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Durable Goods Inventories and the Volatility of Production: Explaining the Less Volatile U.S. Economy

by

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Abstract

Despite the important role played by durable goods production and inventory investment in the business cycle, theoretical models featuring durable goods inventories are rarely available in the literature. This paper provides a simple dynamic optimization model of durable goods inventories and applies the model to analyzing the behavior of durable goods production and sales. It shows that small change in demand shocks can have large effect on the volatility of production relative to that of sales. The more durable is the good, the stronger the effect is. Calibrated exercise shows that the well documented dramatic reduction of output volatility in the U.S. economy since 1984 may be attributable to a decrease in the persistence of demand shocks. The analysis complements and reinforces the analysis of Ramey and Vine (2003).

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1 Introduction

Since 1984 the variance of U.S. output growth has decreased by four times compared to that over the post war period ending in 1983 (e.g., see McConnell and Perez-Quiros, 2000).[1] Finding out what caused such a dramatic decline in output fluctuations has important implications both for policy analysis and for business cycle research. On the empirical ground, a body of literature is been rapidly developed to scrutinize and reconfirm this stylized fact (e.g., see Blanchard and Simon 2001, Kahn, McConnell and Perez-Quiros 2001, Kim, Nelson, and Piger 2003, Stock and Watson 2002, and Ramey and Vine 2003). The consensus is that the durable goods sector appears to be far more important than the nondurable goods sector in contributing to the volatility reduction in GDP. On the theoretical ground, many interesting hypotheses are proposed to explain this structural change in the U.S. economy. Most prominently, Kahn, McConnel and Perez-Quiros (2001) argue that improvements in information technology and inventory management are the chief source of this volatility reduction. Key pieces of evidence in support of this argument are the sharp decline in the durable-goods inventory-to-sales ratio since 1984 and the corresponding sharp decline in the variance of production relative to the variance of sales. Ramey and Vine (2003) argue that the reduction in output fluctuations is due to a structural change in the nature of demand shocks to consumer durables, especially automobiles. In particular, they argue that a small decrease in the volatility of sales may lead to a large decrease in the volatility of production if there are nonconvexities in the cost function. Key empirical evidence in support of this explanation are the facts that 1) a key component of durable goods output – motor vehicles – does not show any sign of a declining inventory-to-sales ratio; in fact the inventory-to-sales ratio for motor vehicles has been remarkably stable for the entire post war period, which is inconsistent with the story provided by Kahn, McConnel and Perez-Quiros (2001); and 2) the magnitude of shocks to durable goods sales and the dynamic processes that propagate these shocks have changed since 1984 (in particular, automobile sales since 1984 have been much less persistent than those prior to 1984).

[1] Also see Kim and Nelson (1999).

Remay and Vine (2003) essentially argue that, although the observed reduction in the volatility of GDP is more prominent for production than for sales, this alone does not exclude the possibility that the structural change in the U.S. is demand driven, if a small change in demand can case a big change in production via a multiplier effect. The analytical approach taken by Ramey and Vine (2003), however, relies on a partial equilibrium linear-quadratic inventory model. A characteristic of this approach is that goods price in the model is exogenous, hence incapable of responding to demand and supply. For this reason, partial equilibrium models may distort the predicted relative volatilities of sales and production, since sales can also be affected by production and inventories via price changes. Another characteristic of this approach is that it is hard to differentiate nondurable goods from durable goods in the model, since the demand side is not endogenously modeled. The durability of goods is a user's measure, not a supplier's measure, hence it requires an explicit model of demand. Such a distinction is important since the empirical evidence shows that the volatility reduction is much stronger for durable goods than for nondurables and services.

This paper provides a general equilibrium model of durable goods inventories in which durable goods price is endogenously determined by demand and supply. I apply this model to studying the hypothesis of Ramey and Vine (2003) in a more general environment. I show that a small decrease in the persistence of demand shocks can lead to a large decrease in the volatility of production relative to the volatility of sales and that this effect is much stronger for durables goods than for nondurable goods, even in a perfectly competitive economy without nonconvexities in production costs. Consistent with Ramey and Vine (2003), calibration exercises show that the observed volatility reduction in durable goods output can be explained by a decrease in the persistence of demand shocks.

