.$$

$$\left.\left.(1-a)\alpha\right]f(\lambda)g(R)\right\} + \mu_6\left\{\sum_R \sum_{\lambda \in \Lambda_\lambda} u'(c_2)\left(\left(\frac{a\alpha}{P_1(\theta)} + 1 - \alpha\right)R + (1-a)\alpha \right.\right.$$

$$-c_2^B\right)f_p(R)g(R) - \sum_R \sum_{\lambda \in \Lambda_\lambda} u'\left(c_2^W\right)\left(\left(\frac{a\alpha}{P_1(\theta)} + 1 - \alpha\right)R + (1-a)\alpha - c_1^B\right) \times$$

$$f_p(R)g(R) - \sum_R \sum_{\lambda \in \Lambda \smallsetminus \Lambda_\lambda} \underline{\underline{\lambda}}\frac{1}{\lambda}u'\left(c_2^W\right)\left(\left(\frac{a\alpha}{P_1(\theta)} + 1 - \alpha\right)R + (1-a)\alpha - c_1^B\right) \times$$

$$f_p(R)g(R)\right\} + \mu_4 - \mu_4' = 0, \ \mu_4, \mu_4' \geq 0$$

$$(1-\pi)\left\{\sum_R \sum_{\lambda \in \Lambda_\lambda} \left[\lambda u'(c_1)(1 - P_1) + (1-\lambda)u'(c_2)\left(\left(\frac{a}{P_1} - 1\right)R \right.\right.\right. \tag{9}$$

$$\left.\left.+1 - a\right]f(\lambda)g(R) + \sum_R \sum_{\lambda \in \Lambda \smallsetminus \Lambda_\lambda} \left[\underline{\lambda}u'(c_1)(1-P_1) + \left(\lambda - \underline{\lambda}\right)u'\left(c_1^N\right)\left(1 - P_1^L\right) + \right.\right.$$

$$(1-\lambda)u'\left(c_2^N\right)\left(\left(\frac{a}{P_1^L} - 1\right)R + 1 - a\right)\right] f(\lambda)g(R)\right\} + \pi\left\{\sum_R \sum_{\lambda \in \Lambda} \left[\underline{\lambda}\left(\lambda u'(c_1)\right)\right.\right.$$

$$\left(1 - P_1^L\right) + \left(1 - \lambda\right) u'\left(c_2^S\right) \left(\left(\frac{a}{P_1^L} - 1\right) R + 1 - a\right)\right) + \left(1 - \underline{\underline{\lambda}}\right) \left(\lambda u'\left(c_1^N\right) \times$$

$$\left(1 - P_1^L\right) + \left(1 - \lambda\right) u'\left(c_2^N\right) \left(\left(\frac{a}{P_1^L} - 1\right) R + 1 - a\right)\right)\right] f\left(\lambda\right) g\left(R\right)\right\} + \mu_6 \times$$

$$\left\{\sum_R \sum_{\lambda \in \Lambda_\lambda} u'\left(c_2\right) \left(\left(\frac{a}{P_1} - 1\right) R + 1 - a\right) f_p\left(R\right) g\left(R\right) - \sum_R \sum_{\lambda \in \Lambda_\lambda} u'\left(c_2^W\right) \times\right.$$

$$\left(\left(\frac{a}{P_1} - 1\right) R + 1 - a\right) f_p\left(R\right) g\left(R\right) - \sum_R \sum_{\lambda \in \Lambda \smallsetminus \Lambda_\lambda} \underline{\underline{\lambda}} \frac{1}{\lambda} u'\left(c_2^W\right) \left(\left(\frac{a}{P_1} - 1\right) R + \right.$$

$$\left. 1 - a\right) f_p\left(R\right) g\left(R\right)\right\} - \mu_5 = 0 \text{ and } \mu_5 \geq 0.$$

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