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Endogenous Policy Reform: Learning versus Flexibility in Industrial Policy Design for Open Economies

by

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Abstract

This paper studies government support which targets industries capable of learning. Monitoring procedures are implemented to prevent rent-seekers from non-learning industries. But this involves bureaucratic red tape that reduces flexibility in response to world market shocks. The main thrust of this study is that, as an economy develops, the balance shifts endogenously between the need for bureaucratic monitoring and the desirability for flexibility. An optimal strategy may be to support learning, with a certain degree of bureaucracy at an early stage when necessary, but to liberalize the policy toward laissez faire, as the economy matures. The results appear be consistent with empirical evidences in Asian countries.

Key Words: targeted industrial policy, dynamic learning externalities, rent-seeking activities, monitoring, bureaucratic red tape, variable market demand, production flexibility

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1 Introduction

In the early stage of development, many countries, particularly those in Asia, adopt industrial policies to support industries enjoying priority. Later on, policy makers emphasize the need to liberalize and approach laissez faire (see, e.g., Rao (1996) for India, Kuo (1995) for Taiwan, SaKong (1993) for Korea, and Komiya (1988) for Japan). Industrial policies typically undergo a continuous sequence of policy reforms over time. In general, they consist of government support and bureaucratic control. This paper focuses on three distinct aspects of industrial policies. The first aspect is that government support promotes learning, and the second is that bureaucratic control is imposed against rent-seekers. As the third aspect, which we emphasize the most in this study, bureaucratic control induces red tape in the sense that it makes private sectors less flexible in response to world market shocks. There are two main questions not yet resolved in the literature: (1) How do these aspects relate to each other? (2) Are the observed policy reforms the correction of past errors by the present decision makers, or do such reforms reflect the optimal transition of the policy design to adapt to the changed circumstances? This paper addresses such questions in both intuitive and analytical terms.

The need for production flexibility in a world with variable demand has been emphasized in the recent literature (see, e.g., Beckman (1990), Killick (1995), and Deyo and Doner (2001)). As case studies, Amsden (1985) and Subrathan (1991) comment on the ability to provide fast delivery as critical to success in the competitive international machine tool market. Industrial policies often reduce flexibility due to complex bureaucratic procedures. Desai (1985) points out that complex state controls over industries in pre-reform India have rendered them uncompetitive in the world market despite their various latent strength. This fact suggests that the decline of flexibility due to bureaucracy can hamper the potential economic development. On the other hand, even at some cost to flexibility, some degree of bureaucratic control may be needed against rent-seekers, who abuse developmental policy for individual interest. Thus, balancing the issues of flexibility, rent-seeking and bureaucracy is critically important for policy makers.

To our knowledge, few studies have analyzed policy reforms taking into account the loss in flexibility due to bureaucracy in the economics literature.[1] Lin and Wan (1996) examined empirical evidences from Asian countries and showed that strict bureaucracy engenders a loss in flexibility in a static setting. In the present paper, an industrial policy refers to the support for the firms that induce dynamic learning externalities. In order to describe this phenomenon, modelling theoretically such an issue in a dynamic setting is vital to examine the optimal design of an industrial policy.

There have been on-going debates about the effectiveness of industrial policies. At least from the theoretical perspective, our results agree with Itoh, Kiyono, Okuno-Fujiwara, and Suzumura (1991) in that temporary industrial policies may be valuable. Moreover, the development process, viewed as a episode of 'catching-up' in 'late-industrialization,' may call for an endogenous policy sequence, with justified government participation in early stages and continued liberalization toward laissez faire. Balancing the shifting weights of benefit against cost of government interventions, an optimal policy sequence has its own rhythm and timing for policy reforms. Therefore, the nature of policy reform might not be a transition from past error to present sagacity, but the adaptation to the change in circumstance.

Our model also shows whether a certain degree of bureaucracy is deemed tolerable under an industrial policy at an early stage of development. The degree of bureaucracy depends on two important issues; the first is the rent-seeking issue in which rent-seekers make an industrial policy less effective, and the second is the flexibility issue in which bureaucratic red tape hurts flexibility in the world market with variable demand. Elaborate bureaucratic procedures preclude the rent-seeking issue but give rise to the flexibility issue. Thus, elaborate bureaucracy can be justified if the rent-seeking issue is relatively significant, but not if the flexibility issue is relatively significant. This result has important policy implication. The lack of competitiveness of export industries in pre-reform India (Desai (1985)) may be attributed to strict administrative controls under frequent market shifts in the world market.

Moreover, we find it plausible that policy makers should seek to reduce in complexity of

[1] Although there are many studies on aspects of production flexibility under demand uncertainty (e.g., Turnovsky (1973) and Epstein (1978)), the impacts of demand variability, which directly affects the flexibility, on the optimal policy are hardly examined in the context of dynamic settings.

bureaucratic procedures before an ultimate transition to laissez faire. An administrative reform may also be an optimal action reflecting the change in circumstance. This finding may be consistent with the fact that in many countries, administrative reforms to simplify bureaucratic procedures have become the slogans of the day even though industry targeting still holds.

The rest of the paper is organized as follows. Section 2 defines what constitutes benefits and costs of bureaucracy in the development context and analyzes what serves as the force for the change in the balance of the costs and the benefits. In Section 3, we presents a dynamic model for a small open economy with industries capable of learning, the presence of potential rent-seekers and variable market demands. We also introduce government actions consisting of both government support and bureaucratic control. Section 4 derives the optimal policy for the model and shows some important results. Casting benefits and costs of an industrial policy in analytic terms, we argue that each policy package may be appropriate to proper circumstances. Section 5 provides a summary as conclusion. All proofs are in the Appendix.

2 Benefit and Cost of Bureaucracy

To lay the foundations for subsequent analysis, this section examines the interdependence among the three aspects of industrial policies introduced in the previous section. These aspects are closely related to benefits and costs of bureaucracy. In the present context, the objective of industrial policies is to promote industries with dynamic learning effect that meets the Kemp criterion (Kemp (1960)).[2] We assume that all industries desire government support for their own interest, but some industries would contribute to more learning when supported. The pace of learning may be less than ideal without an industrial policy.

First, we clarify the benefit of bureaucracy. The ubiquity of bureaucratic procedures is the result of neither coincidence nor malice. The administrative problem is that of potential

[2] For an early study on learning-by-doing, in which technological progress is the serendipitous by-product of experience gained in the production, see, e.g., Arrow (1962). The direction of an industrial policy is decided by various criteria, developed by Mill, Bastable, Kemp and Negishi. See, e.g., Itoh, Kiyono, Okuno-Fujiwara, and Suzumura (1991). In particular, Kemp (1960) showed a crucial condition for an industrial policy, known as the Kemp criterion, requiring that benefits from technological progress over time cannot be obtained by private incentives due to dynamic learning externalities. This condition justifies a targeted industrial policy, whereby some industries with such learning effect should be targeted while others remain non-targeted.

misclassification of commodities. This problem arises because of industry targeting. If there is any ambiguity about the appropriate classification for a commodity, private firms in non-targeted industries have an incentive to make it appear to fit into a targeted category (see, e.g., Corden (1990) for a tariff case).[3] Information asymmetry between the administrator and applicants would allow some non-learning industries to receive support. This makes an industrial policy less effective and might lower an attainable welfare through policy dilution. Bureaucratic controls play an important role in precluding this problem. We call such control as bureaucratic monitoring. Bureaucratic monitoring takes the form of bureaucratic routines (e.g., requiring documentation of the merits of the claim). The purpose of bureaucratic monitoring in this paper is not so much to prevent shirking by the agent, a concern in standard principal-agent problems, but rather to make the policy of industrial promotion more effective by excluding from government assistance those whose claims are without merit.

Second, we identify the cost of bureaucracy. In the present study, flexibility is defined as the adaptability in modifying designs and the capability of timely response to market variability. Bureaucratic monitoring brings about the loss in flexibility. Private firms waste their time and energy due to complex bureaucratic procedures. Thus, the degree of monitoring determines the cost of bureaucracy.[4] This element is quite important in the globalized era. There have been many case studies on the impact of bureaucratic monitoring on flexibility. Roy (1996) states that in some cases hundreds of documents must be submitted to complete a single transaction in pre-reform India. Subrathan (1991) also mentions that there is a 6-10 months delay in getting approval of licenses to import components for production in Indian's machine tool industry. Appelbaum and Smith (2001) state that a relative lack of regulation contributes to making Chinese factories in the garment industry so flexible.

Finally, we discuss about the change of benefit and cost of bureaucracy over time. Matching the elements of benefit and cost, we can arrive at an optimal policy package, consisting of govern-

[3] As another type of rent-seeking activities, there is the concept of 'rent-seeking', in which government and bureaucracy are easily subject to persuasion and influence from businessmen affected by tariffs and quotas. See, e.g., Krueger (1974) and Bhagwati (1982).

[4] The administrative cost may also be significant, but we ignore it to focus on the effect of bureaucratic monitoring on flexibility.

ment support and bureaucratic monitoring, under given any circumstance. For the developing countries, learning often means the catching up with advanced countries, through tapping into the 'technology backlog,' that is, the technology gap. As development proceeds, the scope for learning is reduced (see, e.g., Mansfield, Rapoport, Romero, Villani, Wagner, and Husic (1977) and Kuznets (1982)).[5] Since elaborate bureaucracy is imposed to enhance learning by avoiding policy dilution, as the scope of learning declines, so does the benefit of bureaucracy. In contrast, the nature of the cost of bureaucracy does not change over time. So far as bureaucracy is concerned, proportionately speaking, the economy may face a reduction in benefit but not in cost over time. As circumstances change, so does the optimal policy. Thus, it is quite important to examine optimal policy sequence in a dynamic setting.

3 The Model

3.1 Basic Elements

Consider a small open economy with an infinite period and a continuum of commodities.[6] Commodities are indexed on the interval $[0, 1]$. Commodity z, which is produced in industry z, is associated with each point on the interval. Each industry can be categorized as either a learning industry or a non-learning industry. Learning industries are represented as the subset $[0, K]$ of interval $[0, 1]$, and non-learning industries as the subset $(K, 1]$, where $K \in (0, 1)$ is fixed. The international price of all commodities is nomalized to unity, and the total labor input in the economy is fixed at $\bar{L} > 0$ in all periods.

Human capital H_t is a public good for production benefitting all producers equally. Only the production in learning industries contributes to human capital accumulation through learning-by-doing. Specifically, human capital in the next period depends on its current level and the current labor employment in learning industries, and the state equation for human capital is

[5] It may be considered that each industry has bounded learning, introduced by Young (1993), requiring that no knowledge can be gained by learning in highly matured industries.

[6] We consider a discrete-time model, instead of a continuous-time model. A unit of time should be introduced in order to capture clearly the situation in which a style change in market demands has the negative effect on the production flexibility, explained carefully in the later part.

given as:

$$H_{t+1} = h(L_{A,t}, H_t), \tag{1}$$

where $L_{A,t}$ is the labor employment in learning industries at period t, and $h(\cdot,\cdot)$ is the learning function with $h(\cdot,\cdot) > 0$, $h_L > 0$, $h_H > 0$ and $\lim_{H\to 1-} h(\cdot, H) = 1$. These conditions ensure that human capital is bounded and rises toward its asymptotic limit of unity, and that its increment is increasing in $L_{A,t}$. This specification captures the situation in which the dynamic learning externalities emphasized in the Kemp criterion become larger with an increase in $L_{A,t}$ and are diminishing over time. Figure 1 illustrates these properties when $L_{A,t}$ is fixed at a certain level, L^1 and L^2 with $L^1 > L^2$, in any period.

Taking human capital H_t as given, $x_{z,t}$, the output of commodity z in period t, is competitively produced using labor $l_{z,t}$ and its commodity-specific factor $a_{z,t}$ under a constant-returns technology, as in the framework of the Ricardo-Viner model. Labor can move freely among industries, while the commodity-specific factors cannot. Output of commodity z is given as:

$$x_{z,t} = F(l_{z,t}, a_{z,t}|H_t, \epsilon_{z,t}) \equiv \epsilon_{z,t} H_t f(l_{z,t}, a_{z,t}), \tag{2}$$

where the output-augmenting efficiency index has two components, an economy-wide component H_t and an industry-specific productivity index $\epsilon_{z,t} \in (0, 1]$. We assume for simplicity that for all z, there is the same function f, which is strictly increasing in both arguments, exhibiting constant-returns. Learning effect, which forms human capital H_t, is entirely an economy-wide phenomenon, completely external to firms so that perfect competition continues to prevail.

In this model, the central assumption to capture the variability market demand is that each commodity $z \in [0, 1]$ is 'stylish' in the following sense. There are two alternative styles. In every period only one style is demanded in each industry, and output is worthless unless it is in accordance with the style 'a la mode'. Nature may decide to have a style change at a given probability $\theta_z \in (0, 1]$. θ_z is the parameter for the frequency of demand shock. It captures the degree of demand variability for commodity z. The style is more variable with an increase in θ_z.

We suppose that the degree of demand variability is identical for all industries, i.e., $\theta_z = \theta \in (0, 1]$ for all $z \in [0, 1]$. Let $\Delta_{z,t}$ denote a random variable such that[7]

$$\Delta_{z,t} = \begin{cases} 1 & \text{if the style of commodity } z \text{ changes from period } t-1 \text{ to period } t \\ 0 & \text{otherwise.} \end{cases}$$

Then, $P[\Delta_{z,t} = 1] = \theta$ and $P[\Delta_{z,t} = 0] = 1 - \theta$. Since style change is costly, we assume that for any commodity $z \in [0, 1]$,

$$\epsilon_{z,t} = \begin{cases} 1 & \text{if } \Delta_{z,t} = 0 \\ 1 - \bar{\eta}_{z,t} \in (0, 1] & \text{if } \Delta_{z,t} = 1, \end{cases}$$

where $\bar{\eta}_{z,t} \in [0, 1)$ is the proportional loss in the industry-specific productivity due to style change.

3.2 Government Activities

The government always keeps a balanced budget. It has two policy tools: first, promotion support for learning industries as government support, and second, elaborate monitoring as bureaucratic control. For promotion support, the government imposes a tax on all industries equally to raise a revenue of amount $\tau > 0$ and grants equal support for sales promotion to eligible industries with an administrative cost $c \in (0, \tau)$.[8] Eligibility is given to all approved producers who have applied for the support. Due to asymmetric information between the government and applicants, some non-learning industries may win government approval in rent-seeking. To counter this, the government may install bureaucratic procedures for elaborate monitoring. For simplicity, we assume that the procedure is costless in resources except that the bureaucratic process is detrimental to the ability of firms' response to demand variability.

Let (S_t, M_t) denote the government action in period t consisting of the two binary choices of

[7] Assume that $\Delta_{z,t}$ is independent of z and t. The model is based on a special case of Markovian process. Although we can use a generalized Makovian process, the model would be much complicated.

[8] The parameter c can be considered as the cost of tax collection fee. This cost may be realistically a significant factor.

whether to support and whether to adopt elaborate monitoring, respectively, where[9]

$$S_t = \begin{cases} 1 & \text{if promotion support is adopted} \\ 0 & \text{otherwise,} \end{cases} \qquad M_t = \begin{cases} 1 & \text{elaborate monitoring is adopted} \\ 0 & \text{otherwise,} \end{cases}$$

and S_t and M_t are called the support action and the monitoring action, respectively.

3.2.1 Eligibility of Promotion Support

Elaborate monitoring ($M_t = 1$) enables the government to limit approval to firms only in learning industries. In the absence of such monitoring ($M_t = 0$), the government cannot exclude some part of rent-seeking firms in non-learning industries from sharing support. Let $(K, K(1+\lambda)]$ denote the set of non-learning industries that obtain government support under elaborate monitoring. The parameter $\lambda \in [0, 1/K - 1)$ captures the degree of rent-seeking activities in non-learning industries under elaborate monitoring. Promotion support for learning industries is more diluted into non-learning industries with an increase in the degree of rent-seeking activities λ. Then, given monitoring action M_t, the set of eligible industries $D(M_t) \subset [0, 1]$ is represented as

$$D(M_t) = [0, K + \lambda K(1 - M_t)].$$

3.2.2 Effect of Government Action on Industry-Specific Factor

The objective of promotion support is to increase labor productivity in eligible industries $D(M_t)$. To capture this, we suppose that promotion support leads to net monetary transfer over all industries and influences the resources available to various firms in the form of the supply of the

[9]For my objective to clarify endogenous policy transition, I assume that in each of the two policies the government can make only a binary choice.

industry-specific factor according to the following formula.[10]

$$a_{z,t} \equiv a_z(S_t, M_t) = \begin{cases} a_0 + S_t[\delta(M_t) - \tau] & \text{if } z \in D(M_t) \\ a_0 - S_t\tau & \text{if } z \in [0,1] \setminus D(M_t), \end{cases} \tag{3}$$

where $a_0 > 0$ is the industry-specific factor in each industry without promotion support, and τ is the tax imposed on each industry.[11] They are assumed to be identical for all industries. $\delta(M_t) > 0$ is the after-tax contribution to the industry-specific factor in each eligible industry by such a transfer under promotion support.

Because of a balanced budget of the government in every period, the total tax revenue minus the administrative cost of promotion support, $\tau - c$, must equal the budget size of promotion support. Since the industry-specific factor in eligible industry $z \in D(M_t)$ increases by $\delta(M_t)$ after the taxation, the budget size is $\delta(M_t)|D(M_t)|$, where $|D(M_t)| = K + \lambda K(1 - M_t)$. Then,

$$\delta(M_t) = \frac{\tau - c}{K} \quad \text{if } M_t = 1; \qquad \delta(M_t) = \frac{\tau - c}{K(1+\lambda)} \quad \text{if } M_t = 0. \tag{4}$$

Remark 1 The industry-specific factor in eligible industries increases by receiving promotion support, while that in non-eligible industries decreases by taxation. Moreover, there exists policy dilution due to asymmetric information without elaborate monitoring. Non-elaborate monitoring dilutes promotion support into part of non-learning industries that are undeserved for government assistance by spreading out the extent of eligible industries ($D(1) = [0, K] \subset [0, K(1 + \lambda)] = D(0)$). At the same time, it reduces the effect of promotion support on the industry-specific factor for all eligible industries ($\delta(1) = \frac{\tau-c}{K} > \frac{\tau-c}{K(1+\lambda)} = \delta(0)$). Even though the production in non-learning industries $z \in (K, K(1 + \lambda)]$ seems like the ones in learning industries, these

[10]Although an industrial policy is usually complicated and its form is thus difficult to describe briefly, its primary objective can be said to increase labor productivity so as to promote the targeted industry. This is commonly accomplished in part by such instruments as taxes, exchange rates, rationing and subsidies. Many countries have come to employ a wider range of instruments, including support programs for sector-specific R&D and the provision of information. For example, the Japanese government initiated the Very Large Scale Integrate (VLSI) Semiconductor Project to foster the semiconductor industry in front of the threat by IBM (Imai (1984)). Also, exhibitions such as electronic-show, which are partly supported by public sector, play an important role in providing industry-specific information.

[11]For simplicity, we assume that τ is optimally derived from some institutional reasons.

productions contribute nothing to human capital accumulation. Figure 2 shows the relationship between the government action (S_t, M_t) and the industry-specific factor $a_{z,t}$.

3.2.3 Effect of Government Action on Industry-Specific Productivity Index

Although elaborate monitoring prevents some firms from perverting promotion support, it does not come free in that too much red tape hurts production flexibility in response to a style change. Demand changes in eligible industries are more costly under elaborate monitoring than under non-elaborate monitoring.[12] To clarify this, we suppose that under elaborate monitoring, only eligible firms would apply for support. Only those who are applying for support would lose flexibility due to elaborate monitoring. The lack of flexibility matters only if there is a style change. Specifically, when style of commodity $z \in [0, K]$ in learning industries changes under elaborate monitoring ($\Delta_{z,t} = 1$ and $M_t = 1$), there is a loss in the industry-specific productivity index, $\bar{\eta}_{z,t} = \eta \in (0, 1)$. The effect of elaborate monitoring is identical for all learning industries. For given government action (S_t, M_t) and the realization of style change $\Delta_{z,t}$, the industry-specific productivity index is given as:[13]

$$\epsilon_{z,t} \equiv \epsilon_z(S_t, M_t, \Delta_{z,t}) = \begin{cases} 1 - \eta S_t M_t \Delta_{z,t} & \text{if } z \in [0, K] \\ 1 & \text{if } z \in (K, 1]. \end{cases}$$

Without promotion support ($S_t = 0$), monitoring is pointless so that there will be no red tape and no loss in the industry-specific productivity index, i.e., $\epsilon_{z,t} = 1$.