My model is related to the model of Kahn, McConnel and Perez-Quiros (2001), which also uses a general equilibrium approach. An important difference, however, is that I introduce durable goods inventories into the model via a stockout-avoidance motive, following Abel (1985) and Kahn (1987); whereas Kahn, McConnel and Perez-Quiros (2001) introduce durable goods inventories into their model by putting inventories in the utility function, which fails to make a distinction between durable consumption goods and inventory goods. Such a distinction, however, is important because inventories are not the same thing as purchased goods: the former affects the market transaction price from a supply side whereas the latter does so from a demand side.

The rest of this note is organized as follows. The model and its implications for production volatility are presented in section 2. Concluding remarks are presented in section 3.

2 The Model

Assume that the instantaneous utility function, $u(c)$, is strictly concave in the service provided by a stock of durable goods (c) and that the service flow is proportional to the stock of the goods. Also assume that production decision in period t must be made before demand in period t is known, so that firm has an incentive to accumulate inventories to avoid possible stockouts (see Able 1985 and Kahn 1987). A representative household chooses consumption demand for durable goods (taking price as given) to maximize life-time utilities, subject to the resource constraint that discounted life-time consumption must not exceed discounted life-time labor income plus initial wealth. To simplify the analysis, physical capital is left out of the story. Hence in equilibrium household wealth is simply the stock of inventories in the economy. A representative firm chooses production and inventory investment to maximize profits (taking market prices as given). To simplify the analysis I assume a constant returns to scale production function with labor as the only production factor, which implies a linear cost function for the firm.

Applying the welfare theorems, competitive equilibrium in this model can be derived by solving a social planner's problem in which a planner chooses sequences of production, $\{y_t\}_{t=0}^{\infty}$, purchase of durable consumption goods, $\{c_t - (1-\delta)c_{t-1}\}_{t=0}^{\infty}$, and inventory investment, $\{s_t - s_{t-1}\}_{t=0}^{\infty}$, to solve

$$\max_{\{y_t\}} E_{-1}\left\{\max_{\{c_t,s_t\}} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t,\theta_t\right)-ay_t\right]\right\}\right\}$$

subject to

$$[c_t - (1-\delta)c_{t-1}] + [s_t - s_{t-1}] \leq y_t \tag{1}$$

$$s_t \geq 0 \tag{2}$$

where the operator E_t denotes expectation based on information available in period t and θ represents shocks to preferences that generate urges to consume. Assume $u'_\theta > 0$ and $u''_{c\theta} > 0$, hence a positive shock to θ creates an urge to consume by increasing the marginal utility of consumption. The competitive market price for durable goods is the Lagrangian multiplier associated with the resource constraint (1). The rate of depreciation for durable goods is δ. For simplicity and without loss of generality, the depreciation rate for inventories is assumed to be zero. The cost of production, ay_t, is modeled as a disutility since labor is used to produce output. The linearity of the cost function is meant to keep the model tractable.

Denoting λ and π as the Lagrangian multipliers associated with the resource constraint (1) and the nonnegativity constraint on inventory (2) respectively, the first order conditions with respect to $\{y_t, c_t, s_t\}$ are given by:

$$a = E_{t-1}\lambda_t \tag{3}$$

$$u'(c_t, \theta_t) = \lambda_t - \beta(1-\delta)E_t\lambda_{t+1} \tag{4}$$

$$\lambda_t = \beta E_t \lambda_{t+1} + \pi_t \tag{5}$$

Utilizing (3), equations (4) and (5) can be simplified respectively to

$$u'(c_t, \theta_t) + \beta(1-\delta)a = \lambda_t \tag{6}$$

$$\lambda_t = \beta a + \pi_t. \tag{7}$$

According to (6), the shadow value (competitive price) of one unit of durable goods equals its marginal utility plus the market value of the nondepreciated units, $(1-\delta)$, measured by the production cost the agent gets avoid to pay in the next period, βa. According to (7), the value of one unit of inventory equals the discounted production cost the agent gets avoid to pay next period (βa), plus the shadow value of the slackness constraint (π), which is zero if the constraint does not bind. Combining (6) and (7), we have $u'(c, \theta) \geq \beta\delta a$, implying that the optimal stock of durable goods measured by its marginal utility is bounded below by the discounted user's cost of durable goods, $\beta\delta a$.[2]

[2]Thus, the nonnegativity constraint on inventories acts like a borrowing constraint on durable consumption goods in a competitive rental market.

To obtain closed-form decision rules, assume that the utility function is given by,

$$u(c,\theta) = \frac{(c-\theta)^{1-\alpha}}{1-\alpha}, \quad \alpha \geq 0.$$

Hence the marginal utility of consumption is given by $(c-\theta)^{-\alpha}$. To derive the decision rules of the model, consider two possibilities: the demand shock is below "normal" and the demand shock is above "normal".

Case A: If demand is below normal, then the nonnegativity constraint on inventories does not bind. Hence $\pi_t = 0$ and $s_t \geq 0$. Equation (7) implies that the shadow price of goods is constant[3],

$$\lambda_t = \beta a.$$

Hence equation (6) implies

$$(c_t - \theta_t)^{-\alpha} = \beta\delta a,$$

which gives the optimal consumption policy under case A,

$$c_t = \theta_t + (\beta\delta a)^{-\alpha}.$$

The resource constraint (1) then implies

$$s_t = y_t + s_{t-1} + (1-\delta)c_{t-1} - \theta_t - (\beta\delta a)^{-\alpha}.$$

The threshold preference shock is then determined by the constraint, $s_t \geq 0$, which implies

$$\theta_t \leq y_t + s_{t-1} + (1-\delta)c_{t-1} - (\beta\delta a)^{-\alpha}. \tag{8}$$

Case B: If demand is above normal, then the nonnegativity constraint on inventories binds. Hence $\pi_t > 0$ and $s_t = 0$. The resource constraint (1) implies that optimal consumption policy is given by

$$c_t = y_t + s_{t-1} + (1-\delta)c_{t-1}. \tag{9}$$

To determine the optimal production policy, we can utilize equation (3). Denote $f()$ as the probability density function of innovations in demand (ε) with support

[3] This implies that goods price is downward sticky in an inventory economy. See Blinder (1982), Amihud and Mendelson (1983) for more discussions on this issue.

$[A, B]$, then

$$\begin{aligned} a &= E_{t-1}\lambda_t \qquad (10) \\ &= \int_A^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^{B} \left[u'(c_t, \theta_t) + \beta(1-\delta)a\right] f(\varepsilon) d\varepsilon \end{aligned}$$

where the cutoff demand shock that determines the probability of stocking out, $z(y)$, is implied by (8). Assuming that preference shocks follow a stationary AR(1) process,

$$\theta_t = \gamma + \rho\theta_{t-1} + \varepsilon_t,$$

then (8) can be written as

$$\begin{aligned} \varepsilon_t &\leq y_t + s_{t-1} + (1-\delta)c_{t-1} - (\beta\delta a)^{-\alpha} - E_{t-1}\theta_t \\ &\equiv z(y_t). \end{aligned}$$

The interpretation of (10) is straightforward. The expected value of λ is a probability distribution of two terms: $\lambda = \beta a$ if the realized demand shock is small so that supply exceeds demand ($\pi = 0$); and $\lambda = u'(c, \theta) + \beta(1-\delta)a$ if the realized demand shock is large so that there is a stockout ($\pi > 0$). In the later case the optimal level of consumption is given by (9). More precisely, the left-hand side of (10) is the cost of producing one extra unit of goods today, a. The marginal benefit of having one extra unit of goods available tomorrow is given by the right-hand side of (10) with two possibilities. First, in the event of no stockout due to a low demand, the firm gets to save on the marginal cost of production by postponing production for one period. The present value of this term is βa. This event happens with probability $\int_A^{z(y)} f(\varepsilon) d\varepsilon$. Second, in the event of a stockout due to a high demand, the firm gets to sell the product (i.e., consumption takes place). The value of this term is the marginal utility of consumption plus the present market value of the nondepreciated units, $u'(c, \theta) + \beta(1-\delta)a$, where c is determined by (9). This event happens with probability $\int_{z(y)}^{B} f(\varepsilon) d\varepsilon$.