Remark 2 Since the probability of style change is identical at θ for all industries, the expected

[12] As an example, we may consider the situation in which, under the red tape, output cannot be delivered 'just in time', by regular procedure so that overtime pay must be paid to workers to fulfill the order.

[13] To focus on the case in which elaborate monitoring induces red tape, suppose that the industry-specific productivity index never changes at $\bar{\eta}_{z,t} = 0$ for any non-learning industry $z \in (K, 1]$ under elaborate monitoring ($M_t = 1$) even when its style changes. Without such monitoring ($M_t = 0$), the index never changes at $\bar{\eta}_{z,t} = 0$ for all industries $z \in [0, 1]$ irrespective of whether to have a style change.

industry-specific productivity index is given as:

$$E(\epsilon_{z,t}) = E[\epsilon_z(S_t, M_t, \Delta_{z,t})] = \begin{cases} 1 - \theta\eta S_t M_t & \text{if } z \in [0, K] \\ 1 & \text{if } z \in (K, 1]. \end{cases} \tag{5}$$

The value of $\theta\eta$ represents the expected negative effect in learning industries through red tape. Figure 3 shows the effect of government action on the expected productivity index $E(\epsilon_{z,t})$.

3.3 Government's Dynamic Problem

The government maximizes the expected sum of national income over time with discount rate $\beta \in (0, 1)$.[14] To analyze an optimal policy, we consider the following steps in each period t:

Step 1 Given human capital H_t, the government chooses its action $(S_t, M_t) \in \{0, 1\}^2$. The decision determines the industry-specific factor $a_{z,t} \equiv a_z(S_t, M_t)$ and the expected industry-specific productivity index $E(\epsilon_{z,t}) \equiv E(\epsilon_z(S_t, M_t, \Delta_{z,t}))$ in each industry z. The government has no information about the style of commodity $z \in [0, 1]$ in that period, except the probability θ of style change.

Step 2 The equilibrium labor employment $l^*_{z,t} \equiv l_z(H_t, S_t, M_t)$ in each industry z is determined in the perfectly competitive market.[15] This yields the total employment in learning industries

$$L^*_A(H_t, S_t, M_t) = \int_0^K l^*_z(H_t, S_t, M_t)dz, \tag{6}$$

and the next period's human capital $H_{t+1} = h(L^*_A(H_t, S_t, M_t), H_t)$.

Step 3 Nature decides whether or not there will be a style change $(\Delta_{z,t})$, which in turn determines the industry-specific productivity index $\epsilon_{z,t} \equiv \epsilon_z(S_t, M_t, \Delta_{z,t})$. Using the already de-

[14] Assume that the government is risk-neutral.

[15] Note that competitive firms must choose labor employment facing uncertainty related to a style change in the current period. The actual profit for firms depends on the realization about a style change. We assume that the industry-specific factor is owned by firms so that its rent is included in the profit. If the negative effect of a style change on the profit is small enough, each existing firm never goes bankruptcy because of its positive profit.

cided labor employment $l^*_{z,t}$ and the industry-specific factor $a_{z,t}$ with the output-augmenting efficiency index $\epsilon_{z,t}H_t$, each industry z produces output $x^*_{z,t} \equiv x^*_z(H_t, S_t, M_t, \Delta_{z,t}) = \epsilon_{z,t}H_t f(l^*_{z,t}, a_{z,t})$. Then current period's national income

$$g^*_t = \int_0^1 x^*_z(H_t, S_t, M_t, \Delta_{z,t})dz,$$

Step 4 The first stage in period $t+1$ starts.

The expected national income before the realization of style change is given as:

$$g^e_t \equiv g^e(H_t, S_t, M_t) = Eg^*_t = \int_0^1 E[x^*_z(H_t, S_t, M_t, \Delta_{z,t})]dz. \tag{7}$$

Then, the government's dynamic decision problem at the first stage of each period is to solve for:

$$\max_{\{(S_t, M_t)\in\{0,1\}^2\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \beta^t g^*_t\right] = \sum_{t=0}^{\infty} \beta^t g^e(H_t, S_t, M_t), \tag{8}$$

subject to $H_{t+1} = h(L^*_A(H_t, S_t, M_t), H_t)$ and $H_0 \in (0,1)$ as given. We denote by $v(H_0)$ the value function of this problem.

4 Analysis

This section first introduces some assumptions. After that, we derive a temporal market equilibrium, taking human capital H_t and government action (S_t, M_t) as given. Then, an optimal policy is sketched out in the framework of a dynamic programming. Finally, we discuss some economic implications that are indicated by our results.

4.1 Assumptions

Reallocation of the industry-specific factor over all industries may influence current national income. Thus, the government may have an incentive to reallocate by adopting promotion support for reasons different from dynamic learning effect. In order to focus on learning, we wish to neutralize the effect of the reallocation of the industry-specific factor on current national income.[16] Therefore, we assume that all industries share identical linear homogeneous production technology except the industry-specific productivity index.[17] To make the model tractable, the following assumption is made.

Assumption 1 The production technology for each commodity is described as the Cobb-Douglas function, i.e., $x_{z,t} = \epsilon_{z,t} H_t l_{z,t}^{\mu} a_{z,t}^{1-\mu}$ with $\mu \in (0,1)$.[18]

Remark 3 This assumption requires that promotion support with the administrative cost of promotion support must reduce current national income irrespective of the allocation, and the government has no incentive to reallocate the industry-specific factors without dynamic learning effect.

The next assumption is related to the state equation for human capital.

Assumption 2 $h(L_{A,t}, H_t) = 1 - [1 - \nu(L_{A,t})](1 - H_t)$ with $\nu(L) \in (0,1)$ and $\nu'(L) > 0$ for any $L \geq 0$.

The state equation (1) for H is rewritten as:

$$\frac{1 - H_{t+1}}{1 - H_t} = 1 - \nu(L_{A,t}).$$

The value of ν represents the speed of technological progress. The condition of $\nu' > 0$ requires that the total labor employment in learning industries contributes to the acceleration of human capital.

[16] The neutralization here means that current national income is independent of the allocation of the industry-specific factor.

[17] To see this, suppose that the industry-specific factor and the price of output are identical for all industries. We also suppose that only the government can reallocate the industry-specific factor a_z over two industries $z \in \{1, 2\}$ without cost. We furthermore suppose that the sum of the industry-specific factors in the two industries is fixed at $a = a_1 + a_2$, that the total labor input is fixed at $\bar{L} = l_1 + l_2$, and that the production technology for each industry is identical with $f(l_i, a_i) = l_i^{\mu} a_i^{1-\mu}$. Then, it is easy to find that national income is $\bar{L}^{\mu} a^{1-\mu}$, independent of the allocation of the industry-specific factors. Thus, Under the assumption of the linear homogeneous technology without dynamic learning effect in learning industries, if there is an administrative cost of the reallocation, the government has no incentive to reallocate the industry-specific factor.

[18] The Cobb-Douglas function $f(l, a) = l^{\mu} a^{1-\mu}$ is linear homogeneous. The linear homogeneous function is always homothetic.

Remark 4 Given H_0 and $\{L_{A,l}\}_{l=0}^{t-1}$, human capital in period t can be represented as

$$H_t = 1 - (1 - H_0)\prod_{s=0}^{t-1}[1 - \nu(L_{A,s})], \tag{9}$$

which implies that human capital is strictly increasing over time and its asymptotic limit is unity as t goes to infinity since $\nu(L_{A,s}) \in (0, 1)$.

4.2 Temporal Equilibrium

A temporal market equilibrium in period t is determined taking human capital H_t and government action (S_t, M_t) as given. When labor allocation is determined under perfect competition, firms have no information about the realization of a style change and act as expected profit maximizers so that the wage rate w_t equals the expected marginal value of product with respect to labor. By equation (2), the equilibrium outcome in any period t must satisfy:

$$E(\epsilon_{z,t})H_t f_l(l_{z,t}, a_{z,t}) = w_t, \tag{10}$$

for any $z \in [0, 1]$, with the labor market clearing condition:

$$\int_0^1 l_{z,t}dz = \bar{L}. \tag{11}$$

Government action (S_t, M_t) may affect the equilibrium by changing the industry-specific factor $a_{z,t}$ and the expected industry-specific productivity index $E(\epsilon_{z,t})$.

There are three possible actions to be taken place by the government in each period; [action I] laissez faire, [action II] promotion support without elaborate monitoring, and [action III] promotion support with elaborate monitoring.[19] Let $\mathcal{G} \equiv \{G^1, G^2, G^3\}$ denote the set of all possible government actions, where $G^1 \equiv (0, 0)$, $G^2 \equiv (1, 0)$ and $G^3 \equiv (1, 1)$ represent actions I, II, and III, respectively. The expected national income $g^e(H_t, S_t, M_t)$ and the total labor employment in learning industries $L_A^*(H_t, S_t, M_t)$ in the equilibrium are two important values

[19] The possibility of elaborate monitoring with promotion support can be excluded since there is no objective for monitoring without promotion support (any firm does not apply for support).

for policy makers to maximize the sum of the expected national income over time. The latter is related to future national income since it is the only source to accumulate human capital from the current period to the next through the state equation. By equations (3), (4), (5), (10) and (11), the following result in the equilibrium is obtained.

Lemma 1 *For each $G_t = (S_t, M_t) \in \mathcal{G}$, the total labor employment in learning industries and the expected national income are given as*

$$L^*_{A,t}(H_t, S_t, M_t) = \begin{cases} K\bar{L} & \textit{if } (S_t, M_t) = (0,0) \\ \frac{a_0 - \tau + (\tau - c)/[K(1+\lambda)]}{a_0 - c} K\bar{L} & \textit{if } (S_t, M_t) = (1,0) \\ \frac{(1-\theta\eta)^{1/(1-\mu)}(a_0 - \tau + (\tau - c)/K)}{B} K\bar{L} & \textit{if } (S_t, M_t) = (1,1), \end{cases} \quad (12)$$

and

$$g^e_t(H_t, S_t, M_t) = \begin{cases} H_t\bar{L}^\mu a_0^{1-\mu} & \textit{if } (S_t, M_t) = (0,0) \\ H_t\bar{L}^\mu (a_0 - c)^{1-\mu} & \textit{if } (S_t, M_t) = (1,0) \\ H_t\bar{L}^\mu B^{1-\mu} & \textit{if } (S_t, M_t) = (1,1), \end{cases} \quad (13)$$

respectively, where $B \equiv [K(1-\theta\eta)^{1/(1-\mu)} + 1 - K](a_0 - \tau) + K(1-\theta\eta)^{1/(1-\mu)}(\tau - c)$.

Notice that $L^*_{A,t}(H_t, 1, 0)$ depends on λ, and $L^*_{A,t}(H_t, 1, 1)$ and $g^e_t(H_t, 1, 1)$ depend on θ. By equations (12) and (13), we obtain the following result.

Corollary 1 A rise in rent-seeking activities reduce dynamic learning effect by altering labor allocation, i.e.,

$$\frac{\partial L^*_{A,t}}{\partial \lambda} = -\frac{(\tau - c)\bar{L}}{(a_0 - c)(1+\lambda)^2} < 0.$$ [20]

On the other hand, a rise in demand variability reduces dynamic learning effect as well as the expected national income, i.e.,

$$\frac{\partial L^*_{A,t}}{\partial \theta} = -\frac{(1-K)(a_0-\tau)(a_0-\tau+(\tau-c)/K)K\bar{L}}{A[(KA+1-K)(a_0-\tau)+A(\tau-c)]^2}\frac{\eta A^\mu}{1-\mu} < 0;$$
$$\frac{\partial g^e_t}{\partial \theta} = -\mu\eta H_t K(a_0-c)A^\mu\left[\frac{B}{\bar{L}}\right]^\mu < 0,$$

where $A = (1-\theta\eta)^{1/(1-\mu)}$.

Remark 5 By equations (12) and (13), for any government action $G_t \equiv (S_t, M_t) \in \mathcal{G}$, the equilibrium labor employment $L^*_{A,t}$ in learning industries is independent of human capital $H_t \in (0,1)$, and the equilibrium expected national income g^e_t is proportional to human capital H_t. These are due to the specification in which the production technology has the output-augmenting efficiency index, $\epsilon_{z,t}H_t$. Then, the equilibrium labor employment can be rewritten as $L^*_A(S_t, M_t) = L^*_A(G_t)$ with the element H_t dropped out, and also the expected national income can be rewritten as $g^e_t(H_t, S_t, M_t) = H_t R(S_t, M_t) = H_t R(G_t)$, where $R(\cdot)$ is the expected national income per unit of human capital and is independent of H_t.

4.3 Comparison among Actions I, II and III

This subsection examines the results of actions I, II and III (G^1, G^2 and G^3). Promotion support reallocates the industry-specific factors over all industries with the administrative cost c. This paper focuses on two issues: first, the *rent-seeking issue* in which rent-seekers in non-learning industries receive government assistance under non-elaborate monitoring, and second, the *flexibility issue* in which demand variability reduces flexibility under elaborate monitoring. The rent-seeking issue is more significant with larger degree of rent-seeking activities, λ, while the flexibility issue is more significant with larger demand variability, θ. By equations (12) and (13), the following results are obtained.

Lemma 2 *$R(G^1) > R(G^2) > R(G^3)$ and $L^*_A(G^1) < L^*_A(G^2)$.*

Remark 6 A laissez faire policy always attains the highest expected national income ($R(G^1) > \max\{R(G^2), R(G^3)\}$). This is because the economy incurs administrative cost $c > 0$ under promotion support. Therefore, promotion support can never be justified without dynamic learning effect. Moreover, under promotion support, elaborate monitoring always attains smaller expected national income than non-elaborate monitoring ($R(G^2) > R(G^3)$) since elaborate monitoring induces the loss in the expected industry-specific productivity index that reduces the expected national income.

Remark 7 Promotion support without elaborate monitoring causes learning industries to face a higher labor productivity and select a higher employment than a laissez faire policy, in spite that some rent-seekers in non-learning industries reduce the effectiveness of the support ($L_A^*(G^1) < L_A^*(G^2)$). On the other hand, promotion support with elaborate monitoring yields a higher industry-specific factor than without such monitoring. But at the same time it also reduces the expected industry-specific productivity index in learning industries. An increase in the industry-specific factor induces a higher labor productivity, but a decrease in the industry-specific productivity index induces a lower labor productivity. The industry-specific factor under non-elaborate monitoring depends on the degree of rent-seeking activity, λ, and the expected industry-specific productivity index under elaborate monitoring depends on demand variability, θ. Thus, the two parameters, λ and θ, are crucial to determine which monitoring scheme induces a higher labor productivity and a higher employment in learning industries.

The following result is related to the total labor employment under G^2 and G^3.

Lemma 3 *For any $\lambda \in (0, 1/K-1)$, there exists a unique value $\psi(\lambda) \in (0, 1]$ such that $L_A^*(G^2) < L_A^*(G^3)$ for any $\theta \in (0, \psi(\lambda))$ and $L_A^*(G^2) > L_A^*(G^3)$ for any $\theta \in (\psi(\lambda), 1]$. Furthermore, the critical value $\psi(\lambda)$ is increasing in λ if $\psi(\lambda) < 1$.*

Remark 8 Promotion support induces larger learning effect under elaborate monitoring if the rent-seeking issue is relatively significant, i.e., λ is relatively large compared to θ. In contrast, it induces larger learning effect under non-elaborate monitoring if the flexibility issue is relatively significant, i.e., θ is relatively large compared to λ.

Lemmas 2 and 3 show the trade-off relation in the sense that promotion support reduces the attainable welfare in the current period, but it may increase future welfare by speeding up human capital accumulation. Figure 4 illustrates the relationship between $L_A^*(G^2)$ and $L_A^*(G^3)$ for each pair of λ and θ. The critical value $\theta = \psi(\lambda)$ is represented by OLM. Any pair (λ, θ) in area OLMN yields $L_A^*(G^2) < L_A^*(G^3)$, while any pair in area OPL yields $L_A^*(G^2) > L_A^*(G^3)$.

4.4 Dynamic Decision Problem

This subsection examines government's dynamic decision problem (8), in which the government has a trinary choice in each period.[21] Promotion support may make an economy enjoy larger dynamic learning effect through an increase in labor employment in learning industries. Since learning effect is diminishing over time, so is the benefit of promotion support. However, at the same time, promotion support incurs an administrative cost. The nature of the cost does not depend on time and learning potentials. Thus, the balance between the need for learning effect and the desirability for laissez faire changes endogenously.

Elaborate monitoring also has its benefit and cost under promotion support. The benefit comes from the preclusion of rent-seeking activities. Since such activities reduce the progress of human capital accumulation, and since human capital approaches to its upper bound (diminishing learning potentials over time), the benefit diminishes over time. In contrast, the cost is from the loss in flexibility. The nature of the cost does not depend on time and learning potentials since the loss of flexibility reduces the productivity all the time. Thus, the balance between the need for the preclusion of rent-seeking activities and the desirability for flexibility changes endogenously.

The optimal government action in period t depends only on H_t, and $G(H)$ denotes the function mapping the state variable $H \in (0,1)$ into the optimal government action in $\mathcal{G}$.[22] There are two cases according to the result in Lemma 3. The first case, say case A, is the one with $\theta \in (\psi(\lambda), 1]$ corresponding to area OLP in Figure 4. The second case, say case B, is the one

[21]The state variable H is independent of the realization of style change and this problem has the Markov property. This property is due to the specification in which in each period the total labor inputs in learning industries is determined before Nature decides whether or not to change style, i.e., each competitive firm commits to use some labor inputs before the realization of style uncertainty, and hence next period's level of human capital is also independent of the realization (even though being dependent on probability θ of style change).

[22]The dynamic problem is autonomous since its dependence on time is merely through the discount term, which is assumed to be constant at $\beta \in (0,1)$.

with $\theta \in [0, \psi(\lambda))$ corresponding to area OLMN in Figure 4. On the (ν, R) plane in Figures 5, 6 and 7 (the upper part), points A, E and C represent $(\nu(L_A^*(G^1)), R(G^1))$, $(\nu(L_A^*(G^2)), R(G^2))$ and $(\nu(L_A^*(G^3)), R(G^3))$, respectively. Lemma 3 requires that point C is within area OBEF if $\theta \in (\psi(\lambda), 1]$, while point C is within area EFHG if $\theta \in [0, \psi(\lambda))$.

We further divide case B into two subcases according to the following way. On the (ν, R) plane in Figures 6 and 7 (the upper part), we divide area EFHG into two subareas according to the line that passes through both points A and E. The first is subcase B-I in which point C is within area EIG above line AI, and the second is subcase B-II in which point C is within area EFHI below line AI. Figures 6 and 7 illustrates subcases B-I and B-II, respectively. Then, regarding the relationship between each of the two subcases and the values of (θ, λ), the following preliminary result is obtained.

Lemma 4 *Consider line* AI *in Figures 5, 6 and 7. For any* $\lambda \in (0, 1/K - 1)$*, there exists a unique value* $\varphi(\lambda) \in (0, \psi(\lambda)]$ *such that* $(\nu(L_A^*(G^3)), R(G^3))$ *is above line* AI *for any* $\theta \in (0, \varphi(\lambda))$*, and* $(\nu(L_A^*(G^3)), R(G^3))$ *is below line* AI *for any* $\theta \in (\varphi(\lambda), \psi(\lambda))$*. And also, the critical value* $\varphi(\lambda)$ *is increasing in* λ*.*

Remark 9 Using Lemmas 3 and 4, for any $\lambda \in (0, 1/K-1)$, the range $(0, 1]$ of demand variability θ is divided into three subranges; $(0, \varphi(\lambda))$, $(\varphi(\lambda), \psi(\lambda))$ and $(\psi(\lambda), 1]$, which correspond to subcase B-I, subcase B-II and case A, respectively. Case A corresponds to the situation in which the flexibility issue is more significant. Subcase B-I corresponds to the situation in which the rent-seeking issue is relatively significant compared to the flexibility issue. Subcase B-II shows the intermediate situation between case A and subcase B-I (see Figure 4).[23]

[23] Figures 4, 5, 6 and 7 illustrate these three categories for the better understanding. Dotted curve OT in Figure 4 represents the function $\theta = \varphi(\lambda)$ that is monotonically increasing in λ and divides area B into area B-I and area B-II. Curve OLM represents the function $\theta = \psi(\lambda)$ that is also monotonically increasing in λ and divides the whole area into area B and area A. Figure 5, as explained before, shows case A that corresponds area A in Figure 4. In contrast, Figure 6 shows subcase B-I that corresponds to area B-I in Figure 4, and the result of action III (point C) is inside area EIG. Finally, Figure 7 indicates subcase B-II that corresponds to area B-II in Figure 4, and the result of action III (point C) is inside EFHI.