Clearly, the probability of stocking out, $\int_{z(y)}^{B} f(\varepsilon) d\varepsilon$, is determined by the level of production (y) committed one period in advance. The larger is y, the larger $z(y)$

is, hence the smaller the probability of stocking out is. Since $u'(c, \theta) > \beta\delta a$ in case of stocking out, (10) shows that an optimal cutoff point, $z(y) \in [A, B]$, exists and it is unique given the monotonicity of the marginal utility function, $u'(c)$. This cutoff point $z(y)$ is also the optimal target level of inventories determined by the firm, which depends on the probability distribution of demand shocks and other structural parameters in general, such as $\{a, \beta, \delta\}$.

Proposition 1 *The optimal inventory target (the cutoff demand) is a constant:*

$$z(y_t) = \sigma,$$

where σ depends positively on the variance of demand shocks.

Proof. Rewrite (10) as (utilizing equation 9):

$$\begin{aligned} a &= \int_A^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^B \left[(c_t - \theta_t)^{-\alpha} + \beta(1-\delta)a\right] f(\varepsilon) d\varepsilon \\ &= \int_A^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^B \left\{[(y_t + s_{t-1} + (1-\delta)c_{t-1}) - \theta_t]^{-\alpha} + \beta(1-\delta)a\right\} f(\varepsilon) d\varepsilon \\ &= \int_A^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^B \left\{\left[z(y_t) + (\beta\delta a)^{-1} - \varepsilon_t\right]^{-\alpha} + \beta(1-\delta)a\right\} f(\varepsilon) d\varepsilon, \end{aligned}$$

where the last equality utilized the definition of $z(y)$. This can be simplified to:

$$(1-\beta)a = \int_{z(y_t)}^B \left\{\left[z(y_t) + (\beta\delta a)^{-1} - \varepsilon_t\right]^{-\alpha} - \beta\delta a\right\} f(\varepsilon) d\varepsilon. \tag{11}$$

Clearly, the right-hand side of (11) is monotonically decreasing in z and it is an implicit function in the form, $g(z_t, \Omega) = 0$, where Ω is a set of parameters. Hence, the solution for $z(y)$ is unique and it must be a constant, σ, which solves $g(\sigma, \Omega) = 0$ or

$$(1-\beta)a = \int_{\sigma}^B \left\{\left[\sigma + (\beta\delta a)^{-1} - \varepsilon_t\right]^{-\alpha} - \beta\delta a\right\} f(\varepsilon) d\varepsilon. \tag{11$'$}$$

Now, consider an increase in the variance of ε that preserves the mean (i.e., an increase in B). (11$'$) indicates that σ must also increase in order to maintain the equality.■

Proposition 2 *The optimal decision rules for inventory holdings, durable goods sales and production are given respectively by*

$$s_t = \sigma - \min\{\sigma, \varepsilon_t\}$$

$$c_t - (1-\delta)c_{t-1} = [1-(1-\delta)L]\left(E_{t-1}\theta_t + (\beta\delta a)^{-\alpha} + \min\{\sigma, \varepsilon_t\}\right)$$

$$y_t = [1-(1-\delta)L]\left(E_{t-1}\theta_t + (\beta\delta a)^{-\alpha}\right) + \delta \min\{\sigma, \varepsilon_{t-1}\}$$

where L denotes the lag operator.

Proof. Utilizing the identity, $\theta_t = \varepsilon_t + E_{t-1}\theta_t$, and the identity, $\sigma = y_t + s_{t-1} + (1-\delta)c_{t-1} - (\beta\delta a)^{-\alpha} - E_{t-1}\theta_t$, case A and case B discussed above indicate that inventory holdings are given by the rule,

$$s_t = \begin{cases} \sigma - \varepsilon_t & \text{if } \varepsilon_t \le \sigma \\ 0 & \text{if } \varepsilon_t > \sigma \end{cases} = \max\{0, \sigma - \varepsilon_t\} = \sigma - \min\{\sigma, \varepsilon_t\},$$

and the optimal consumption stock is determined by the rule,

$$\begin{aligned} c_t &= \begin{cases} \theta_t + (\beta\delta a)^{-\alpha} & \text{if } \varepsilon_t \le \sigma \\ y_t + s_{t-1} + (1-\delta)c_{t-1} & \text{if } \varepsilon_t > \sigma \end{cases} \\ &= \begin{cases} E_{t-1}\theta_t + (\beta\delta a)^{-\alpha} + \varepsilon_t & \text{if } \varepsilon_t \le \sigma \\ E_{t-1}\theta_t + (\beta\delta a)^{-\alpha} + \sigma & \text{if } \varepsilon_t > \sigma \end{cases} \\ &= E_{t-1}\theta_t + (\beta\delta a)^{-\alpha} + \min\{\sigma, \varepsilon_t\}. \end{aligned}$$

The sales of durable consumption goods are thus determined by $(1-(1-\delta)L)\,c_t$. Furthermore, we have

$$y_t = \sigma + E_{t-1}\theta_t + (\beta\delta a)^{-\alpha} - s_{t-1} - (1-\delta)c_{t-1}.$$

Substituting out $s_{t-1}$and c_{t-1} in y_t following the decision rules for s_t and c_t and simplifying give the rule for production.■

Notice that when goods are nondurable ($\delta = 1$), the decision rules in proposition (2) become identical to those obtained by Kahn (1987) up to a constant, $(\beta\delta a)^{-\alpha}$. This shows that although Kahn's (1987) analysis is based on a partial equilibrium model, his result continues to hold in general equilibrium (for the case $\delta = 1$) where demand is endogenous and the equilibrium price (λ) can respond to demand and supply. The reason for this is that the competitive price is downward sticky in general equilibrium because firms opt to hold inventories rather than to decrease price when the marginal utility of consumption is low (i.e., $\lambda_t = \beta a$ when $\pi_t = 0$). Equilibrium price becomes variable (i.e., it goes up) only when demand (θ) is high enough (i.e., $\pi_t > 0$ in the event of a stockout). Hence, the simplifying assumption of an exogenously constant price in Kahn's (1987) partial equilibrium model has no severe consequence for implications of optimal production and inventory behavior. The same implication may carry over to the partial equilibrium model of Ramey and Vine (2003).

Proposition 3 *The variance of production decreases as the persistence of demand shocks falls.*

Proof. Denote $x_t \equiv E_{t-1}\theta_t + (\beta\delta a)^{-\alpha}$ and $v_t \equiv \min\{\sigma, \varepsilon_{t-1}\}$. Also denote $P \equiv \Pr[\varepsilon \leq \sigma]$. Note that the covariances, $cov(x_t, v_t) = P \times cov(x_t, \varepsilon_{t-1}) = P\rho\sigma_\varepsilon^2$ and $cov(x_{t-1}, v_t) = 0$. The decision rule for production can be rewritten as

$$y_t = x_t - (1-\delta)x_{t-1} + \delta v_t,$$

and the variance of production is then given by

$$\begin{aligned}
\sigma_y^2 &= \sigma_x^2 + (1-\delta)^2\sigma_x^2 - 2(1-\delta)cov(x_t, x_{t-1}) + \delta^2\sigma_v^2 + 2\delta cov(x_t, v_t) \\
&= \left[1 + (1-\delta)^2 - 2(1-\delta)\rho\right]\sigma_x^2 + \delta^2\sigma_v^2 + 2\delta P\rho\sigma_\varepsilon^2.
\end{aligned}$$

Since $\sigma_x^2 = \frac{\rho^2}{1-\rho^2}\sigma_\varepsilon^2$ and $\sigma_v^2 = P^2\sigma_\varepsilon^2$, we have

$$\sigma_y^2 = \left[1 + (1-\delta)^2 - 2(1-\delta)\rho\right]\frac{\rho^2}{1-\rho^2}\sigma_\varepsilon^2 + \delta^2 P^2\sigma_\varepsilon^2 + 2\delta P\rho\sigma_\varepsilon^2.$$