4.4.1 Main Results

We now show the main results in each of the three cases, case A, subcases B-I and B-II. Let $m_i = [1 - \beta(1 - \nu(L_A^*(G^i)))]/[1 - \beta(1 - \nu(L_A^*(G^1)))] > 1$ and $r_i = R(G^1)/R(G^i) > 1$ for each $i \in \{2, 3\}$.

Case A : The Flexibility Issue is Relatively Significant

Proposition 1 *Suppose that the flexibility issue is relatively significant such that $\theta \in (\psi(\lambda), 1]$.* (1) *The adoption of promotion support without elaborate monitoring is optimal for $H \in (0, \hat{H})$, and a laissez faire policy is optimal for $H \in (\hat{H}, 1)$, where*

$$\hat{H} \equiv \hat{H}(\lambda) = 1 - \frac{r_2 - 1}{r_2 m_2 - 1} \in (0, 1). \tag{14}$$

(2) *The critical value $\hat{H}$ is decreasing in λ.*

Remark 10 (Policy Liberalization) Since human capital converges to its upper-bound, the effect of promotion support on learning effect is diminishing over time.[24] This implies that the merit of promotion support is diminishing over time with the nature of its cost unchanged. Thus, promotion support becomes less attractive over time. If the initial level of human capital is less than $\hat{H}$, then the government should adopt the policy reform from the promotion support regime to the laissez faire regime. That is, the optimal policy calls for endogenous liberalization in some future period (see Figure 5 (the lower part)).

Remark 11 (Bureaucracy) Elaborate monitoring has large negative impact on the industry-specific productivity index. Since the flexibility issue is relatively more significant compared to the rent-seeking issue, the cost of elaborate monitoring is relatively high compared to its benefit. Thus, elaborate monitoring cannot be justified.

[24] Proposition 1 partially comes from the result that our original dynamic problem can be reducible to an optimal stopping problem in which the government decides the timing of changing from action II to action I if the initial level of human capital is small enough so that action II is optimal in the initial period. For the discussion of an optimal stopping problem, see Dixit and Pindyck (1994).

Remark 12 (Comparative Statics) A rise in the degree of rent-seeking activities makes promotion support less effective. In this case, the reform to a laissez faire policy should be implemented at an earlier stage of human capital accumulation.

Subcase B-I : The Rent-Seeking Issue is Relatively Significant

Proposition 2 *Suppose that the rent-seeking issue is relatively significant such that $\theta \in (0, \varphi(\lambda))$.*
(1) *The adoption of promotion support with elaborate monitoring is optimal for $H \in (0, \tilde{H})$, and a laissez faire policy is optimal for $H \in (\tilde{H}, 1)$, where*

$$\tilde{H} = \tilde{H}(\theta) = 1 - \frac{r_3 - 1}{r_3 m_3 - 1} \in (0, 1). \tag{15}$$

(2) *The critical value $\tilde{H}$ is decreasing in θ.*

Remark 13 (Policy Liberalization) As in case A, the result is also due to the property of diminishing learning effect. If the initial level of human capital is less than $\tilde{H}$, then the government should adopt the policy reform from the promotion support regime to the laissez faire regime in some future period, i.e., the optimal policy calls for endogenous policy liberalization (see Figure 6 (the lower part)).

Remark 14 (Bureaucracy) In contrast to case A, elaborate monitoring should be accompanied with promotion support in the early stage. Although elaborate monitoring reduces flexibility, it makes promotion support significantly more effective by avoiding serious rent-seeking. The benefit of elaborate monitoring is relatively high compared to its cost.

Remark 15 (Comparative Statics) The policy reform to a laissez faire policy should be implemented at an earlier stage of human capital accumulation as demand variability becomes larger. The increase in demand variability θ reduces the speed of human capital accumulation as well as the expected national income, making a laissez faire policy become more attractive.

Subcase B-II : Intermediate Case Subcase B-II corresponds to the case in which (θ, λ) is in the intermediate case between subcase B-I and case A. Then the following preliminary result is obtained.

Lemma 5 *Suppose that* $\theta \in (\varphi(\lambda), \psi(\lambda))$. *Let* $\hat{H} = 1 - (r_2 - 1)/(r_2 m_2 - 1)$. *Then,*

(1) $G(H) = G^1 = (0, 0)$ *for any* $H \in (\hat{H}, 1)$;

(2) *there exists some* $\varepsilon \in (0, \hat{H})$ *such that* $G(H) = G^2 = (1, 0)$ *for any* $H \in (\hat{H} - \varepsilon, \hat{H})$;

(3) *there exists some* $\xi \in (0, \hat{H})$ *such that* $G(H) = G^3 = (1, 1)$ *for any* $H \in (0, \xi)$.

Remark 16 The first part of this result states that a laissez faire policy is optimal at a later stage of human capital accumulation as in the previous cases. The second says that promotion support without elaborate monitoring is optimal if human capital is less than, but close enough to $\hat{H}$. The third means that if human capital is small enough, promotion support with elaborate monitoring is optimal.

Notice that in subcase B-II, the complete description of the optimal government action over the domain of the state variable H may be complex. The assumption of $\theta \in (\varphi(\lambda), \psi(\lambda))$ is insufficient to guarantee that there exist two critical values H_1 and H_2 such that $G(H) = G^3$ for $H \in (0, H_1)$, $G(H) = G^2$ for $H \in (H_1, H_2)$, and $G(H) = G^1$ for $H \in (H_2, 1)$.[25] To understand

[25] There may be a case in which, focusing on the range $(a, 1) \subset (0, 1)$ of H, action II is optimal for any H in its lower range (a, a_1), action III is optimal for any H in its middle range (a_1, a_2), action II is *again* optimal for any H in its higher range (a_2, a_3), and action I is finally optimal for any H in its highest range $(a_3, 1)$. To understand this, consider a numerical example, where $(R_1, R_2, R_3) = (1.00, 0.81, 0.80)$ and $(\nu_1, \nu_2, \nu_3) = (0.10, 0.39, 0.40)$ with $\beta = 0.95$, where $R_i \equiv R(G^i)$ and $\nu_i \equiv \nu(L_A^*(G^i))$ for $i \in \{1, 2, 3\}$. Let us restrict ourselves into the range $H \in (0.840, 1.000)$. Then we can derive the optimal action: $G(H) = G^2$ for $H \in (0.840, 0.850)$, $G(H) = G^3$ for $H \in (0.850, 0.868)$, $G(H) = G^2$ for $H \in (0.868, 0.909)$, and $G(H) = G^1$ for $H \in (0.909, 1.000)$. In fact, the optimal action may not change monotonically, as human capital H increases. It changes from G^2 to G^3 at $H_1 = 0.850$, from G^3 to G^2 again at $H_2 = 0.868$, and from G^2 to G^1 at $H_3 = 0.909$. This implies that there are three critical values of H, at each of which the policy change should be considered, and that the range in which G^2 is optimal is divided into two regions, $(0.840, 0.850)$ and $(0.868, 0.909)$.

However, it does not mean that the optimal policy (sequence of actions) is $G^2 \to G^3 \to G^2 \to G^1$. Define by $\mathbf{G}(H)$ the function mapping the state variable $H \in (0, 1)$ into the optimal sequence of government actions from that period. The function $G : (0, 1) \to \mathcal{G} \equiv \{G^1, G^2, G^3\}$ is a function mapping the state variable H into the optimal government action at that period, while the function $\mathbf{G} : (0, 1) \to \mathcal{G}^{\chi_0}$ mapping the state variable $H \in (0, 1)$ into the optimal infinite sequence of government actions from that period, where $\mathcal{G}^{\chi_0}$ is the policy space consisting of countably infinite sequence of elements in $\mathcal{G}$. In this example, the optimal policy sequence is $\mathbf{G}(H) = \{2^{(2)}, 1^{(\infty)}\}$ for any $H \in (0.840, 0.850)$, $\mathbf{G}(H) = \{3^{(1)}, 1^{(\infty)}\}$ for any $H \in (0.850, 0.868)$, $\mathbf{G}(H) = \{2^{(1)}, 1^{(\infty)}\}$ for any $H \in (0.868, 0.909)$, and $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (0.909, 1.000)$, where $i^{(l)}$ is the finite sequence $\{G^i, \cdots, G^i\}$ with l elements, and $1^{(\infty)}$ is the infinite sequence $\{G^1, G^1, \cdots\}$. $\{3^{(m)}, 2^{(n)}, 1^{(\infty)}\}$ means that the government adopts G^3 m times (periods), G^2 n times (periods) and then G^1 forever. This implies that if the initial human capital is between 0.840 and 0.850, then the path of human capital under the optimal action skips over the range $(0.850, 0.868)$, where G^3 is optimal.

subcase B-II more, we consider an optimal sequence of government actions. Let

$$H' = 1 - \frac{(1-\beta)[R(G^2) - R(G^3)]}{[1-\beta(1-\nu(L_A^*(G^3))]R(G^2) - [1-\beta(1-\nu(L_A^*(G^2))]R(G^3)}.$$

Then, the following preliminary result is obtained.

Lemma 6 *Suppose that $\theta \in (\varphi(\lambda), \psi(\lambda))$ and $\bar{H} < H'$. For any $H \in (0,1)$, neither G^2 nor G^3 follows G^1, and G^3 never follows G^2 in the optimal sequence of government actions.*

Remark 17 Once a laissez faire policy is optimal in some period, promotion support cannot be optimal in any future period irrespective of monitoring action. Moreover, once promotion support without elaborate monitoring is optimal, elaborate monitoring cannot be justified in any future period.

By Lemmas 5 and 6, the following main result in subcase B-II is obtained.

Proposition 3 *Suppose that $\theta \in (\varphi(\lambda), \psi(\lambda))$ and $\bar{H} < H'$. Suppose also that the adoption of promotion support with elaborate monitoring is optimal for a given initial human capital $H \in (0,1)$. Then, at most two policy reforms should be adopted in some future periods, i.e., $G^3 \to G^1$ or $G^3 \to G^2 \to G^1$, where $G^i \to G^j$ represents the policy reform from G^i to G^j.*

Remark 18 (Two Policy Reforms) In contrast to the previous cases, case A and subcase B-I, where one endogenous policy reform should be implemented, the important point here is that under some conditions, there may be two endogenous policy reforms, $G^3 \to G^2 \to G^1$. Figure 7 (the lower part) illustrates a case in which the policy reform should be implemented in periods T^* and T^{**}.

Remark 19 The balance between the benefit and the cost of monitoring changes endogenously over time. In this case, the benefit dominates the cost in the early stage when learning potentials are large, while the cost dominates the benefit in the later stage when learning potentials get small.[26] That is, the rent-seeking issue is significant in the early stage, while the flexibility issue is

[26]Case A corresponds to a situation in which the cost always dominates the benefit until the period when promotion support cannot be justified. Conversely, subcase B-I corresponds to a situation in which the benefit always dominates the cost until the period when promotion support cannot be justified.

significant in the later stage. Noticing that ultimately promotion support cannot be justified, the policy makers should adopt two endogenous policy reforms; the first is the administrative reform from elaborate monitoring to non-elaborate monitoring with learning industries continuously supported, and the second is the reform to a laissez faire policy.

Remark 20 (Numerical Example)

Consider an example in which $\bar{L} = 1$, $a_0 = 1$, $K = 0.2$, $\bar{\delta} = 0.25$, $\eta = 0.1$, $\mu = 0.5$, $\tau = 0.1$, $\lambda = 0.3$, $\theta = 0.25$ and $\nu(L) = L$. The expected national income per unit of human capital and the speed of human capital accumulation under government actions, G^1, G^2 and G^3, are represented as $(R(G^1), \nu(L_A^*(G^1)) = (1, 0.2)$, $(R(G^2), \nu(L_A^*(G^2)) = (0.9747, 0.23)$, and $(R(G^3), \nu(L_A^*(G^3)) = (0.9688, 0.2329)$, respectively. It is easy to show that this is the case of $\theta \in (\varphi(\lambda), \psi(\lambda))$, i.e., subcase B-II. Suppose that the initial human capital is $H_0 = 0.1$. Using Lemmas 5 and 6, the optimal sequence of government actions is

$$\{\underbrace{G^3, G^3, G^3, G^3}_{4 \text{ periods}}, \underbrace{G^2, G^2}_{2 \text{ periods}}, \underbrace{G^1, G^1, \ldots}_{\text{infinitely many periods}}\},$$

which implies that there are two endogenous policy reforms.

Finally we deduce the following result that is satisfied in all cases, case A, subcase B-I and subcase B-II.

Corollary 2 *Let* $\check{H} \equiv \max\{\hat{H}, \tilde{H}\}$, *where* $\hat{H} = 1 - (r_2 - 1)/(r_2 m_2 - 1)$ *and* $\tilde{H} = 1 - (r_3 - 1)/(r_3 m_3 - 1)$. *Then, initially the adoption of promotion support is optimal for any* $H \in (0, \check{H})$, *and ultimately the laissez faire regime is optimal for any* $H \in (\check{H}, 1)$.

4.5 Discussions

4.5.1 Policy Reform from Industrial Policies to Laissez Faire

In the development process of many countries (especially those in Asia), there is a continuous sequence of policy reforms, leading from the implementation of industrial policies to the gradual adoption of laissez faire. At the same time, there have been on-going debates about the effectiveness of industrial policies. Taking Japan for example, Miwa (2004) states that targeted industrial policies constitute unfair government intervention and play little role in development. In their view, an industrial policy represents past errors and liberalizing reform is its subsequent correction. In contrast, researchers like Amsden (2001) regards the protection of infant industries as necessary. Others like Itoh, Kiyono, Okuno-Fujiwara, and Suzumura (1991) maintain that although past industrial policies may not be error-free, such policies as the adoption of temporary protection to launch an industry can be helpful, at least in theory.

In order to evaluate the effectiveness of industrial policies, an investigation of economic history is necessary. This study is not an investigation of history, but a theoretic inquiry. From the theoretical point of view, our results agree with Itoh, Kiyono, Okuno-Fujiwara, and Suzumura (1991) in that temporary industrial policies may be valuable. But more than that, the development process, viewed as an episode of 'catching-up' in 'late-industrialization,' calls for an endogenous sequence of policy reforms, with justified government participation in early stages and continued liberalization toward laissez faire. Therefore, it is not a case of present wisdom correcting past errors. Balancing the benefit and the cost of industrial policies, reforms also have their optimal timing.

4.5.2 Cost and Benefit of Bureaucracy

At a more specific level, it is well recognized that industrial policies may nurture knowledge capital and overcome coordination failures. But there is inherent cost due to the distortion that industrial policies induce. What has not been examined analytically in the past literature is that these bureaucratic practices accompanying industrial policies would incur the cost of losing production flexibility as a result. This is studied in our inquiry. In cases, A, B-I and B-II, we have highlighted the balance of cost and benefit related to industrial policies and bureaucratic procedures.

There has been a lot of studies that emphasize the significance of production flexibility for achieving success across a wide rage of industries, especially quality- and fashion-sensitive industries (see, e.g., Deyo and Doner (2001), Appelbaum and Smith (2001), and Dicken (1992)).

In particular, globalization may intensify the sensitivity since international markets are characterized by ruthless competition and ongoing innovation. In such a situation, the flexibility issue is significant. The cost of monitoring may be large so that elaborate monitoring cannot be justified, as in case A. This finding has important policy implication since practicing elaborate monitoring could be problematical in the case that the market of a targeted industry is highly variable (e.g., machine tool industry and electronic industry). It may be considered that the lack of competitiveness of Indian exports in pre-reform India can be attributed to strict regulatory controls under frequent market shifts in the international market.

On the other hand, the rent-seeking issue may depend partially on state governance. If state governance is not mature, as in some developing countries, it is much more difficult to prevent undeserved firms from obtaining government assistance. Since the assistance is not uniform for all industries under a targeted industrial policy, there is doubt about the appropriate classification for a good. In this situation, it may be easy for firms in non-targeted industries to make it appear to fit into a targeted category. Thus, the rent-seeking issue is significant, and the benefit of elaborate monitoring is large. Although seemingly inefficient, complex bureaucratic procedures might be necessary to make industrial policies effective, as in subcase B-I.

The model also implied that administrative reform is not necessarily a correction of past errors, but may be an optimal action of reflecting to the change in circumstance. The desirability for flexibility becomes more crucial compared to the need for the preclusion of rent-seeking activities over time. In subcase B-II, which is the case between the previous two cases, the policy makers seek to reduce in complexity of bureaucratic procedure before an ultimate transition to laissez faire. This may be consistent with the fact that in many countries, administrative reforms to simplify bureaucratic procedures have become the slogans of the day even though industry targeting still holds.

5 Conclusion

Targeted industrial policies are usually accompanied with some bureaucratic controls, which often bring about bureaucratic red tape. This paper focused on bureaucratic monitoring as the controls to prevent sub-optimal rent-seeking activities. A key problem of bureaucratic monitoring is that it reduces production flexibility in a globalized world with variable market demands. We developed a dynamic model that explicitly includes the negative effect of monitoring on flexibility and addressed policy transition to laissez faire.

In this paper the policy in each period consists of two components; (1) government support, which promotes dynamic learning, and (2) a bureaucratic control, which constitutes monitoring. The analysis calls for endogenous liberalization in the sense that a targeted industrial policy should be implemented at an earlier stage of human capital accumulation, and then a laissez faire policy should be adopted at a later stage. The result might be consistent with the empirical evidence in Asian countries.

It was also shown that whether or not to couple elaborate monitoring with a targeted industrial policy at an earlier stage depends on the degrees of rent-seeking activities and demand variability. When demand variability is relatively large, elaborate monitoring cannot be justified contrary to the traditional protectionist's stance. In contrast, when rent-seeking activities prevail for some reasons, elaborate monitoring may induce higher welfare over time. The paper also found it plausible that the government should adopt two endogenous policy transitions: the first is the administrative reform from elaborate monitoring into a simplified one, with learning industries continuously supported, and the second is the reform to a laissez faire policy.

This paper tried to shed light into the motivation of a government to adopt bureaucratic monitoring in the presence of industrial targeting. Hopefully this will help to interpret economic history as well as to design government policies. As a final remark, we have not studied the role of innovation in the development process. This problem seems important since the aim of some industrial policies may be to promote innovation by domestic firms. Although there are some similarities between innovative process and learning process, it is important to understand the difference: innovation of technologies is not the by-product of production experience. That topic should be explained separately in the future.

6 Appendix

In this Appendix, we will first explain the optimal control problem with a trinary choice, and show an important result which we use in the proof of results in Section 4. After that, we will show the proofs of all Lemmas and Propositions in Section 4.

6.1 Optimal Control Problem with Trinary Choice

6.1.1 Setup

Consider the situation in which there are three distinct actions, one of which must be chosen in each period. Let $\mathcal{G} \equiv \{G^1, G^2, G^3\}$ denote the set of all feasible actions in each period, and let $\mathcal{G}^{\chi_0} \equiv \{G^1, G^2, G^3\}^{\chi_0}$ denote the set of all sequences whose element is in $\mathcal{G}$. Let $H_t \in (0,1)$ denote the state variable in period t. The reward in period t, which depends on H_t and $G_t \in \mathcal{G}$, is represented as reward function $H_t R(G_t)$. The state variable in period $t+1$ depends on H_t and variable $\nu(L_A^*(G_t)) \in (0,1)$, which is determined by G_t, and the state equation for the state variable is described by

$$H_{t+1} = 1 - (1 - H_t)[1 - \nu(L_A^*(G_t))].$$

The discrete-time dynamic optimization problem with discount rate $\beta \in (0,1)$ is given by:

$$\max_{\{G_t \in \mathcal{G}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t H_t R(G_t),$$

subject to the above state equation and H_0 as given. For simplicity, for $i \in \{1,2,3\}$, let

$$R_i \equiv R(G^i), \quad \nu_i \equiv \nu(L_A^*(G^i)) \quad \text{and} \quad M_i \equiv 1 - \nu_i,$$

and we assume that $R_1 > R_2 > R_3$ and $1 > M_1 > M_2 > M_3$ as in case B in Section 4. We denote by $v(H)$ the value function of this problem.