To show that $\frac{\partial\sigma_y^2}{\partial\rho} > 0$, we need only to show that the first term is increasing in ρ. Differentiating the first term with respect to ρ gives

$$-2(1-\delta)\frac{\rho^2}{1-\rho^2} + \left[1 + (1-\delta)^2 - 2(1-\delta)\rho\right]\frac{2\rho}{(1-\rho^2)^2},$$

which is positive if and only if

$$(1-\delta)\rho(1-\rho^2) < \left[1+(1-\delta)^2 - 2(1-\delta)\rho\right],$$

which can be simplified to

$$\delta^2 + a(1-\delta) > 0,$$

where $a \equiv (1-\rho)\left[2-\rho(1+\rho)\right]$. Since $a > 0$, the above inequality always holds for any value of $\delta \in [0,1]$.■

Proposition 4 *The relative volatility of production to sales decreases as the persistence of demand shocks falls (i.e., as ρ decreases). In particular, the variance ratio of production to sales can decrease from bigger than one to less than one if goods are durable. The more durable is the good, the easier it is for this structural change to take place as ρ decreases.*

Proof. Denote $x_t \equiv (1-(1-\delta)L)\left(E_{t-1}\theta_t + (\beta\delta a)^{-\alpha}\right)$ and $v_t \equiv \min\{\sigma, \varepsilon_{t-1}\}$. Denote durable goods sales by

$$\begin{aligned} q_t &\equiv c_t - (1-\delta)c_{t-1} \\ &= y_t + v_{t+1} - v_t. \end{aligned}$$

Denote $P \equiv \Pr[\varepsilon \le \sigma]$. Note that $cov(x_t, v_t) = P\rho\sigma_\varepsilon^2$. Since $y_t = x_t + \delta v_t$, we also have

$$cov(y_t, v_t) = cov(x_t, v_t) + \delta\sigma_v^2 = P\rho\sigma_\varepsilon^2 + \delta\sigma_v^2,$$

$$cov(y_t, v_{t+1}) = cov(y_{t-1}, v_t) = 0.$$

Hence, the variance of durable goods sales is given by

$$\begin{aligned} \sigma_q^2 &= \sigma_y^2 + 2\sigma_v^2 - 2cov(y_t, v_t) \\ &= \sigma_y^2 + 2(1-\delta)\sigma_v^2 - 2P\rho\sigma_\varepsilon^2. \end{aligned}$$

Since $\sigma_v^2 = P^2\sigma_\varepsilon^2$, we have

$$\sigma_y^2 - \sigma_q^2 = 2P\left[\rho + (\delta-1)P\right]\sigma_\varepsilon^2, \tag{12}$$

which increases with ρ, suggesting that the variability of production relative to that of sales decreases as the persistence of shocks falls. Furthermore, the variance of production can become less than the variance of sales (i.e., $\sigma_y^2 - \sigma_q^2 < 0$) if $\rho < (1-\delta)P$. Clearly, the inequality, $\rho < (1-\delta)P$, is easier to satisfy the smaller δ is. On the other hand, this inequality is impossible to satisfy when $\delta = 1$ under the restriction $\rho \geq 0$ (i.e., as long as demand shocks are positively serially correlated).$\blacksquare$

Equation (12) shows that when $\delta = 1$, we always have $\sigma_y^2 > \sigma_q^2$ as long as $\rho > 0$. Namely, production is more volatile than sales. This replicates the result of Kahn (1987). However, when goods are durable (e.g., $\delta = 0$), it becomes possible for the volatility of production to be less volatile than sales if the persistence of preference shocks (ρ) is low enough (e.g., $\rho < P$). This indicates that a decrease in the persistence of demand shocks could have a much stronger effect on the relative volatility of production to sales for durables than for nondurables.

It is generally known that production is more volatile than sales for both durable and nondurable goods (e.g., see Blinder 1986, Blinder and Maccini 1991, and Ramey and West 1999). But recently the literature has also shown that since 1984, while the absolute volatilities of production for both durables and nondurables have decreased, it is only the durable goods (e.g., motor vehicles) for which the volatility of production has decreased by so much so that the variance ratio of production to sales has become less than one (e.g., $\sigma_y^2/\sigma_q^2 = 0.6$; see Ramey and Vine, 2003). These empirical facts are consistent with the properties and implications of the model.

The intuition behind the model's implications is as follows. When there exists a motive to avoid possible stockouts due to production lags and demand uncertainty, firms opt to produce output according to both the expected future demand and a target level of inventories. On the consumer side, a positive preference shock implies high marginal utility of consumption in the current period and, as shocks are persistent, in the future as well, so the household will want to increase both current and future consumption. As plans for current production cannot be altered, any rise in current sales must be satisfied entirely by a reduction in inventories. On its own, this implies a one-for-one rise in the production committed for the next period to replenish the depleted inventory stock. If, in addition, consumption is nondurable (so sales equals consumption in each period), then the shock implies raised future sales.

In anticipation of this, the firm will further raise its planned production for next period. As a result, the variance of production exceeds that of sales, and inventory co-moves with production at business-cycle frequencies.[4] However, if consumption goods are durable, increased consumption in the current period raises the household's stock of consumption goods available for subsequent periods, reducing the anticipated increase in future sales, and hence the response in production. Thus, holding the persistence of demand shocks constant, durability mitigates the volatility of production. Consequently, when the persistence of demand shocks falls it has a stronger effect on durables than on nondurables with regard to the volatility of production relative to that of sales.

To get a quantitative sense of what this implies for the reduction of production volatility for goods that are highly durable, such as passenger cars, assume $\delta = 0.025$ (the average half life of cars is about 7 years or 28 quarters, which implies $\delta \approx 0.025$) and consider the change in the variance ratios of production to sales when ρ decreases from 0.85 to 0.3 (which is consistent with the empirical estimates suggested by Ramey and Vine 2003, table 7). Assuming that $P\,(\equiv \Pr[\varepsilon \leq \sigma]) \approx 0.5$, then the predicted variance ratios of production to sales are given by[5]

$$\frac{\sigma_y^2}{\sigma_q^2} = \left[1 - \frac{2P\left[\rho + (\delta - 1)P\right]}{\left[1 + (1-\delta)^2 - 2(1-\delta)\rho\right]\frac{\rho^2}{1-\rho^2} + \delta^2 P^2 + 2\delta P \rho}\right]^{-1} = \begin{cases} 1.9, & \text{if } \rho = 0.85 \\ 1.0, & \text{if } \rho = 0.5 \\ 0.4, & \text{if } \rho = 0.3 \end{cases}$$

Ramey and Vine (2003) report that the variance ratio of production to sales for motor vehicles has decreased from 2.1 to 0.6 since 1984. These empirical facts are consistent with the predictions of the simple general equilibrium model.

[4] See Wen (2002) for analysis on inventory movements at high and low business cycle frequencies.

[5] Notice that the variance of the innovations in demand (σ_ε^2) does not affect the volatility ratio of production to sales directly except indirectly through its effect on the parameter $P = \Pr[\varepsilon \leq \sigma]$, where the target inventory level (σ) positively depends on σ_ε^2. Since $\sigma > 0$, we have $P > 0.5$. However, as P increases, the variance ratio reduction is even more dramatic. For example, when $P = 0.7$, the variance ratio changes from 1.4 to 0.2 as ρ decreases from 0.9 to 0.3, a 7 fold reduction.

3 Concluding Remarks

Despite the important role of durable goods production and inventory investment played in the business cycle, theoretical models of durable goods inventories are rarely available in the literature. Thus many important empirical issues relating to inventories cannot be rigorously addressed. This note provides a simple dynamic general equilibrium model of durable goods inventories and applies the model to analyzing a prominent feature of the post war U.S. economy.

The fact that the U.S. economy has become less volatile since the early 1980's has sparked immense interests in searching for its causes. The empirical evidence strongly suggests that a volatility reduction in the durable goods sector since the early 1980's holds the key for the decline in GDP volatility. This structural change could be technology driven (as advocated by Kahn, McConnell and Perez-Quiros, 2001), or it could be demand driven (as advocated by Ramey and Vine, 2003). A crucial question, which each of these theories must answer, is why this structural change is more prominent for durable goods sector than for nondurable goods sector?