6.1.2 Some Preliminaries

Let $V(H, g)$ denote the value when action sequence $g \in \mathcal{G}^{\chi_0}$ is chosen with $H \in (0,1)$ as given. We denote by $\{i^{(k)}\}$ the finite sequence $\{G^i, \cdots, G^i\}$, the number elements is k. Then, we have that, for $m \in \{1, 2, \cdots\}$ and for $i \neq j \in \{2, 3\}$,

$$\begin{aligned}
&V(H;\{1^{(\infty)}\}) = V(H;\{2^{(0)},1^{(\infty)}\}) = V(H;\{3^{(0)},1^{(\infty)}\}) = \frac{R_1}{1-\beta} - (1-H)\frac{R_1}{1-\beta M_1};\\
&V(H;\{i^{(m)},1^{(\infty)}\}) = \left[R_i\frac{1-\beta^m}{1-\beta} + R_1\frac{\beta^m}{1-\beta}\right] - (1-H)\left[R_i\frac{1-(\beta M_i)^m}{1-\beta M_i} + R_1\frac{(\beta M_i)^m}{1-\beta M_1}\right];\\
&V(H;\{1^{(1)},i^{(m)},1^{(\infty)}\}) = R_1 + \frac{\beta}{1-\beta}[R_i + \beta^m(R_1 - R_i)]\\
&\quad - (1-H)\left[R_1 + \beta M_1\left(\frac{R_i}{1-\beta M_i} + (\beta M_i)^m\left[\frac{R_1}{1-\beta M_1} - \frac{R_i}{1-\beta M_i}\right]\right)\right];
\end{aligned}$$

$$
\begin{aligned}
V(H;\{i^{(1)},j^{(m)},1^{(\infty)}\}) = R_i + &\frac{\beta}{1-\beta}[R_j + \beta^m (R_1 - R_j)] \\
-(1-H)\left[R_i + \beta M_i\left(\frac{R_j}{1-\beta M_j} + (\beta M_j)^m\left[\frac{R_1}{1-\beta M_1} - \frac{R_j}{1-\beta M_j}\right]\right)\right].
\end{aligned}
$$

We first deduce the following preliminary results regarding the function V:

Claim 1 *For any $m \in \{0, 1, \cdots\}$ and any $i \in \{2, 3\}$,*

$$
\frac{\partial V(H;\{i^{(m+1)},1^{(\infty)}\})}{\partial H} < \frac{\partial V(H;\{i^{(m)},1^{(\infty)}\})}{\partial H}.
$$

Proof of Claim 1 Since $M_1 > M_i$ for $i \in \{2, 3\}$, it must hold that, for $m = 0$,

$$
\begin{aligned}
\frac{\partial V(H;\{1^{(\infty)}\})}{\partial H} - \frac{\partial V(H;\{i^{(1)},1^{(\infty)}\})}{\partial H} &= \frac{R_1}{1-\beta M_1} - \left[R_1 \frac{\beta M_i}{1-\beta M_1} + R_i\right] \\
&= \frac{1-\beta M_i}{1-\beta M_1} R_1 - R_i > R_1 - R_i > 0.
\end{aligned}
$$

For $m \in \{1, 2, \ldots\}$, it must hold that

$$
\frac{\partial V(H;\{i^{(m)},1^{(\infty)}\})}{\partial H} - \frac{\partial V(H;\{i^{(m+1)},1^{(\infty)}\})}{\partial H} = (\beta M_i)^m (1-\beta M_i)\left[\frac{R_1}{1-\beta M_1} - \frac{R_i}{1-\beta M_i}\right] > 0.
$$

Thus, for any $m \in \{0, 1, 2, \ldots\}$ and any $i \in \{2, 3\}$, Claim 1 holds. ■

Claim 2 *For any $m \in \{0, 1, \cdots\}$ and any $i \in \{2, 3\}$,*

$$
\frac{\partial V(H;\{3^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H} < \frac{\partial V(H;\{2^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H} < \frac{\partial V(H;\{1^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H}.
$$

Proof of Claim 2 For $j \in \{2, 3\}$,

$$
\begin{aligned}
&\frac{\partial V(H;\{(j-1)^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H} - \frac{\partial V(H;\{j^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H} \\
=&(R_{j-1} - R_j) + \beta(M_{j-1} - M_j)\left[R_1 \frac{(\beta M_i)^m}{1-\beta M_1} + R_i \frac{1-(\beta M_i)^m}{1-\beta M_i}\right] > 0,
\end{aligned}
$$

which yields the desired result. ■

Claim 3 *For any $m \in \{0, 1, \cdots\}$ and any $i \in \{2, 3\}$,*

$$
\frac{\partial V(H;\{i^{(m)},1^{(\infty)}\})}{\partial H} < \frac{\partial V(H;\{1^{(1)},i^{(m)},1^{(\infty)}\})}{\partial H} \leq \frac{\partial V(H;\{1^{(\infty)}\})}{\partial H},
$$

with equality if $m = 0$.

Proof of Claim 3 For any $m \in \{0, 1, \cdots\}$ and any $i \in \{2, 3\}$, it must hold that

$$\begin{aligned}
&\frac{\partial V(H; \{1^{(1)}, i^{(m)}, 1^{(\infty)}\})}{\partial H} - \frac{\partial V(H; \{i^{(m)}, 1^{(\infty)}\})}{\partial H} \\
=&\left(R_1 + \beta M_1 \left[R_1 \frac{(\beta M_i)^m}{1 - \beta M_1} + R_i \frac{1 - (\beta M_i)^m}{1 - \beta M_i} \right] \right) - \left(R_1 \frac{(\beta M_i)^m}{1 - \beta M_1} + R_i \frac{1 - (\beta M_i)^m}{1 - \beta M_i} \right) \\
=&(1 - \beta M_1)[1 - (\beta M_i)^m] \left(\frac{R_1}{1 - \beta M_1} - \frac{R_i}{1 - \beta M_i} \right) > 0,
\end{aligned}$$

which yields the desired result. ■

Let $\mathbf{G} : (0, 1) \to \mathcal{G}^{\chi_0}$ denote the policy function mapping the state space $(0, 1)$ into the action sequence space $\mathcal{G}^{\chi_0}$, which consists of countably infinite sequence. We apply the concept of the backwards iteration. For any $m \in \{0, 1, \cdots\}$ and any $i \in \{2, 3\}$, let $H_i^{m,m+1}$ such that

$$V(H_i^{m,m+1}; \{i^{(m)}, 1^{(\infty)}\}) = V(H_i^{m,m+1}; \{i^{(m+1)}, 1^{(\infty)}\}).$$

Then we deduce the following result:

Claim 4 *Suppose that $V(H_i^{m,m+1}; \{i^{(m)}, 1^{(\infty)}\}) = V(H_i^{m,m+1}; \{i^{(m+1)}, 1^{(\infty)}\})$. Then, it must hold that*

$$H_i^{m,m+1} = 1 - \frac{\frac{R_1}{R_i} - 1}{M_i^m \left[\frac{1 - \beta M_i}{1 - \beta M_1} \frac{R_1}{R_i} - 1 \right]} \Leftrightarrow 1 - H_i^{m,m+1} = \frac{R_1 - R_i}{M_i^m (1 - \beta M_i)} \left[\frac{R_1}{1 - \beta M_1} - \frac{R_i}{1 - \beta M_i} \right]^{-1}.$$

Proof of Claim 4 We can derive this result directly using the function V. ■

6.1.3 Proposition and Proof

In this section we present the main proposition, and then show their proof by applying the backwards iteration. We first show the preliminary result in the case in which the state variable H_t is close enough to its asymptotic limit.

Lemma 7 *There exists some value $\bar{H} \in (0, 1)$, close enough to unity, such that $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (\bar{H}, 1)$*

Proof of Lemma 7 Suppose, to get a contradiction, that $\{1^{(\infty)}\}$ is not an optimal policy for any $H \in (0, 1)$. Pick any $H_{\bar{t}} \in (0, 1)$ as the state variable in period $\bar{t}$ such that

$$H_{\bar{t}} = 1 - \epsilon, \quad \text{where} \quad 0 < \epsilon < \frac{R_1 - R_2}{R_1}(1 - \beta),$$

and the optimal action in period $\bar{t}$ is not G^1. Without loss of generality, we assume that $G(H_{\bar{t}}) = G^2$. Then, it must hold that $H_s \in (1 - \epsilon, 1)$ for all $s > \bar{t}$, which implies that

$$v(H_{\bar{t}}) = R_2 + \max_{\{G_s \in \mathcal{G}\}_{s=\bar{t}+1}^{\infty}} \sum_{s=\bar{t}+1}^{\infty} \beta^{s-\bar{t}} H_s R(G_s) < R_2 + \max_{\{G_s \in \mathcal{G}\}_{s=\bar{t}+1}^{\infty}} \sum_{s=\bar{t}+1}^{\infty} \beta^{s-\bar{t}} R(G_s) = R_2 + \frac{\beta}{1 - \beta} R_1,$$

since $R_1 > R_2 > R_3$ and $H_s \in (0,1)$. Also, since $H_{\bar{t}} \in [1-\epsilon, 1)$, $v(H_{\bar{t}})$ must satisfy

$$v(H_{\bar{t}}) = \max_{\{G_s \in \mathcal{G}\}_{s=\bar{t}}^{\infty}} \sum_{s=\bar{t}}^{\infty} \beta^{s-\bar{t}} H_s R(G_s) \geq (1-\epsilon) \max_{\{G_s \in \mathcal{G}\}_{s=\bar{t}}^{\infty}} \sum_{s=\bar{t}}^{\infty} \beta^{s-\bar{t}} R(G_s) = (1-\epsilon)\frac{R_1}{1-\beta}.$$

But, since $0 < \epsilon < (R_1 - R_2)(1-\beta)/R_1$ by the assumption, we have that

$$(1-\epsilon)\frac{R_1}{1-\beta} > \left[1 - \frac{(R_1 - R_2)(1-\beta)}{R_1}\right]\frac{R_1}{1-\beta} = R_2 + \frac{\beta}{1-\beta}R_1,$$

which contradicts to the fact that

$$(1-\epsilon)\frac{R_1}{1-\beta} \leq v(H_{\bar{t}}) < R_2 + \frac{\beta}{1-\beta}R_1.$$

Thus, for any H close enough to unity, G^1 is optimal, i.e., $\mathbf{G}(H) = \{1^{(\infty)}\}$. ■

By Lemma 7 and Claims 2 and 4, we can deduce the following result:

Lemma 8 $\mathbf{G}(H) = \{1^{(\infty)}\}$ *for any* $H \in (H^1, 1)$, *where* $H^1 \equiv \max\{H_2^{0,1}, H_3^{0,1}\}$.

Proof of Lemma 8 We need to find the action G^i and the level of state variable H^1 such that

$$V(H, \{1^{(\infty)}\}) = \max_{l \in \mathcal{G}^{\times 0}} V(H, l),$$

for any $H \in (H^1, 1)$, and

$$V(H, \{1^{(\infty)}\}) < V(H, \{i^{(1)}, 1^{(\infty)}\}),$$

for any $H \in (0, H^1)$. By Lemma 7 and Claim 2, there exists some value $\bar{H} \in (0,1)$, close to unity, such that $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (\bar{H}, 1)$, and that

$$\frac{\partial V(H; \{3^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{2^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(\infty)}\})}{\partial H},$$

and

$$V(1; \{3^{(1)}, 1^{(\infty)}\}) < V(1; \{2^{(1)}, 1^{(\infty)}\}) < V(1; \{1^{(\infty)}\}).$$

Compare action sequences $\{1^{(\infty)}\}$, $\{2^{(1)}, 1^{(\infty)}\}$ and $\{3^{(1)}, 1^{(\infty)}\}$, and find H^1 such that

$$H^1 = \max\{H_2^{0,1}, H_3^{0,1}\}.$$

By Claim 4, it must hold that

$$\max\{V(H; \{3^{(1)}, 1^{(\infty)}\}), V(H; \{2^{(1)}, 1^{(\infty)}\})\} < V(H; \{1^{(\infty)}\})$$

for $H \in (H_2^{0,1}, 1)$ if $H_3^{0,1} < H_2^{0,1}$ and for $H \in (H_3^{0,1}, 1)$ if $H_2^{0,1} < H_3^{0,1}$. Therefore, setting $H^1 \equiv H_i^{0,1} = \max\{H_2^{0,1}, H_3^{0,1}\}$, we obtain the desired result. ■

We now have two cases: the first is $H_3^{0,1} < H_2^{0,1}$, and the second is $H_2^{0,1} < H_3^{0,1}$. If $H_3^{0,1} < H_2^{0,1}$, then $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (H_2^{0,1}, 1)$. On the other hand, if $H_2^{0,1} < H_3^{0,1}$, then $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (H_3^{0,1}, 1)$. We consider the second case to examine subcase B-I in Section 4. Suppose that $H_2^{0,1} < H_3^{0,1}$. Then, the main result is:

Result 1 *Suppose that $H_2^{0,1} < H_3^{0,1}$. Then, $G(H) = G^3$ if $H \in (0, H_3^{0,1})$, and $G(H) = G^1$ if $H \in (H_3^{0,1}, 1)$.*

To prove this proposition, it is sufficient to show the following lemma:

Lemma 9 *Suppose that $H_2^{0,1} < H_3^{0,1}$. Then, $\mathbf{G}(H) = \{1^{(\infty)}\}$ if $H \in (H_3^{0,1}, 1)$, and $\mathbf{G}(H) = \{3^{(n)}, 1^{(\infty)}\}$ if $H \in (H_3^{n,n+1}, H_3^{n-1,n})$ for any $n \in \{1, 2, \cdots\}$.*

We apply the mathematical induction to this problem. We first examine the case of $n = 1$, and then consider the case of $n = m+1$ assuming that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ if $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. To do so, we deduce the following result.

Claim 5 *Suppose that $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (H_3^{0,1}, 1)$. Then, $\{2^{(m)}, 1^{(\infty)}\}$ is not optimal for any $m \in \{1, 2, \cdots\}$ and any $H \in (0, 1)$.*

Proof of Claim 5 Noting that, by Claim 2,

$$\frac{\partial V(H; \{3^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{2^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(\infty)}\})}{\partial H},$$

and $H_2^{0,1} < H_3^{0,1}$, it must hold that

$$\max\{V(H; \{3^{(1)}, 1^{(\infty)}\}), V(H; \{1^{(\infty)}\})\} > V(H; \{2^{(1)}, 1^{(\infty)}\}),$$

which implies that $\{2^{(1)}, 1^{(\infty)}\}$ is not optimal. Next we will show that if $\{2^{(1)}, 1^{(\infty)}\}$ is not optimal, then $\{2^{(m)}, 1^{(\infty)}\}$ is not optimal for any $m \in \{1, 2, \cdots\}$. To see this, suppose, to get a contradiction, that there exists some positive integer $\bar{m} > 1$ such that $\{2^{(\bar{m})}, 1^{(\infty)}\}$ is optimal for some $\bar{H} \in (0, 1)$. Then, for any $m \in \{1, \cdots, \bar{m} - 1\}$, there exists some $\tilde{H}_m \in (0, 1)$ such that $\{2^{(m)}, 1^{(\infty)}\}$ is optimal, which contradicts to the fact that $\{2^{(1)}, 1^{(\infty)}\}$ is not optimal. ∎

We now examine the case of $n = 1$.

Claim 6 *$\{i^{(1)}, 3^{(1)}, 1^{(\infty)}\}$ is not optimal for any $H \in (0, 1)$ and for any $i \in \{1, 2\}$.*

Proof of Claim 6 We will show that

$$\max\{V(H; \{3^{(1)}, 1^{(\infty)}\}), V(H; \{1^{(\infty)}\})\} > V(H; \{1^{(1)}, 3^{(1)}, 1^{(\infty)}\}),$$

Suppose, to get a contradiction, that $\{1^{(1)}, 3^{(1)}, 1^{(\infty)}\}$ is optimal for some $H \in (0, 1)$. Since

$$\frac{\partial V(H; \{3^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(1)}, 3^{(1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(\infty)}\})}{\partial H},$$

by Claim 3, there exists some $\bar{H} \in (H_3^{0,1}, 1)$ such that

$$V(\bar{H}; \{1^{(\infty)}\}) < V(\bar{H}; \{1^{(1)}, 3^{(1)}, 1^{(\infty)}\}),$$

which contradicts to the assumption that $\mathbf{G}(H)=\{1^{(\infty)}\}$ for any $H\in(H_3^{0,1},1)$. ■

Claim 7 *Suppose that* $\mathbf{G}(H)=\{1^{(\infty)}\}$ *for any* $H\in(H_3^{0,1},1)$. *Then,*

$$V(H_3^{0,1};\{3^{(1)},1^{(\infty)}\})=V(H_3^{0,1};\{1^{(\infty)}\})\geq V(H_3^{0,1};\{2^{(1)},3^{(1)},1^{(\infty)}\}).$$

Proof of Claim 7 Suppose, to get a contradiction, that

$$V(H_3^{0,1};\{3^{(1)},1^{(\infty)}\})<V(H_3^{0,1};\{2^{(1)},3^{(1)},1^{(\infty)}\}).$$

Then, there exists some $\bar{H}\in(H_3^{0,1},1)$ such that $\{2^{(1)},3^{(1)},1^{(\infty)}\}$ is optimal for $\bar{H}$, which contradicts to the fact that $\{1^{(\infty)}\}$ is optimal for $H\in(H_3^{0,1},1)$. ■

Claim 8 *Suppose that* $\mathbf{G}(H)=\{1^{(\infty)}\}$ *for any* $H\in(H_3^{0,1},1)$. *Then, for any* $H\in(H_3^{1,2},H_3^{0,1})$,

$$V(H;\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H;\{3^{(1)},1^{(\infty)}\})$$

if $V(H_3^{1,2};\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H_3^{1,2};\{3^{(1)},1^{(\infty)}\})$.

Proof of Claim 8 By Claim 7, it must hold that $V(H_3^{0,1};\{3^{(1)},1^{(\infty)}\})\geq V(H_3^{0,1};\{2^{(1)},3^{(1)},1^{(\infty)}\})$. Note that $V(H;\cdot)$ is an affine function. By the assumption, we have that $V(H_3^{1,2};\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H_3^{1,2};\{3^{(1)},1^{(\infty)}\})$. Therefore, we derive the desired result. ■

Claim 9 *Suppose that* $\mathbf{G}(H)=\{1^{(\infty)}\}$ *for any* $H\in(H_3^{0,1},1)$. *Then,* $\mathbf{G}(H)=\{3^{(1)},1^{(\infty)}\}$ *for any* $H\in(H_3^{1,2},H_3^{0,1})$ *if* $V(H_3^{1,2};\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H_3^{1,2};\{3^{(1)},1^{(\infty)}\})$.