Using general equilibrium analysis, this paper shows that small changes in the demand shock process can lead to large changes in the volatility of production and inventory investment, and that this effect is especially strong on durable goods. In particular, it is shown that the dramatic decline in the volatility of durable goods production relative to the volatility of durable goods sales in the U.S. can be explained by a fall in the persistence of shocks to consumer preferences. The analysis complements and reinforces the analysis of Ramey and Vine (2003). If the proposed theory is correct, it implies that the observed decline in GDP volatility since 1984 may not become a permanent feature of the U.S. economy, as it depends on changes in the nature of exogenous shocks.

While these implications of the simple general equilibrium model are consistent with data, further work is clearly needed, especially to validate and to refine the definition of demand shocks. In the model, changes in demand are caused by shocks to preferences. Such shocks are not observable, hence cannot be directly measured. A natural next step in this line of research is to find a way to determine whether the changes in demand shocks are truly exogenous. It is possible, for example, that the assumed change in the demand shock process since 1984 reflect households' responses to a changing macro economic environment, such as changes in the government monetary policy or in the financial system that have eased credit availability or borrowing constraints (e.g., see Blanchard and Simon, 2001). Due to the extreme simplicity of the general equilibrium model, endogenous responses from demand to environment changes may have been captured instead in the model as exogenous preference shocks. This possibility is worth to be further explored.[6] But the general equilibrium framework provided in this paper may offer a natural vehicle for carrying out further analysis on lines like this.

[6] See Antinolfi and Wen (2003) for preliminary analysis along this line.

References

[1] Abel, A., 1985, Inventories, stock-outs and production smoothing, Review of Economic Studies LII, 283-293.

[2] Amihud, Y. and H. Mendelson, 1983, Price smoothing and inventory, Review of Economic Studies L, 87-98.

[3] Antinolfi, G. and Y. Wen, 2003, The Decline in the Volatility of GDP since 1984: A Financial Explanation. Working Paper (in progress).

[4] Blanchard, O. and J. Simon, 2001, The long and large decline in U.S. output volatility, Brookings Papers on Economic Activity, 2001:1, 135-164.

[5] Blinder, A., 1982, Inventories and sticky prices: More on the microfoundations of macroeconomics, American Economic Review 72, 334-348.

[6] Blinder, A., 1986, Can the production smoothing model of inventory behavior be saved? The Quarterly Journal of Economics CI (August), 431-453.

[7] Blinder, A. and L. Maccini, 1991, Taking stock: A critical assessment of recent research on inventories, Journal of Economic Perspectives 5 (Winter), 73-96.

[8] Eichenbaum, M., 1989, Some empirical evidence on the production level and production cost smoothing models of inventory investment, American Economic Review 79 (4), 853-64.

[9] Feldstein, M. and A. Auerbach, 1976, Inventory behavior in durable-goods manufacturing: The target-adjustment model, Brooking Papers on Economic Activity, vol. 1976, 351-396.

[10] Kahn, J., 1987, Inventories and the volatility of production, American Economic Review 77 (September), 667-679.

[11] Kahn, J., M. McConnell and G. Perez-Quiros, 2001, Inventories and the information revolution: Implications for output volatility, Mimeo, Federal Reserve Bank of New York.

[12] Kim, CJ. and C. Nelson, 1999, Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle, The Review of Economics and Statistics 81(4), 608-616.

[13] Kim, CJ., C. Nelson and J. Piger, 2003, The less volatile U.S. economy: A Bayesian investigation of timing, breadth, and potential explanations, Working Paper.

[14] McConnell, M. and G. Perez-Quiros, 2000, Output fluctuations in the United States: What has changed since the Early 1980's? American Economic Review 90 (5), 1464-76.

[15] Ramey, V., 1991, Nonconvex costs and the behavior of inventories, Journal of Political Economy 99 (no. 2), 306-334.

[16] Ramey, V. and D. Vine, 2003, Tracking the source of the decline in GDP volatility: An analysis of the automobile industry, Working Paper, UC San Diego.

[17] Ramey, V. and K. West, 1999, Inventories, chapter 13 in *Handbook of Macroeconomics*, Vol. 1B, North-Holland.

[18] Stock, J. and M. Watson, 2002, Has the business cycle changed and why? NBER Working Paper #9127.

[19] Wen, Y., 2002, Understanding the inventory cycle, Working Paper 02-04, Department of Economics, Cornell University.