Proof of Claim 9 Suppose that $V(H_3^{1,2};\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H_3^{1,2};\{3^{(1)},1^{(\infty)}\})$. Then, by Claim 8, $\{2^{(1)},3^{(1)},1^{(\infty)}\}$ cannot be optimal for any $H\in(H_3^{1,2},H_3^{0,1})$. Also, by Claim 6, $\{1^{(1)},3^{(1)},1^{(\infty)}\}$ cannot be optimal for any $H\in(H_3^{1,2},H_3^{0,1})$. Note also that $\mathbf{G}(H)=\{1^{(\infty)}\}$ for any $H\in(H_3^{0,1},1)$, that

$$\max\{V(H;\{1^{(\infty)}\}),V(H;\{3^{(2)},1^{(\infty)}\})\}<V(H;\{3^{(1)},1^{(\infty)}\}),$$

for any $H\in(H_3^{1,2},H_3^{0,1})$, that $H_{t+1}=1-M_3(1-H)\in(H_3^{0,1},1)$ for $H\in(H_3^{1,2},H_3^{0,1})$, and that $\{2^{(m)},1^{(\infty)}\}$ is not optimal by Claim 5. Therefore, it must hold that $\mathbf{G}(H)=\{3^{(1)},1^{(\infty)}\}$ for any $H\in(H_3^{1,2},H_3^{0,1})$. ■

Claim 10 *Suppose that* $H_2^{0,1}<H_3^{0,1}$. *Then,* $V(H_3^{1,2};\{2^{(1)},3^{(1)},1^{(\infty)}\})<V(H_3^{1,2};\{3^{(1)},1^{(\infty)}\})$.

Proof of Claim 10 Define the function

$$\Gamma(H)\equiv V(H;\{3^{(1)},1^{(\infty)}\})-V(H;\{2^{(1)},3^{(1)},1^{(\infty)}\}).$$

We need to show that $\Gamma(H_3^{1,2}) > 0$ if $H_2^{0,1} < H_3^{0,1}$. Note that

$$\Gamma(H) = (R_3 - R_2) + \beta(R_1 - R_3) + (1-H)[R_2 - R_3 - \beta M_3 A_1(1-\beta M_2) + \beta M_2 A_3(1-\beta M_3)],$$

where $A_i = R_i/(1-\beta M_i)$ for $i \in \{2,3\}$. Since

$$1 - H_3^{1,2} = \frac{R_1 - R_3}{M_3(1-\beta M_3)(A_1 - A_3)},$$

we have that

$$\begin{aligned}\Gamma(H_3^{1,2}) &= -(R_2 - R_3) + \frac{R_1 - R_3}{M_3}\left[1 - \beta(M_2 - M_3) - \frac{(1-\beta M_2)(A_1 - A_2)}{(1-\beta M_3)(A_1 - A_3)}\right] \\ &= -(R_2 - R_3) + \frac{R_1 - R_3}{M_3}\left[1 - \beta(M_2 - M_3) - \frac{(1-\beta M_2)R_1 - (1-\beta M_1)R_2}{(1-\beta M_3)R_1 - (1-\beta M_1)R_3}\right].\end{aligned}$$

Given (R_1, M_1) and (R_3, M_3), define $\bar{\Gamma}(R_2, M_2|(R_1, M_1), (R_3, M_3)) \equiv \Gamma(H_3^{1,2})$. Then, it is obvious that $\bar{\Gamma}(R_2, M_2|(R_1, M_1), (R_3, M_3)) = 0$ is as an affine function with respect to (R_2, M_2). Since

$$\begin{aligned}&\bar{\Gamma}(R_3, M_3) = 0; \\ &\bar{\Gamma}(R_1, M_1) = \frac{R_1 - R_3}{M_3}[(1 - M_3) - \beta(M_1 - M_3)] > \frac{R_1 - R_3}{M_3}(M_1 - M_3)(1-\beta) > 0; \\ &\bar{\Gamma}(R_1, M_3) = -(R_1 - R_3) < 0;\end{aligned}$$

it is easy to show that the affine function $\bar{\Gamma}(R_2, M_2) = 0$ passes through (R_3, ν_3) and some point between (R_1, ν_1) and (R_1, ν_3) on the $(R_2, \nu_2)-$plane, and that any point in the area, denoted by $\mathbf{\Gamma}$, below the line $\bar{\Gamma}(R_2, M_2) = 0$ satisfies $\bar{\Gamma}(R_2, M_2) > 0$. Let

$$\mathbf{\Lambda} \equiv \{(R_2, \nu_2) \mid H_2^{0,1} < H_3^{0,1},\ R_1 > R_2 > R_3 \text{ and } \nu_1 < \nu_2 < \nu_3\}$$

denote the set of feasible pairs of R_2 and ν_2 that satisfies $H_2^{0,1} < H_3^{0,1}$. Since this set $\mathbf{\Lambda}$ is represented by the triangle whose points are (R_1, ν_1), (R_3, ν_3) and (R_3, ν_1), we have that $\mathbf{\Lambda} \subset \mathbf{\Gamma}$. Therefore, it must hold that if $H_2^{0,1} < H_3^{0,1}$, then $\Gamma(H_3^{1,2}) > 0$ or

$$V(H_3^{1,2}; \{2^{(1)}, 3^{(1)}, 1^{(\infty)}\}) < V(H_3^{1,2}; \{3^{(1)}, 1^{(\infty)}\}),$$

which is the desired result. ■

From the above analysis, we deduce the following result related to the case of $n = 1$ in Lemma 9.

Lemma 10 *Suppose that $H_2^{0,1} < H_3^{0,1}$. Then, $\mathbf{G}(H) = \{3^{(1)}, 1^{(\infty)}\}$ for any $H \in (H_3^{1,2}, H_3^{0,1})$.*

Proof of Lemma 10 From Claims 9 and 10, we derive the desired result. ■

We now suppose that

$$\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\} \quad \text{for any} \quad H \in (H_3^{m,m+1}, H_3^{m-1,m}),$$

with $H_2^{0,1} < H_3^{0,1}$. Then, we consider the case of $n = m + 1$.

Claim 11 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,* $\{1^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ *and* $\{2^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ *cannot be an optimal policy for any* $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$.

Proof of Claim 11 Consider the policy $\{1^{(1)}, 3^{(m)}, 1^{(\infty)}\}$. Suppose, to get a contradiction, that $\{1^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ is an optimal policy for some $\bar{H} \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. By Claims 1 and 3, it must hold that

$$\frac{\partial V(H; \{3^{(m+1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{3^{(m)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(1)}, 3^{(m)}, 1^{(\infty)}\})}{\partial H}.$$

Then, there exists some $\hat{H} \in (H_3^{m,m+1}, H_3^{m-1,m})$ such that $\{1^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ is optimal, which contradicts to the assumption that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. Similarly, consider the policy $\{2^{(1)}, 3^{(m)}, 1^{(\infty)}\}$. Suppose, to get a contradiction, that $\{2^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ is an optimal policy for some $\bar{H} \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. By Claim 2, it must hold that

$$\frac{\partial V(H; \{3^{(m+1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{2^{(1)}, 3^{(m)}, 1^{(\infty)}\})}{\partial H}.$$

Then, there exists some $\hat{H} \in (H_3^{m,m+1}, H_3^{m-1,m})$ such that $\{2^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ is optimal, which contradicts to the assumption that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. ■

Claim 12 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then, the optimal policy can be represented as* $\{l^*, 3^{(m+1)}, 1^{(\infty)}\}$ *for any* $H \in (0, H_3^{m,m+1})$, *where* l^* *is a finite sequence, possibly empty, of elements from* $\mathcal{G}$.

Proof of Claim 12 From Claim 11, we already know that $\{1^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ and $\{2^{(1)}, 3^{(m)}, 1^{(\infty)}\}$ cannot be an optimal policy for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. Since $M_1 > M_2 > M_3$, there must exist some $\bar{t}$ such that $H_{\bar{t}} \in (H_3^{m+1,m+2}, H_3^{m,m+1})$ if the initial state variable is less than $H_3^{m+1,m+2}$. In this case, the optimal policy can be represented as $\{l, 3^{(m+1)}, 1^{(\infty)}\}$, where l is a non-empty finite sequence of elements from $\mathcal{G}$. On the other hand, if the initial state variable is between $H_3^{m+1,m+2}$ and $H_3^{m,m+1}$, the optimal policy can be represented as $\{l, 3^{(m+1)}, 1^{(\infty)}\}$, where l is a finite sequence of elements from $\mathcal{G}$, but may be empty. ■

Claim 13 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,* $\{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}$ *is not an optimal policy for any* $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$.

Proof of Claim 13 By Claim 12, we know that the optimal policy can be represented as $\{l^*, 3^{(m+1)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$, where l^* is a finite sequence, possibly empty, of elements from $\mathcal{G}$. We will show that, for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$,

$$V(H; \{3^{(m+1)}, 1^{(\infty)}\}) > V(H; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}),$$

Suppose, to get a contradiction, that $\{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}$ is optimal for some $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. Since

$$\frac{\partial V(H; \{3^{(m+1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(1)}, 3^{(m)}, 1^{(\infty)}\})}{\partial H},$$

by Claim 2, there exists some $\bar{H} \in (H_3^{m,m+1}, H_3^{m-1,m})$ such that

$$V(\bar{H}; \{3^{(m)}, 1^{(\infty)}\}) < V(\bar{H}; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}),$$

which contradicts to the assumption that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. ■

Claim 14 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,*

$$V(H_3^{m,m+1}; \{3^{(m+1)}, 1^{(\infty)}\}) = V(H_3^{m,m+1}; \{3^{(m)}, 1^{(\infty)}\}) \geq V(H_3^{m,m+1}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

Proof of Claim 14 Suppose, to get a contradiction, that

$$V(H_3^{m,m+1}; \{3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m,m+1}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

Then, there exists some $\bar{H} \in (H_3^{m,m+1}, H_3^{m-1,m})$ such that $\{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}$ is optimal for $\bar{H}$, which contradicts to the fact that $\{3^{(m)}, 1^{(\infty)}\}$ is optimal for $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. ■

Claim 15 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then, for any* $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$,

$$V(H; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H; \{3^{(m+1)}, 1^{(\infty)}\})$$

if $V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\})$.

Proof of Claim 15 By Claim 14, it must hold that

$$V(H_3^{m,m+1}; \{3^{(m+1)}, 1^{(\infty)}\}) \geq V(H_3^{m,m+1}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

Note that $V(H; \cdot)$ is an affine function. By the assumption, we have $V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\})$. Therefore, we derive the desired result. ■

Claim 16 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,* $\mathbf{G}(H) = \{3^{(m+1)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$ *if* $V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\})$.

Proof of Claim 16 Suppose that $V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\})$. Then, by Claim 15, $\{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}$ cannot be optimal for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. Also, by Claim 13, $\{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}$ cannot be optimal for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. Note also that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$, that

$$\max\{V(H; \{3^{(m)}, 1^{(\infty)}\}), V(H; \{3^{(m+2)}, 1^{(\infty)}\})\} < V(H; \{3^{(m+1)}, 1^{(\infty)}\}),$$

for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$, and $H' = 1-M_3(1-H) \in (H_3^{m,m+1}, 1)$ for $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. Thus, it must hold that $\mathbf{G}(H) = \{3^{(m+1)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. ■

Claim 17 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,*

$$V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\}).$$

Proof of Claim 17 Define the function

$$\Gamma_m(H) \equiv V(H; \{3^{(m+1)}, 1^{(\infty)}\}) - V(H; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

We need to show that $\Gamma_m(H_3^{m+1,m+2}) > 0$ if $H_2^{0,1} < H_3^{0,1}$. Note that

$$\begin{aligned}\Gamma_m(H) =& \beta^{m+1}(R_1 - R_2) - (1-\beta^{m+1})(R_2 - R_3) \\ &+ (1-H)[R_2 - (1-\beta M_2)[(\beta M_3)^{m+1}A_1 + (1-(\beta M_3)^{m+1})A_3],\end{aligned}$$

where $A_i = R_i/(1-\beta M_i)$ for $i \in \{1,2,3\}$. Since

$$1 - H_3^{m+1,m+2} = \frac{R_1 - R_3}{M_3^{m+1}(1-\beta M_3)(A_1 - A_3)},$$

we have that

$$\begin{aligned}\Gamma_m(H_3^{m+1,m+2}) =& \beta^{m+1}(R_1 - R_2) - (1-\beta^{m+1})(R_2 - R_3) \\ &+ \frac{(R_1 - R_3)(1-\beta M_2)}{M_3^{m+1}(1-\beta M_3)(A_1 - A_3)}[A_2 - (\beta M_3)^{m+1}A_1 - (1-(\beta M_3)^{m+1})A_3].\end{aligned}$$

For given (R_1, M_1) and (R_3, M_3), $\Gamma_m(H_3^{m+1,m+2}) \equiv \bar{\Gamma}_m(R_2, M_2 : R_1, M_1, R_3, M_3) = 0$ is regarded as an affine function with respect to (R_2, M_2) or (R_2, ν_2). Note that

$$\begin{aligned}\bar{\Gamma}_m(R_3, M_3) =& \beta^{m+1}(R_1 - R_3) - \beta^{m+1}(R_1 - R_3) = 0 \quad \text{and} \\ \bar{\Gamma}_m(R_3, M_1) =& \beta^{m+1}(R_1 - R_3) \\ &+ \frac{(R_1 - R_3)}{M_3^{m+1}(1-\beta M_3)(A_1 - A_3)}\{R_3 - (1-\beta M_1)[(\beta M_3)^{m+1}A_1 + (1-(\beta M_3)^{m+1})A_3]\} \\ =& \frac{\beta(R_1 - R_3)(M_1 - M_3)}{M_3^{m+1}(1-\beta M_3)(A_1 - A_3)}[(\beta M_3)^{m+1}A_1 + (1-(\beta M_3)^{m+1})A_3] > 0.\end{aligned}$$

Also, it must hold that $\bar{\Gamma}_m(R_1, M_1) > 0$. To see this, note that

$$\bar{\Gamma}_m(R_1, M_1) = V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\}) - V(H_3^{m+1,m+2}; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

Suppose, to get a contradiction, that $\bar{\Gamma}_m(R_1, M_1) < 0$, or

$$V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\}) < V(H_3^{m+1,m+2}; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}).$$

Since

$$\frac{\partial V(H; \{3^{(m+1)}, 1^{(\infty)}\})}{\partial H} < \frac{\partial V(H; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\})}{\partial H}$$

by Claim 3, there exists some $\bar{H} \in (H_3^{m,m+1}, H_3^{m-1,m})$ such that

$$V(\bar{H}; \{3^{(m)}, 1^{(\infty)}\}) < V(\bar{H}; \{1^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}),$$

which contradicts to the assumption that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$. Thus, it must hold that $\bar{\Gamma}_m(R_1, M_1) > 0$. Therefore, since $\bar{\Gamma}_m(R_3, M_3) = 0$, $\bar{\Gamma}_m(R_3, M_1) > 0$ and $\bar{\Gamma}_m(R_1, M_1) > 0$, it is easy to show that the affine function $\bar{\Gamma}_m(R, M) = 0$ does not pass through any point in the triangle that is connected by (R_1, ν_1), (R_3, ν_3) and (R_3, ν_1) on the $(R_2, \nu_2)-$plane. Let

$$\mathbf{\Lambda} \equiv \{(R_2, \nu_2) \mid H_2^{0,1} < H_3^{0,1},\ R_1 > R_2 > R_3 \text{ and } \nu_1 < \nu_2 < \nu_3\}$$

denote the set of feasible pairs of R_2 and ν_2 that satisfies $H_2^{0,1} < H_3^{0,1}$. Then, it must hold that

$$\bar{\Gamma}_m(R_2, M_2) \geq 0 \quad \text{for any} \quad (R_2, M_2) \in \mathbf{\Lambda}.$$

This implies that if $H_2^{0,1} < H_3^{0,1}$, then $\Gamma_m(H_3^{m+1,m+2}) \geq 0$ or

$$V(H_3^{m+1,m+2}; \{2^{(1)}, 3^{(m+1)}, 1^{(\infty)}\}) \leq V(H_3^{m+1,m+2}; \{3^{(m+1)}, 1^{(\infty)}\}),$$

which is the desired result. ■

From the above analysis, we deduce the following result under the assumption that $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$, which corresponds to the case of $n = m + 1$ in Lemma 9.

Lemma 11 *Suppose that* $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ *with* $H_2^{0,1} < H_3^{0,1}$. *Then,* $\mathbf{G}(H) = \{3^{(m+1)}, 1^{(\infty)}\}$ *for any* $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$.

Proof of Lemma 11 From Claims 16 and 17, we derive the desired result. ■

We now summarizes the proof of Lemma 9 as follows:

Proof of Lemma 9 By Lemma 8 and $H_2^{0,1} < H_3^{0,1}$, $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (H_3^{0,1}, 1)$. By Lemma 10, $\mathbf{G}(H) = \{3^{(1)}, 1^{(\infty)}\}$ for any $H \in (H_3^{1,2}, H_3^{0,1})$. By Lemma 11, if $\mathbf{G}(H) = \{3^{(m)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m,m+1}, H_3^{m-1,m})$ with $H_2^{0,1} < H_3^{0,1}$, then $\mathbf{G}(H) = \{3^{(m+1)}, 1^{(\infty)}\}$ for any $H \in (H_3^{m+1,m+2}, H_3^{m,m+1})$. By the method of the mathematical induction, it must hold that if $H_2^{0,1} < H_3^{0,1}$, then $\mathbf{G}(H) = \{3^{(n)}, 1^{(\infty)}\}$ if $H \in (H_3^{n,n+1}, H_3^{n-1,n})$ for any $n \in \{1, 2, \cdots\}$. ■

6.2 Proofs of Lemmas and Propositions in Section 4

Proof of Lemma 1 Suppose that the government chooses action I ($G_t = G^1$) in period t. By equations (3) and (5), the industry-specific factor is $a_{z,t} = a_0$, and the (expected) industry-specific productivity index is $E(\epsilon_{z,t}) = 1$ for any industry $z \in [0,1]$. The conditions (10) and (11) yield the equilibrium wage rate $w_t^* = \mu H(a_0/\bar{L})^{1-\mu}$ and the equilibrium labor employment $l_{z,t}^* = \bar{L}$ in any industry $z \in [0,1]$. Then, the total labor employment in learning industries and the (expected) national income are

$$L_{A,t}^* = K\bar{L}; \qquad g_t^e = H_t \bar{L}^{\mu} a_0^{1-\mu}.$$

Suppose that the government chooses action II ($G_t = G^2$) in period t. There is no red tape and no loss in the industry-specific productivity index under non-elaborate monitoring. However, due to rent-seeking, part of non-learning industries becomes eligible for promotion support so that $D(0) = [0,(1+\lambda)K]$. By equations (3), (4) and (5), the industry-specific factor and the expected industry-specific productivity index are

$$a_{z,t} = \begin{cases} a_0 - \tau + \frac{\tau - c}{K(1+\lambda)} & \text{if } z \in [0, K(1+\lambda)] \\ a_0 - \tau & \text{if } z \in (K(1+\lambda), 1] \end{cases}; \qquad E(\epsilon_{z,t}) = 1 \quad \text{for all } z \in [0,1].$$

The conditions (10) and (11) yield the equilibrium wage rate $w_t^* = \mu H[(a_0 - c)/\bar{L}]^{1-\mu}$ and the equilibrium labor employment

$$l_{z,t}^* = \begin{cases} \frac{a_0 - \tau + (\tau - c)/[K(1+\lambda)]}{a_0 - c} \bar{L} & \text{if } z \in [0, K(1+\lambda)] \\ \frac{a_0 - \tau}{a_0 - c} \bar{L} & \text{if } z \in (K(1+\lambda), 1]. \end{cases}$$

Then, the total labor employment in learning industries and the (expected) national income are

$$L_{A,t}^* = \frac{a_0 - \tau + (\tau - c)/[K(1+\lambda)]}{a_0 - c} K\bar{L}; \qquad g_t^e = H_t \bar{L}^{\mu} (a_0 - c)^{1-\mu}.$$

Suppose that the government chooses action III ($G_t = G^3$) in period t. Since there is no rent-seeking activities under elaborate monitoring, only learning industries are eligible for promotion support so that $D(1) = [0, K]$. However, such monitoring gives rise to red tape that reduces the productivity index in eligible industries. By equations (3), (4) and (5), the industry-specific factor and the expected industry-specific productivity index are

$$a_{z,t} = \begin{cases} a_0 - \tau + \frac{\tau - c}{K} & \text{if } z \in [0, K] \\ a_0 - \tau & \text{if } z \in (K, 1] \end{cases}; \qquad E(\epsilon_{z,t}) = \begin{cases} 1 - \theta\eta & \text{if } z \in [0, K] \\ 1 & \text{if } z \in (K, 1]. \end{cases}$$

The conditions (10) and (11) yield the equilibrium wage rate $w_t^* = \mu H(B/\bar{L})^{1-\mu}$ and the equilibrium labor employment

$$l_{z,t}^* = \begin{cases} \frac{(1-\theta\eta)^{1/(1-\mu)}(a_0 - \tau + (\tau - c)/K)}{B} \bar{L} & \text{if } z \in [0, K] \\ \frac{a_0 - \tau}{B} \bar{L} & \text{if } z \in (K, 1], \end{cases}$$

where $B \equiv [K(1-\theta\eta)^{1/(1-\mu)} + 1 - K](a_0 - \tau) + K(1-\theta\eta)^{1/(1-\mu)}(\tau - c)$. Then, the total labor employment in learning industries and the (expected) national income are

$$L^*_{A,t} = \frac{(1-\theta\eta)^{1/(1-\mu)}(a_0 - \tau + (\tau - c)/K)}{B} K\bar{L}; \qquad g^e_t = H_t \bar{L}^{\mu} B^{1-\mu}. \quad \blacksquare$$

Proof of Corollary 1 Notice that $L^*_{A,t}(H_t, 1, 0)$ depends on λ, and $L^*_{A,t}(H_t, 1, 1)$ and $g^e_t(H_t, 1, 1)$ depend on θ. Differentiating $L^*_{A,t}(H_t, 1, 0)$ with respect to λ and differentiating $L^*_{A,t}(H_t, 1, 1)$ and $g^e_t(H_t, 1, 1)$ with respect to θ yield the desired results. $\blacksquare$

Proof of Lemma 2 We first show that $R(G^1) > R(G^2) > R(G^3)$. Let $\bar{\delta} = \delta(1) = (\tau - c)/K$. Since

$$R(G^1) - R(G^2) = \bar{L}^{\mu}[a_0^{1-\mu} - (a_0 - \tau + K\bar{\delta})^{1-\mu}] > 0,$$

it must hold that $R(G^1) > R(G^2)$. Furthermore, notice that

$$\begin{aligned} &R(G^2) - R(G^3) \\ =&\bar{L}^{\mu}[(a_0 - \tau + K\bar{\delta})^{1-\mu} - \{[K(1-\theta\eta)^{\frac{1}{1-\mu}} + 1 - K](a_0 - \tau) + K(1-\theta\eta)^{\frac{1}{1-\mu}}\bar{\delta}\}^{1-\mu}]. \end{aligned}$$

Since

$$\begin{aligned} &(a_0 - \tau + K\bar{\delta}) - \{[K(1-\theta\eta)^{\frac{1}{1-\mu}} + 1 - K](a_0 - \tau) + K(1-\theta\eta)^{\frac{1}{1-\mu}}\bar{\delta}\} \\ =&K[1 - (1-\theta\eta)^{\frac{1}{1-\mu}}](a_0 - \tau + \bar{\delta}) \\ =&K[1 - (1-\theta\eta)^{\frac{1}{1-\mu}}](a_0 - c + \tau(1-K)/K) > 0, \end{aligned}$$

it must hold that $R(G^2) > R(G^3)$. Thus, $R(G^1) > R(G^2) > R(G^3)$. Next We show that $L^*_A(G^1) < L^*_A(G^2)$. Since $K(1+\lambda) < 1$ and

$$L^*_A(G^2) - L^*_A(G^1) = K\bar{L}\left[\frac{\bar{\delta}(1 - K(1+\lambda))}{(1+\lambda)(a_0 - \tau + K\bar{\delta})}\right] > 0,$$

it must hold that $L^*_A(G^2) > L^*_A(G^1)$. $\blacksquare$

Proof of Lemma 3 It is first shown that for any $\lambda \in (0, 1/K - 1)$, there exists a unique value $\psi(\lambda) \in (0, 1]$ such that $L^*_A(G^2) < L^*_A(G^3)$ for any $\theta \in (0, \psi(\lambda))$ and $L^*_A(G^2) > L^*_A(G^3)$ for any $\theta \in (\psi(\lambda), 1]$. Let $A(\theta) \equiv (1-\theta\eta)^{1/(1-\mu)} \in [(1-\eta)^{\frac{1}{1-\mu}}, 1)$ and let $\bar{\delta} = \delta(1) = (\tau - c)/K$. Then, it must hold that

$$L^*_A(G^3) - L^*_A(G^2) = K\bar{L}[C(\theta) - D(\lambda)],$$

where

$$C(\theta) = \frac{A(\theta)(a_0 - \tau + \bar{\delta})}{(KA(\theta) + 1 - K)(a_0 - \tau) + KA(\theta)\bar{\delta}}; \qquad D(\lambda) = \frac{a_0 - \tau + \bar{\delta}/(1+\lambda)}{a_0 - \tau + K\bar{\delta}}.$$

Notice that

$$C(0) = D(0) = \frac{a_0 - \tau + \bar{\delta}}{a_0 - \tau + K\bar{\delta}}; \qquad D(1/K-1) = 1;$$
$$\frac{\partial C(\theta)}{\partial \theta} = -\frac{(1-K)(a_0-\tau)(a_0-\tau+\bar{\delta})K\bar{L}}{A(\theta)[(KA(\theta)+1-K)(a_0-\tau)+KA(\theta)\bar{\delta}]^2}\frac{\eta(1-\theta\eta)^{\frac{\mu}{1-\mu}}}{1-\mu} < 0;$$
$$\frac{\partial D(\lambda)}{\partial \lambda} = -\frac{\bar{\delta}K\bar{L}}{(a_0-\tau+K\bar{\delta})(1+\lambda)^2} < 0.$$

Pick any $\lambda \in (0, 1/K-1)$. There are two possible cases; $D(\lambda) \le C(1)$ and $D(\lambda) > C(1)$. Suppose that $D(\lambda) \le C(1)$. Since $C(\theta)$ is decreasing in θ, it must hold that $C(\theta) - D(\lambda) > C(1) - D(\lambda) \ge 0$ and hence $L_A^*(G^3) > L_A^*(G^2)$ for any $\theta \in (0,1]$. Next suppose that $D(\lambda) > C(1)$. Note that $D(0) = C(0) > D(\lambda)$. Since $C(\theta)$ is decreasing in θ, there exists a unique value of $\psi(\lambda) \in (0,1)$ such that $C(\theta) > D(\lambda)$ for any $\theta \in (0, \psi(\lambda))$ and $C(\theta) < D(\lambda)$ for any $\theta \in (\psi(\lambda), 1]$. Thus, it must hold that $L_A^*(G^3) > L_A^*(G^2)$ for any $\theta \in (0, \psi(\lambda))$ and $L_A^*(G^3) < L_A^*(G^2)$ for any $\theta \in (\psi(\lambda), 1]$. For convenience, define $\psi(\lambda) = 1$ for any λ satisfying $D(\lambda) \le C(1)$. Then, the desired result is derived. Also, not that if $C(\theta) < D(\lambda)$ or $\psi(\lambda) < 1$, then $C(\psi(\lambda)) = D(\lambda)$. Then, it must hold that $\partial\psi(\lambda)/\partial\lambda = D'(\lambda)/C'(\theta) > 0$, which implies that $\psi(\lambda)$ is increasing in λ. ■

Proof of Lemma 4 The line AI on the (ν, R) plane in Figures 5, 6 and 7 is represented as

$$R - R(G^1) = \frac{R(G^1) - R(G^2)}{\nu(L_A^*(G^1)) - \nu(L_A^*(G^2))}[\nu - \nu(L_A^*(G^1))].$$

Plugging $(\nu, R) = (\nu(L_A^*(G^3)), R(G^3))$ into the above equation, the condition of subcase B-I is that the left-hand side is larger than the right-hand side, while the condition of subcase B-II is that the left-hand side is smaller than the right-hand side. We denote the left-hand side minus the right-hand side by $W(\nu, R)$. Then, to prove the first part, it is enough to show that for any $\lambda \in (0, 1/K-1)$, there exists a unique value $\varphi(\lambda) \in (0, \psi(\lambda)]$ such that $W(\nu(L_A^*(G^3)), R(G^3)) > 0$ for $\theta \in (0, \varphi(\lambda))$ and $W(\nu(L_A^*(G^3)), R(G^3)) < 0$ for $\theta \in (\varphi(\lambda), \psi(\lambda))$. For each $i \in \{2, 3\}$, define

$$\kappa_i \equiv \frac{R(G^1)\nu(L_A^*(G^i)) - R(G^i)\nu(L_A^*(G^1))}{R(G^1) - R(G^i)} \in (0,1).$$

Note that $\kappa_2 \equiv \kappa_2(\lambda)$ and $\kappa_3 \equiv \kappa_3(\theta)$ depend on λ and θ, respectively. Pick any $\lambda \in (0, 1/K-1)$. Define the difference between $\kappa_2(\lambda)$ and $\kappa_3(\theta)$ by

$$D(\theta) \equiv \kappa_3(\theta) - \kappa_2(\lambda) = \frac{R_1\nu_3(\theta) - R_3(\theta)\nu_1}{R_1 - R_3(\theta)} - \kappa_2(\lambda),$$

where $R_1 = R(G^1)$, $R_2 = R(G^2)$, $\nu_1 = \nu(L_A^*(G^1))$ and $\nu_2 = \nu(L_A^*(G^2))$ are independent of θ, and $R(G^3) = R_3(\theta)$ and $\nu_3(\theta) = \nu(L_A^*(G^3))$ are dependent on θ. Then, we have that

$$\frac{\partial D(\theta)}{\partial \theta} = \frac{R_1(\nu_3(\theta) - \nu_1)}{(R_1 - R_3(\theta))^2}\frac{\partial R_3(\theta)}{\partial \theta} + \frac{R_1}{R_1 - R_3(\theta)}\frac{\partial \nu_3(\theta)}{\partial \theta} < 0,$$

since $R_1 > R_3(\theta)$ and $\nu_3(\theta) > \nu_1$ by the assumption of (ν, R), and since $\partial R_3(\theta)/\partial\theta < 0$ and $\partial\nu_3(\theta)/\partial\theta < 0$. Also, we have that

$$D(0) = \frac{(R_1 - R_2)(\nu_3(0) - \nu_2)}{\nu_2 - \nu_1} > 0,$$

since $R_3(0) = R_2$. Thus, for given λ, $D(\theta)$ is decreasing in θ. Note that if $\theta > \psi(\lambda)$, then obviously $D(0) < 0$. Therefore, setting $\varphi(\lambda)$ such that $D(\varphi(\lambda)) = 0$ with $\varphi(\lambda) < \psi(\lambda)$ or $D(\varphi(\lambda)) > 0$ with $\varphi(\lambda) = \psi(\lambda)$, where $\varphi(\lambda) = \min\{1, \psi(\lambda)\}$, it must hold that there exists a unique value $\varphi(\lambda) \in (0, \psi(\lambda))$ such that $D(\theta) > 0$ for $\theta \in (0, \varphi(\lambda))$ and $D(\theta) < 0$ for $\theta \in (\varphi(\lambda), \psi(\lambda))$. Since $D(\theta) \gtreqless 0$ is equivalent to $W \gtreqless 0$, the desired result can be obtained.

We next show that the critical value $\varphi(\lambda)$ is increasing in λ. By $D(\varphi(\lambda)) = 0$, taking the derivative with respect to λ, it must hold that, for any λ such that $\varphi(\lambda) < \psi(\lambda)$,

$$\frac{\partial\varphi(\lambda)}{\partial\lambda} = \frac{1}{R_1 - R_2(\lambda)}\frac{\partial\nu_2(\lambda)}{\partial\lambda} \Big/ \left[\frac{\nu_3(\theta) - \nu_1}{(R_1 - R_3(\theta))^2}\frac{\partial R_3(\theta)}{\partial\theta} + \frac{1}{R_1 - R_3(\theta)}\frac{\partial\nu_3(\theta)}{\partial\theta}\right] > 0,$$

which implies that the critical value $\varphi(\lambda)$ is increasing in λ. ■

Proof of Proposition 1 First, we show the following claim.

Claim 18 *Suppose that there are two government actions $\bar{G}_t = (\bar{S}_t, \bar{M}_t)$ and $\hat{G}_t = (\hat{S}_t, \hat{M}_t)$ such that $L_A^*(\bar{G}_t) < L_A^*(\hat{G}_t)$ and $R(\bar{G}_t) < R(\hat{G}_t)$. Then, $\bar{G}_t$ is never optimal for any $H_t \in (0, 1)$.*

Proof of Claim 18 Using $g^e(H_t, S_t, M_t) = H_t R(S_t, H_t)$, the function $\hat{v}(H_t, S_t, M_t)$ is defined as:

$$\hat{v}(H_t, S_t, M_t) \equiv H_t R(S_t, M_t) + \beta v(H_{t+1}),$$

subject to $H_{s+1} = 1 - [1 - \nu(L_A^*(S_s, M_s))](1 - H_s)$ for $s \in \{t, t+1, \dots\}$. The value of $\hat{v}(H_t, S_t, M_t)$ represents the sum of expected national income from period t under the optimal policy taking government action (S_t, M_t) in period t as given. We need to show that if $L_A(\bar{S}_t, \bar{M}_t) < L_A(\hat{S}_t, \hat{M}_t)$ and $R(\bar{S}_t, \bar{M}_t) < R(\hat{S}_t, \hat{M}_t)$, then it must hold that $\hat{v}(H_t, \bar{S}_t, \bar{M}_t) < \hat{v}(H_t, \hat{S}_t, \hat{M}_t)$ for all $H_t \in (0, 1)$. It is enough to show that:

$$R(\bar{S}_t, \bar{M}_t) < R(\hat{S}_t, \hat{M}_t) \quad \text{and} \quad v(H'_{t+1}) < v(H''_{t+1}),$$

where $H'_{t+1} = h(L_A^*(\bar{S}_t, \bar{M}_t), H_t)$ and $H''_{t+1} = h(L_A^*(\hat{S}_t, \hat{M}_t), H_t)$. By the assumption, we already know that $R(\bar{S}_t, \bar{M}_t) < R(\hat{S}_t, \hat{M}_t)$. The remaining is to show that $v(H'_{t+1}) < v(H''_{t+1})$. Suppose, to get a contradiction, that $v(H'_{t+1}) \geq v(H''_{t+1})$. Let $\{\tilde{S}_{t+s}, \tilde{M}_{t+s}\}_{s=1}^{\infty}$ such that

$$v(H'_{t+1}) = \sum_{s=0}^{\infty} \beta^s H'_{t+1+s} R(\tilde{S}_{t+1+s}, \tilde{M}_{t+1+s}),$$

where $H'_{t+1} = h(L^*_A(\bar{S}_t, \bar{M}_t), H_t)$ and $H'_{l+1} = h(L^*_A(\tilde{S}_l, \tilde{M}_l), H'_l)$ for $l \geq t+2$. Then, since $v(H'_{t+1}) \geq v(H''_{t+1})$, for $\{\tilde{S}_{t+s}, \tilde{M}_{t+s}\}_{s=1}^{\infty}$ it must hold that:

$$v(H'_{t+1}) = \sum_{s=0}^{\infty} \beta^s H'_{t+1+s} R(\tilde{S}_{t+1+s}, \tilde{M}_{t+1+s}) \geq \sum_{s=0}^{\infty} \beta^s H''_{t+1+s} R(\tilde{S}_{t+1+s}, \tilde{M}_{t+1+s}),$$

where $H''_{t+1} = h(L^*_A(\hat{S}_t, \hat{M}_t), H_t)$ and $H''_{l+1} = h(L^*_A(\tilde{S}_l, \tilde{M}_l), H''_l)$ for $l \geq t+2$. By the assumption that $L_A(\bar{S}_t, \bar{M}_t) < L_A(\hat{S}_t, \hat{M}_t)$, it must hold that $H'_{t+1} < H''_{t+1}$. By Assumption 2, we have that $H_{s+1} = 1 - (1 - \nu(L_{A,s}))(1 - H_s)$. Thus, we have that

$$H''_{t+1} > H'_{t+1} \quad \Rightarrow \quad H''_{t+2} - H'_{t+2} = [1 - \nu(L^*_A(\tilde{S}_{t+1}, \tilde{M}_{t+1}))](H''_{t+1} - H'_{t+1}) > 0 \quad \Rightarrow \quad H''_{t+2} > H'_{t+2}.$$

Similarly, we can deduce $H''_{t+l} > H'_{t+l}$ for all $l \geq 1$. Then, it must hold that

$$\sum_{s=0}^{\infty} \beta^s H'_{t+1+s} R(\tilde{S}_{t+1+s}, \tilde{M}_{t+1+s}) < \sum_{s=0}^{\infty} \beta^s H''_{t+1+s} R(\tilde{S}_{t+1+s}, \tilde{M}_{t+1+s}),$$

which contradicts to the assumption that $v(H'_{t+1}) \geq v(H''_{t+1})$. Therefore, it must hold that $v(H'_{t+1}) < v(H''_{t+1})$, and hence $\hat{v}(H_t, \bar{S}_t, \bar{M}_t) < \hat{v}(H_t, \hat{S}_t, \hat{M}_t)$ for all $H_t \in (0, 1)$. ■

The expected national income $g^e_t = H_t R(G_t)$ can be regarded as a current reward, and the total employment $L^*_A(G_t)$ in learning industries as the value related to future rewards through human capital accumulation. Thus, the government prefers a pair of higher g^e_t and higher $L^*_{A,t}$ by choosing its action G_t in each period. If a government action achieves lower expected national income per unit of human capital and lower speed of human capital accumulation than another action, such an action cannot be optimal. We call it a dominated government action.

By Lemma 2, action I (G^1) is never dominated. For any $H_t \in (0, 1)$, define the value when the government chooses action I in all periods by

$$v_1(H_t) = \sum_{s=0}^{\infty} \beta^s H_{t+s} R(G^1), \tag{16}$$

subject to $H_{s+1} = 1 - [1 - \nu(L^*_A(G^1))](1 - H_s)$. And for each $G^i \in \mathcal{G}$ $(i \neq 1)$, define the value when he chooses action i (G^i) in period t and then action I forever from period $t+1$ by

$$v_{i1}(H_t) = H_t R(G^i) + \beta v_1(H_{t+1}), \tag{17}$$

and $H_{t+1} = 1 - [1 - \nu(L^*_A(G^i))](1 - H_t)$ and $H_{t+l+1} = 1 - [1 - \nu(L^*_A(G^1))](1 - H_{t+l})$ for all $l \geq 1$. Using equation (9), these value functions, (16) and (17), can be respectively rewritten as

$$v_1(H) = R(G^1) \left[\frac{1}{1-\beta} - \frac{1-H}{1-\beta M(G^1)} \right], \tag{18}$$

and

$$v_{i1}(H) = \left[R(G^i) + \frac{\beta R(G^1) M(G^i)}{1 - \beta M(G^1)} \right] H + \beta R(G^1) \left[\frac{1}{1-\beta} - \frac{M(G^i)}{1 - \beta M(G^1)} \right]. \tag{19}$$

where $M(G^j) = 1 - \nu(L_A^*(G^j))$ for each $G^j \in \mathcal{G}$.

Consider case A in which the economy has relatively small degree of rent-seeking activities and relatively large demand variability such that $\theta \in (\psi(\lambda), 1]$. From Lemmas 2 and 3 and Claim 18, action III ($G^3 = (1,1)$) is dominated by action II ($G^2 = (1,0)$) so that the government can restrict itself into a binary choice between actions I and II. The cost of elaborate monitoring is always relatively high compared to its benefit so that elaborate monitoring cannot be justified. Figure 5 (the upper part) illustrates this situation on (ν, R) space, where point C representing the result of government action G^3 is within the shaded area, i.e., G^3 is dominated by G^2.

Since $R(G^1) > R(G^2)$ and $\nu(L_A^*(G^1)) < \nu(L_A^*(G^2))$, the government faces the trade-off between the current reward and the future reward that is determined by the total employment in learning industries in each period. Then, the following claim is obtained:

Claim 19 *Suppose that the flexibility issue is relatively significant such that $\theta \in (\psi(\lambda), 1]$. Then, there exists a unique value $\hat{H} \in (0,1)$ such that the adoption of promotion support without elaborate monitoring is optimal for $H \in (0, \hat{H})$, and a laissez faire policy is optimal for $H \in (\hat{H}, 1)$.*

Proof of Claim 19 For this proof, we follow the discussion of the optimal stopping region in Dixit and Pyndyck Dixit and Pindyck (1994). Suppose that $\theta \in (\psi(\lambda), 1]$. We first examine the impossibility of multiple policy changes. To show this, we pay attention to the case of a binary choice between policy A (setting G^1 forever) and policy B (setting G^2 now). Now consider the following Bellman equation:

$$V(H) = \max\{v_1(H), HR(G^2) + \beta V(1 - [1 - \nu(L_A^*(G^2))](1 - H))\}.$$

Policy A is optimal for those values of H for the maximum on the right-hand side is attained at the first argument, that is,

$$v_1(H) > HR(G^2) + \beta V(1 - [1 - \nu(L_A^*(G^2))](1 - H)),$$

and policy B is optimal if the opposite inequality holds. We call the corresponding divisions of the range of H the policy A region and the policy B region, respectively. We will show that there exists a unique value $\hat{H} \in [0,1]$ such that $G(H) = G^2$ for $H \in [0, \hat{H})$ and $G(H) = G^1$ for $H \in (\hat{H}, 1]$, that is, it is impossible to have the case in which the regions could be any sequence of alternating intervals. And there is a clean division of the range into low and high values separated by a threshold, say $\hat{H}$, such that policy A is optimal for $H \in [\hat{H}, 1]$ and policy B is optimal for $H \in [0, \hat{H}]$, i.e., the value function of this problem is equivalent to the original value function; $V(H) = v(H)$ for all $H \in [0,1]$.

Denoting $V(H) - v_1(H)$ by $\Omega(H)$, we have that

$$\begin{aligned}\Omega(H) &= \max\{0,\ HR(G^2) + \beta V(H_2') - v_1(H)\},\\ &= \max\{0,\ [HR(G^2) + \beta v_1(H_2') - v_1(H)] + \beta\Omega(H_2')\ \}.\end{aligned}$$

where $H_2' = 1 - [1 - \nu(L_A^*(G^2))](1 - H)$. Note that

$$\begin{aligned} F(H) &\equiv HR(G^2) + \beta v_1(H_2') - v_1(H) \\ &= -\left[\frac{1-\beta[1-\nu(L_A^*(G^2))]}{1-\beta[1-\nu(L_A^*(G^1))]}R(G^1) - R(G^2)\right]H + \left[\frac{1-\beta[1-\nu(L_A^*(G^2))]}{1-\beta[1-\nu(L_A^*(G^1))]} - 1\right]R(G^1). \end{aligned}$$

The function F is just the difference between the value of waiting for exactly one period before changing the policy, and that of changing right away. Since $F' < 0$, F is strictly decreasing. By decreasing property of F, the solution function $\Omega(H)$ must be decreasing. To see this, note that the second argument of the max operator on the right-hand side consists of two parts. The first is decreasing in H, as we have already shown. The second is decreasing if $\Omega(H)$ is. Thus, starting with a decreasing function, the right-hand side yields another increasing function, Then, the fixed point of the iteration step is itself a decreasing function. Since the second argument is decreasing, there is a unique $\hat{H} \in [0,1]$ such that the second argument is positive if and only if $H \in [0,\hat{H})$. Thus, $G(H) = G^2$ if $H \in [0,\hat{H})$ and $G(H) = G^1$ if $H \in (\hat{H},1]$. ■

The threshold value $\hat{H}$ in Claim 19 must satisfy the condition that the value of setting action I forever equals the value of setting action II now and action I forever, that is,

$$v_1(\hat{H}) = v_{21}(\hat{H}). \tag{20}$$

Let $m_i = [1-\beta M(G^i)]/[1-\beta M(G^1)] = [1-\beta(1-\nu(L_A^*(G^i)))]/[1-\beta(1-\nu(L_A^*(G^1)))] > 1$ and $r_i = R(G^1)/R(G^i) > 1$ for each $i \in \{2,3\}$. By equations (18), (19) and (20), $\hat{H}$ satisfies:

$$\frac{R_1}{1-\beta} - (1-\hat{H})\frac{R_1}{1-\beta M_1} = \left[R_2 + R_1\frac{\beta}{1-\beta}\right] - (1-\hat{H})\left[R_2 + R_1\frac{\beta M_2}{1-\beta M_1}\right].$$

Solving this for $\hat{H}$, we can derive equation (14). And also, since $m_2 > 1$ and $r_2 > 1$, it must hold that $\hat{H} \in (0,1)$. From equation (14), it must hold that

$$\frac{\partial \hat{H}}{\partial r} = -\frac{m-1}{(rm-1)^2} < 0 \quad \text{and} \quad \frac{\partial \hat{H}}{\partial m} = \frac{r(r-1)}{(rm-1)^2} > 0.$$

Since $\partial L_A^*(G^2)/\partial\lambda < 0$ and $\partial R(G^2)/\partial\lambda = 0$, we get $\partial m/\partial\lambda < 0$ and $\partial r/\partial\lambda = 0$. Therefore, it must hold that

$$\frac{\partial \hat{H}(m,r)}{\partial\lambda} = \frac{\partial \hat{H}}{\partial m}\frac{\partial m}{\partial\lambda} + \frac{\partial \hat{H}}{\partial r}\frac{\partial r}{\partial\lambda} < 0,$$

which implies that $\hat{H}$ is decreasing in λ. ■

Proof of Proposition 2 Consider subcase B-I in which the rent-seeking issue is relatively significant so that $\theta \in (0,\varphi(\lambda))$, as shown in area B-I in Figure 4 and Figure 6 (the upper part). After examining the dynamic decision problem with a trinary case, the following claim is obtained.

Claim 20 *Suppose that the rent-seeking issue is relatively significant such that $\theta \in (0,\varphi(\lambda))$. Then, there exists a unique value $\tilde{H} \in (0,1)$ such that the adoption of promotion support with elaborate monitoring is optimal for $H \in (0,\tilde{H})$, and a laissez faire policy is optimal for $H \in$*

$(\tilde{H}, 1)$.

Proof of Claim 20 Suppose that $\theta \in (0, \varphi(\lambda))$. Note that

$$H_2^{0,1} = 1 - \frac{r_2 - 1}{r_2 m_2 - 1} \lesseqgtr 1 - \frac{r_3 - 1}{r_3 m_3 - 1} = H_3^{0,1}$$
$$\Leftrightarrow \ R_3 \gtreqless -\frac{R_1 - R_2}{\nu_2 - \nu_2}\nu_3 + \frac{R_1\nu_2 - R_2\nu_1}{\nu_2 - \nu_1} \quad \Leftrightarrow \quad \kappa_2 \equiv \frac{R_1\nu_2 - R_2\nu_1}{R_1 - R_2} \lesseqgtr \frac{R_1\nu_3 - R_3\nu_1}{R_1 - R_3} \equiv \kappa_3.$$

From Lemma 4, $\kappa_2 < \kappa_3$ if $\theta \in (0, \varphi(\lambda))$. Therefore, if $\theta \in (0, \varphi(\lambda))$, then it must hold that $H_2^{0,1} < H_3^{0,1}$. From Result 1, there exists a unique value $\tilde{H} \in (0, 1)$ such that $G(H) = G^3$ for $H \in (0, \tilde{H})$ and $G(H) = G^1$ for $H \in (\tilde{H}, 1)$. ■

Similarly as in case A, the threshold value $\tilde{H}$ in Claim 20 must satisfy the condition that the value of setting action I forever equals the value of setting action III now and action I forever, that is,

$$v_1(\tilde{H}) = v_{31}(\tilde{H}). \tag{21}$$

Equations (18), (19) and (21) yield equation (15):

$$H_3^{0,1} = \tilde{H} = \tilde{H}(\theta) = 1 - \frac{r_3 - 1}{r_3 m_3 - 1} \in (0, 1).$$

By equation (15), it must hold that

$$\frac{\partial \tilde{H}}{\partial r_3} = -\frac{m_3 - 1}{(r_3 m_3 - 1)^2} < 0 \quad \text{and} \quad \frac{\partial \tilde{H}}{\partial m_3} = \frac{r_3(r_3 - 1)}{(r_3 m_3 - 1)^2} > 0.$$

Note that θ affects both $L_A^*(G^3)$ and $R(G^3)$ when the government action is action III. Since $\partial L_A^*(G^3)/\partial\theta < 0$ and $\partial R(G^3)/\partial\theta < 0$, we have that $\partial m_3/\partial\theta < 0$ and $\partial r_3/\partial\theta > 0$. Therefore, we have that

$$\frac{\partial \tilde{H}(m_3, r_3)}{\partial \theta} = \frac{\partial \tilde{H}}{\partial m_3}\frac{\partial m_3}{\partial \theta} + \frac{\partial \tilde{H}}{\partial r_3}\frac{\partial r_3}{\partial \theta} < 0.$$

Thus, $\tilde{H}$ is decreasing in θ. ■

Proof of Lemma 5 We will first prove that $G(H) = G^1$ for any $H \in (\bar{H}, 1)$. By Lemma 4, if $\theta \in (\varphi(\lambda), \psi(\lambda))$, then $\kappa_2(\lambda) > \kappa_3(\theta)$. By the proof in Claim 20, if $\kappa_2(\lambda) > \kappa_3(\theta)$, then $H_2^{0,1} > H_3^{0,1}$. By Lemma 8, noting that $\bar{H} = H_2^{0,1} > H_3^{0,1}$, it must hold that $\mathbf{G}(H) = \{1^{(\infty)}\}$ for any $H \in (\bar{H}, 1)$, which is the desired result in (1).

Next we will show that there exists some $\varepsilon \in (0, \bar{H})$ such that $G(H) = G^2$ for any $H \in (\bar{H} - \varepsilon, \bar{H})$. From the proof of Lemma 8 and $\bar{H} = H_2^{0,1} > H_3^{0,1}$, it must hold that, for any $H \in (0, \bar{H})$, $\max\{V(H; \{3^{(1)}, 1^{(\infty)}\}), V(H; \{2^{(1)}, 1^{(\infty)}\})\} > V(H; \{1^{(\infty)}\})$. Note that $G(H) = G^1$ for any $H \in (\bar{H}, 1)$, and that $V(\cdot)$ is an affine function. Then, since $\bar{H} > H_3^{0,1}$, it is obvious that there exists some $\varepsilon \in (0, \bar{H})$ such that $V(H; \{3^{(1)}, 1^{(\infty)}\}) < V(H; \{2^{(1)}, 1^{(\infty)}\})$ or $G(H) = G^2$ for any $H \in (\bar{H} - \varepsilon, \bar{H})$ in (2).

Finally, we will show that there exists some $\xi \in (0, \bar{H})$ such that $G(H) = G^3$ for any

$H \in (0, \xi)$. Define

$$\hat{v}(H_t, G^i) = H_t R(G^i) + \beta v(H^i_{t+1}(H_t)),$$

where $H^i_{t+1}(H_t) = 1 - (1 - H_t)[1 - \nu(L^*_A(G^i))]$ for $i \in \{1, 2, 3\}$. It is enough to show that there exists some $\xi \in (0, \bar{H})$ such that

$$\hat{v}(H_t, G^3) > \max\{\hat{v}(H_t, G^1), \hat{v}(H_t, G^2)\},$$

for any $H_t \in (0, \xi)$. For $i \in \{1, 2\}$, define $E_i(H_t) \equiv v(H^3_{t+1}(H_t)) - v(H^i_{t+1}(H_t))$ on $H_t \in (0, \bar{H})$. Let

$$D_1 \equiv \inf_{H_t \in (0, \bar{H})} E_1(H_t).$$

Since $v(H^3_{t+1}(H_t)) > v(H^1_{t+1}(H_t))$ by $H^3_{t+1}(H_t) > H^1_{t+1}(H_t)$, and $H_t \in (0, \bar{H}) \subset (0, 1)$, it must hold that $D_1 > 0$. Let $\xi_1 \equiv \beta D_1/(R_1 - R_3) > 0$. Then, we have that

$$\begin{aligned}\hat{v}(H_t, G^3) - \hat{v}(H_t, G^1) &= \beta[v(H^3_{t+1}(H_t)) - v(H^1_{t+1}(H_t))] - H_t(R_1 - R_3) = \beta E_1(H_t) - H_t(R_1 - R_3)\\ &> \beta D_1 - H_t(R_1 - R_3) = (\xi_1 - H_t)(R_1 - R_3).\end{aligned}$$

Thus, it must hold that $\hat{v}(H_t, G^3) > \hat{v}(H_t, G^1)$ for any $H_t \in (0, \xi_1)$. Similarly, let $\xi_2 \equiv \beta D_2/(R_2 - R_3) > 0$, where

$$D_2 \equiv \inf_{H_t \in (0, \bar{H})} E_2(H_t).$$

Then, it must hold that $\hat{v}(H_t, G^3) > \hat{v}(H_t, G^2)$ for any $H_t \in (0, \xi_2)$. Setting $\xi \equiv \min\{\xi_1, \xi_2\} < \bar{H}$, it must hold that $\hat{v}(H_t, G^3) > \max\{\hat{v}(H_t, G^1), \hat{v}(H_t, G^2)\}$ for any $H_t \in (0, \xi)$. Thus, there exists some $\xi \in (0, \bar{H})$ such that $G(H) = G^3$ for any $H \in (0, \xi)$, which is the desired result in (3). ■

Proof of Lemma 6 Notice that $V(1, \{k^{(1)}, 3^{(n)}, 2^{(m)}, 1^{(\infty)}\}) = A_k$ since

$$V(H, \{k^{(1)}, 3^{(n)}, 2^{(m)}, 1^{(\infty)}\}) = A_k - (1 - H)B_k,$$

where

$$\begin{aligned}A_k &= R_k + \frac{\beta^{m+n+1}}{1-\beta} R_1 + \frac{1-\beta^m}{1-\beta} R_2 + \beta\frac{1-\beta^n}{1-\beta} R_3 > 0;\\ B_k &= R_k + \frac{\beta^{m+n+1} M_2^m M_3^n M_k}{1-\beta M_1} R_1 + \beta^{n+1} M_3^n M_k \frac{1-\beta^m M_2^m}{1-\beta M_2} R_2 + \beta M_k \frac{1-\beta^n M_3^n}{1-\beta M_3} R_3 > 0.\end{aligned}$$

Claim 21 $\{1^{(1)}, 3^{(n)}, 2^{(m)}, 1^{(\infty)}\} = \{1^{(1)}, l\}$ *is not optimal for* $H_0 \in (0, 1)$, *where* $l = \{3^{(n)}, 2^{(m)}, 1^{(\infty)}\}$ *with* $n \in \{1, 2, \ldots\}$ *and* $m \in \{1, 2, \ldots\}$.

Proof of Claim 21 Pick any n and m with, and pick any $H_0 \in (0, 1)$. Suppose, to get a contradiction, that $\{1^{(1)}, l\}$ is optimal for H_0, i.e., $\mathbf{G}(H_0) = \{1^{(1)}, l\}$. Then, it must hold that

$$V(H_0, \{1^{(1)}, l\}) \geq V(H_0, \{l\}).$$

Since $\{l\}$ is optimal for $H_1 = 1 - (1 - H_0)[1 - \nu(L_A^*(G^1))]$, it must hold that

$$V(H_1, \{l\}) \geq V(H_1, \{1^{(1)}, l\}).$$

Since V is an affine function, it must hold that

$$Y(1) \equiv \lim_{H \to 1-} V(H_1, \{l\}) - \lim_{H \to 1-} V(H_1, \{1^{(1)}, l\}) \geq 0.$$

However, it must hold that

$$Y(1) = -(1 - \beta^{n+m})R_1 + (1 - \beta^n)R_3 < -(1 - \beta^n)(R_1 - R_3) < 0,$$

which contradicts to the fact that $Y(1) \geq 0$. Thus, $\{1^{(1)}, 3^{(n)}, 2^{(m)}, 1^{(\infty)}\} = \{1^{(1)}, l\}$ is not optimal for $H_0 \in (0, 1)$. ■

Claim 22 $\{1^{(1)}, i^{(m)}, 1^{(\infty)}\} = \{1^{(1)}, l\}$ *is not optimal for* $H_0 \in (0, 1)$ *with* $m \in \{1, 2, \ldots\}$ *and* $i \in \{2, 3\}$.

Proof of Claim 22 Pick any m with, and pick any $H_0 \in (0, 1)$. Suppose, to get a contradiction, that $\{1^{(1)}, l\}$ is optimal for H_0, i.e., $\mathbf{G}(H_0) = \{1^{(1)}, l\}$. Then, it must hold that

$$V(H_0, \{1^{(1)}, l\}) \geq V(H_0, \{l\}).$$

Since $\{l\}$ is optimal for $H_1 = 1 - (1 - H_0)[1 - \nu(L_A^*(G^1))]$, it must hold that

$$V(H_1, \{l\}) \geq V(H_1, \{1^{(1)}, l\}).$$

Since V is an affine function, it must hold that

$$Y(1) \equiv \lim_{H \to 1-} V(H_1, \{l\}) - \lim_{H \to 1-} V(H_1, \{1^{(1)}, l\}) \geq 0.$$

However, it must hold that

$$Y(1) = -(1 - \beta^n)(R_1 - R_i) < 0,$$

which contradicts to the fact that $Y(1) \geq 0$. Thus, $\{1^{(1)}, i^{(m)}, 1^{(\infty)}\}$ is not optimal for $H_0 \in (0, 1)$ and $i \in \{2, 3\}$. ■

Claim 23 $\{2^{(1)}, 3^{(1)}, l, 1^{(\infty)}\}$ *is not optimal for* $H_0 \in (0, 1)$.

Proof of Claim 23 Suppose, to get a contradiction, that $\{2^{(1)}, 3^{(1)}, l, 1^{(\infty)}\}$ is optimal for H_0, i.e., $\mathbf{G}(H_0) = \{2^{(1)}, 3^{(1)}, l, 1^{(\infty)}\}$. Then it must hold that

$$\begin{aligned} v(H_0) &= H_0 R_2 + \beta[1 - (1 - H_0)M_2]R_3 + \beta^2 v(1 - (1 - H_0)M_2 M_3) \\ &> H_0 R_3 + \beta[1 - (1 - H_0)M_3]R_2 + \beta^2 v(1 - (1 - H_0)M_2 M_3). \end{aligned}$$

Thus, we have that

$$H_0 > 1 - \frac{(1-\beta)(R_2 - R_3)}{(1-\beta M_3)R_2 - (1-\beta M_2)R_3}.$$

Let $H_2 = 1 - (1 - H_0)M_2$. This yields

$$H_2 > 1 - \frac{(1-\beta)M_2(R_2 - R_3)}{(1-\beta M_3)R_2 - (1-\beta M_2)R_3} \equiv H'.$$

Since $G(H_2) = G^3$ by the assumption and $G(H) = G^1$ for any $H \in (\bar{H}, 1)$ by Lemma 5 (1), it must hold that

$$H' < H_2 < \bar{H}.$$

However, it contradicts to the assumption of $\bar{H} < H'$. Therefore, $\{2^{(1)}, 3^{(1)}, l, 1^{(\infty)}\}$ is not optimal for H_0. ■

We now show that the optimal sequence of government actions can be given as $\{3^{(n)}, 2^{(m)}, 1^{(\infty)}\}$ with $n \in \{0, 1, \ldots\}$ and $m \in \{0, 1, \ldots\}$. We first show that once G^1 is optimal, neither G^2 nor G^3 are optimal in any future period. Suppose, to get a contradiction, that $\{1^{(1)}, l, 1^{(\infty)}\}$ is optimal for some $H \in (0, 1)$, where l includes elements of 2 or/and 3. By Claim 22, l can be neither $3^{(m)}$ nor $2^{(m)}$. By Claim 23, G^3 never follows G^2. Thus, l can be represented as $\{3^{(m)}, 2^{(n)}\}$. However, by Claim 21, $\{1^{(1)}, l, 1^{(\infty)}\}$ cannot be optimal. This contradicts to the assumption. Therefore, the optimal sequence of government actions can be represented as $\{3^{(n)}, 2^{(m)}, 1^{(\infty)}\}$. ■

Proof of Proposition 3 By Lemma 5 (1) and $G(H) = G^3$, the optimal sequence of government actions is described as $\mathbf{G}(H) = \{3^{(1)}, l, 1^{(\infty)}\}$, where l is a finite sequence from $\mathcal{G}$. By Lemma 5 (2), l may include an element of 2. By Lemma 5 (3), there exists some $H \in (0, 1)$ such that $G(H) = G^3$. Thus, by Lemma 6, l can be written in the form of $l = \{3^{(n)}, 2^{(m)}\}$, where $n \in \{0, 1, \ldots\}$ and $m \in \{0, 1, \ldots\}$. If $m = 0$, then $\mathbf{G}(H) = \{3^{(n+1)}, 1^{(\infty)}\}$, i.e., exact one policy reform from G^3 to G^1 is adopted in some future period in the optimal sequence. If $m \neq 0$, then $\mathbf{G}(H) = \{3^{(n+1)}, 2^{(m)}, 1^{(\infty)}\}$, i.e., two policy reforms (the one from G^3 to G^2 and then the other from G^2 to G^1) are adopted in some future periods in the optimal sequence. ■

Proof of Corollary 2 From Propositions 1 and 3 and Lemma 5 (1), the desired result is obtained. ■

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Figure 1: Human Capital and Time

(Fixed Labor Inputs in Learning Industries: $L^1 > L^2$)

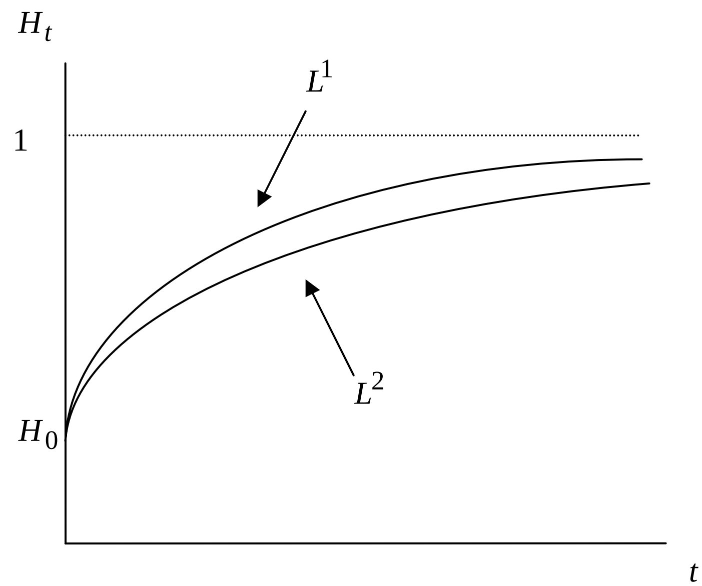
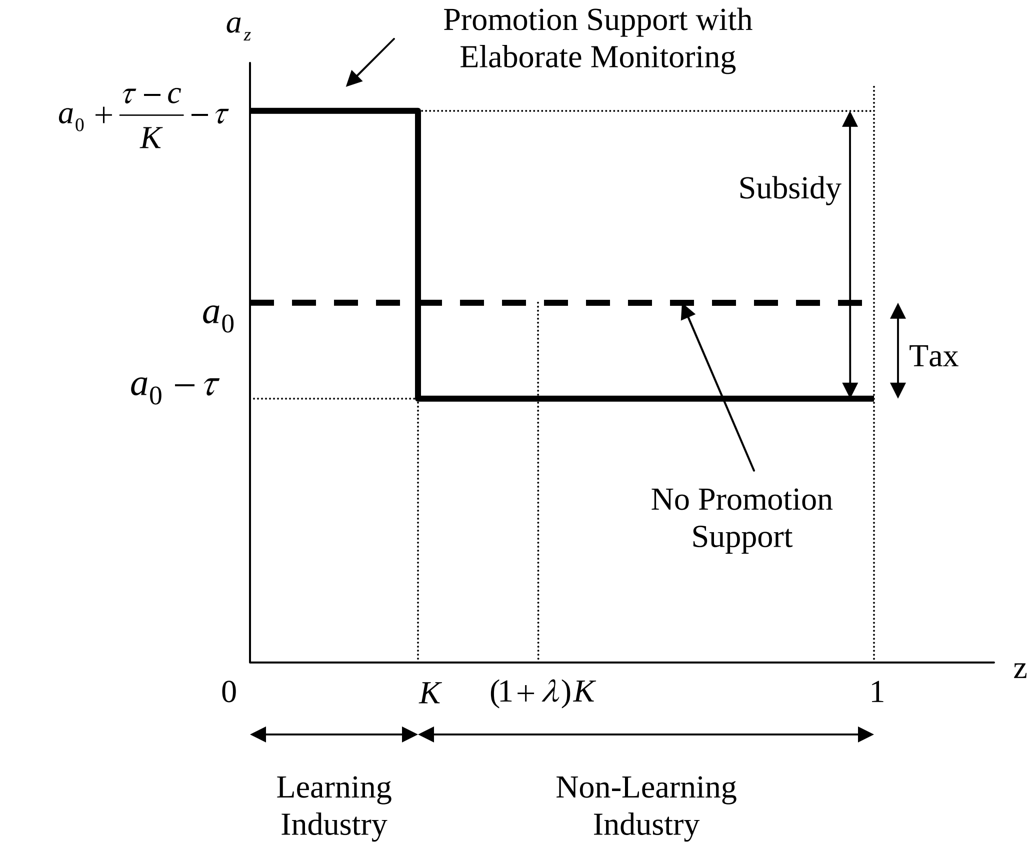


Figure 2: Government Action and Industry-Specific Factor



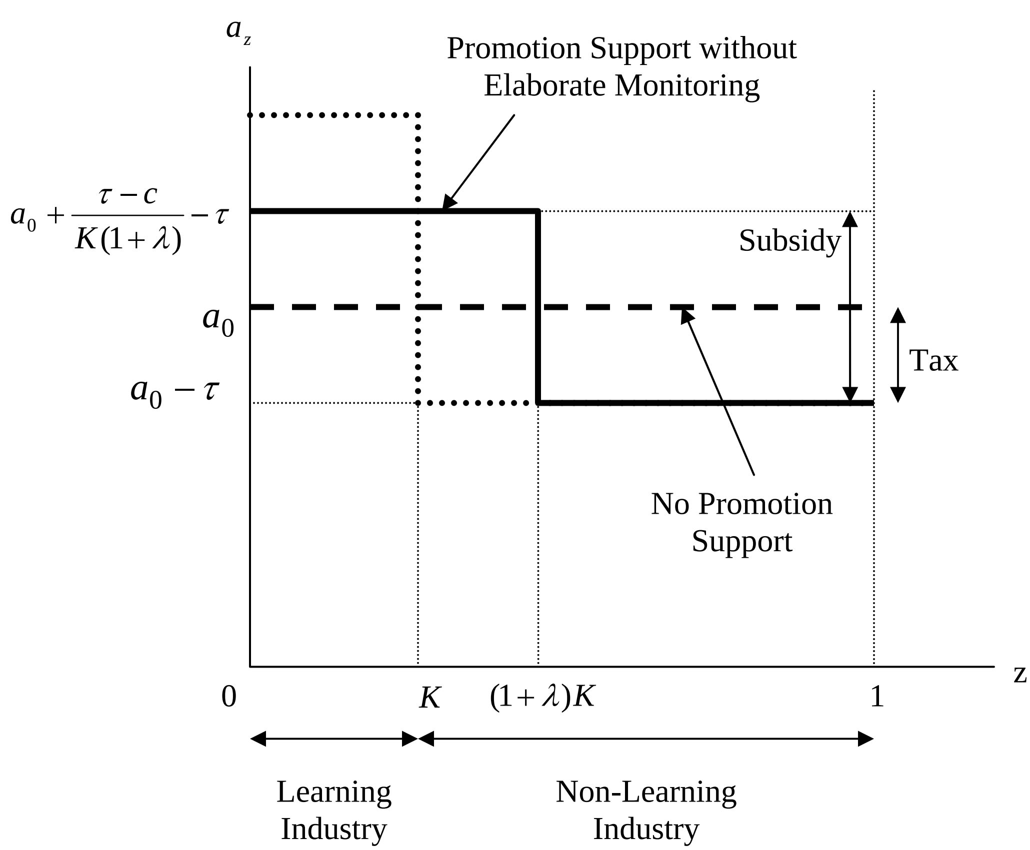


Figure 3: Government Action and Industry-Specific Productivity Index

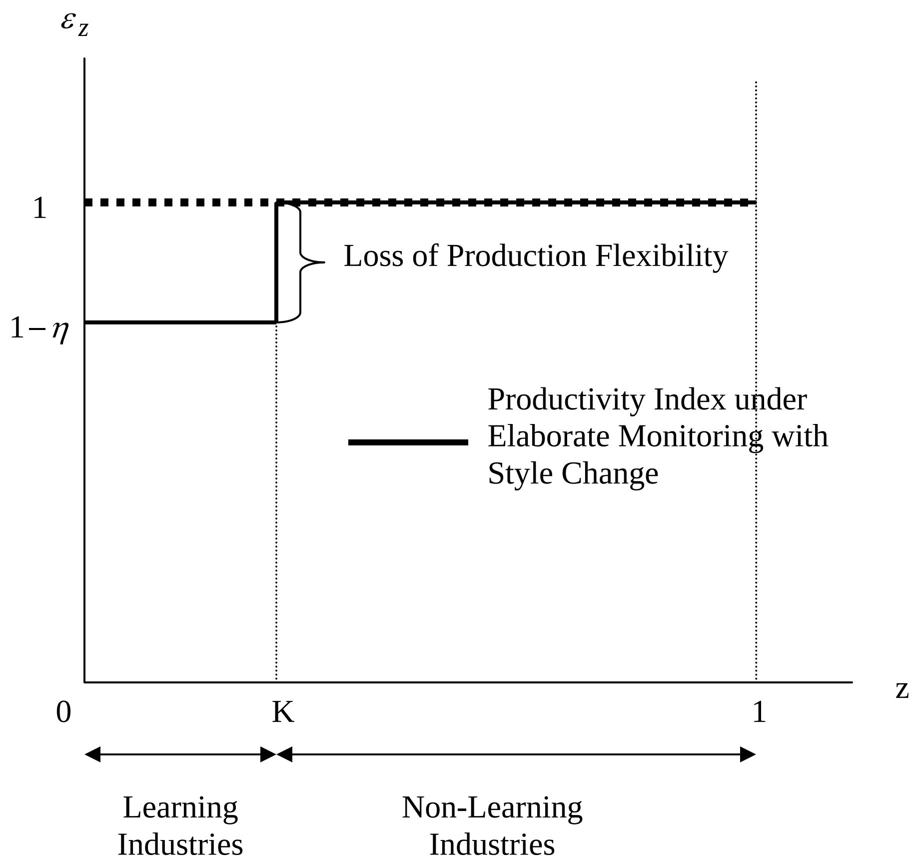


Figure 4: Critical Value under Promotion Support

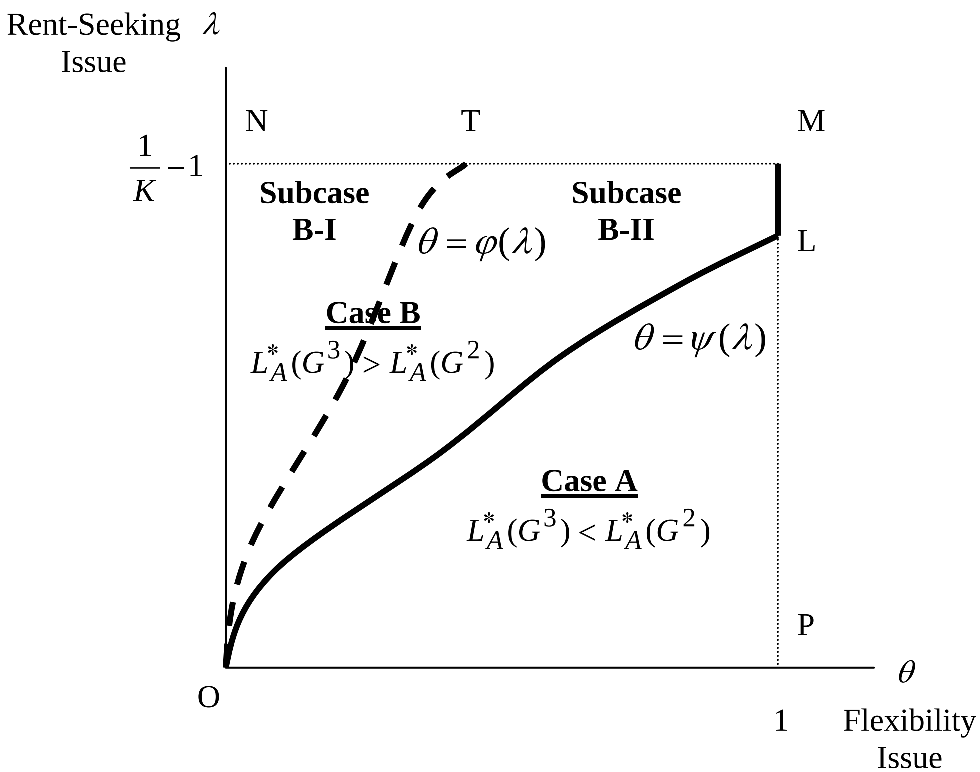
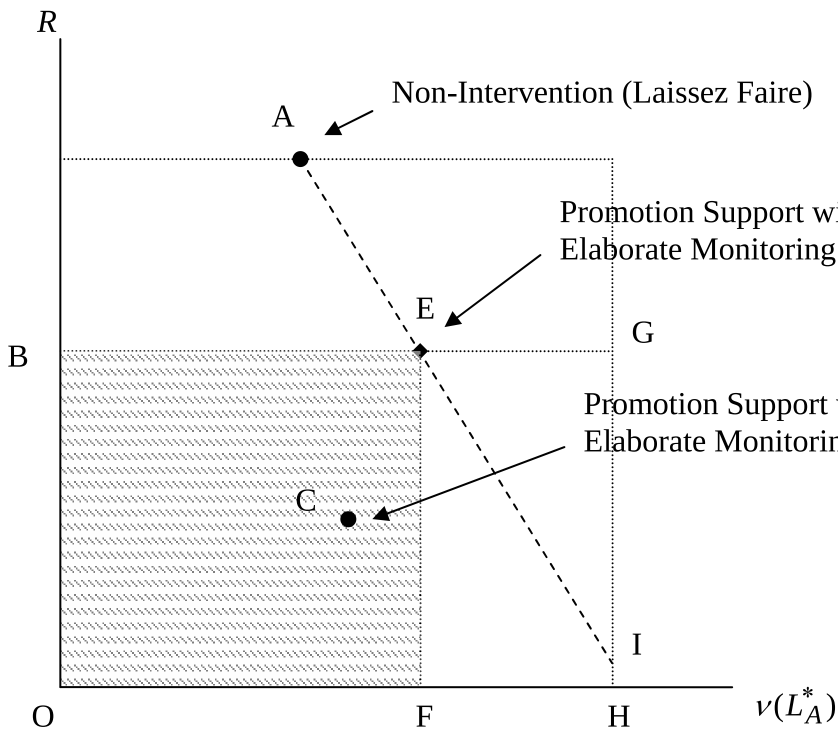


Figure 5

Case A: The Flexibility issue is relatively significant so that $\theta \in (\psi(\lambda),1]$.



<u>Optimal Action</u>

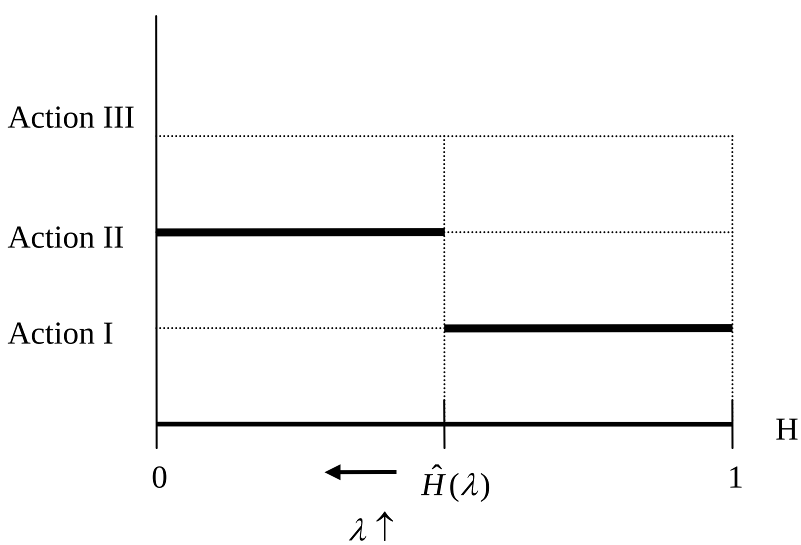
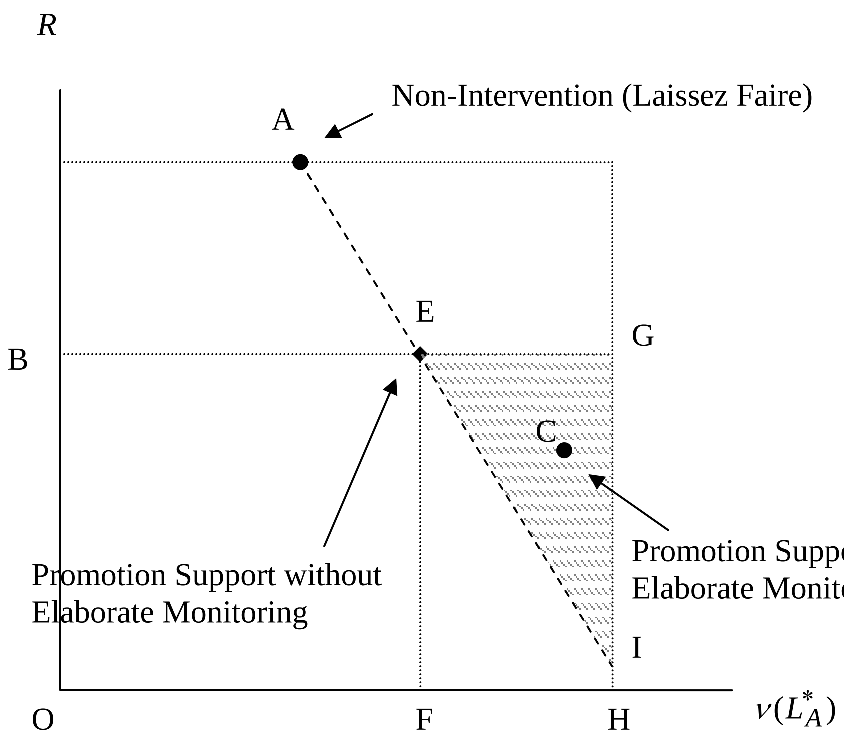


Figure 6

Subcase B-I : The rent-seeking issue is relatively significant so that $\theta \in (0, \varphi(\lambda))$.



Optimal Action

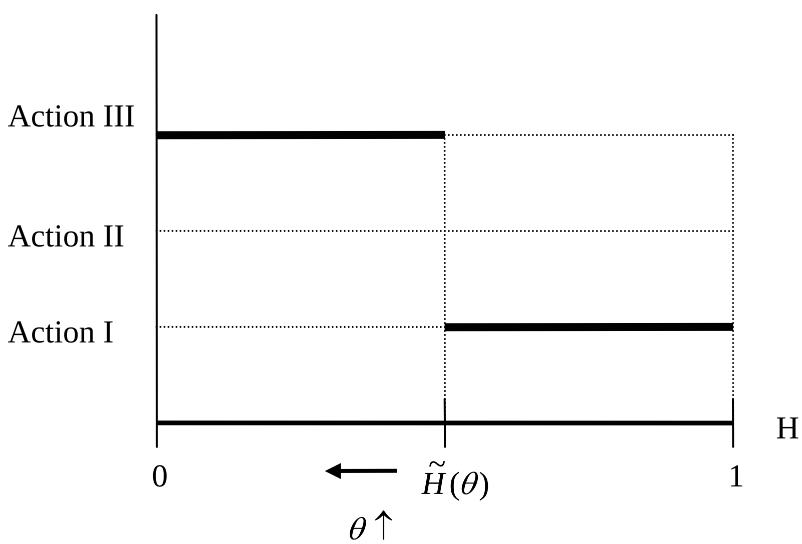


Figure 7

Subcase B-II : The Intermediate Case $\theta \in (\varphi(\lambda), \psi(\lambda))$